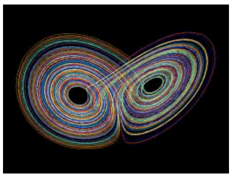
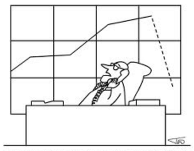
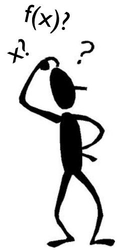




"BI tech support? The predictive analysis system is giving the wrong answer again—can you please fix it?... "

MAT 132

Differential Equations



Equations

For each equation find x (or (x,y) when appropriate) such that

$x^2 + x - 2 = 0$

$x^3 + x + 1 = 0$

$x \sin(y) + e^x = 0$

$x \sin(y) + e^x = 2y$

For each equation, find a function f such that

$\int f(x)\, dx = x^3$ or such that

$f(x)+x.f'(x)+x^2=0$

$x^2 + x$

Compare the following two problems

- Find a number x such that $x^2-3x+1=0$
  - In other words, find a ***number*** with certain properties.
- Find a function f such that f '(x)=3f(x) for all x in R.
  - In other words, find a ***function*** with certain properties.

A differential equation

- Find a function f such that f '(x)=3f(x) for all x in R.

Adding constrains

- Find a function f such that f '(x)=3f(x) for all x in R and f(1) = 3
- Find a function f such that f '(x)=3f(x) for all x in R and f(1) = 3 and f(0)=1.

Definition

- An (ordinary) *differential equation* is an equation in which the unknown is a function and where one or more of the derivatives of this function appears.
- Examples

$$y' = y^2 + 1 + \sin(x)$$

$$\frac{d^2 f}{dt^2} + 3t\frac{df}{dt} = t^5 f$$

Question:

- In the second equation, f is differentiated with respect to t.
- In the first question, what does y' mean?

$$y' = y^2 + 1 + \sin(x)$$
$$\frac{d^2 f}{dt^2} + 3t\frac{df}{dt} = t^5 f$$

Definition The *order* of a differential equation is the order of the highest derivative of the unknown function.

EXAMPLES:

$y' = y^2 + 1 + \sin(x)$, $order = 1$,
x is the independent variable
y=y(x) is the dependent variable

$\frac{d^2 f}{dt^2} + 3t\frac{df}{dt} = t^5 f$ $order = 2$,
t is the independent variable
f=f(t) is the dependent variable

Definition: The solution of a differential equation is a function f which satisfies the equation.

EXAMPLES OF SOLUTIONS

$y' = 3x^2 y, \quad y = Ce^{x^3}$, C is a real number

$\frac{d^2 f}{dt^2} = -f, \quad y = a\cos(t) + b\sin(t)$, a and b are real numbers

Given a differential equation with unknown function y, an ***initial condition*** is an equation of the form $y(t_0)=x_0$, where t_0 and x_0 are numbers

EXAMPLE

- Which of the following functions are solutions of the differential equation y''+y=sin(x)?

$a.\ y = \sin(x)$

$b.\ y = \cos(x)$

$c.\ y = x\sin(x)/2$

$d.\ y = -x\cos(x)/2$

EXAMPLE: Solve the following differential equations

$$y' = 3,\ y(1) = 4$$
$$y' = 3 + x,\ y(1) = 4$$
$$y' = y,\ y(0) = 2$$
$$y' = xy$$
$$xy' = 0,\ y(0) = 7$$
$$xy' = \frac{1}{x}y$$

Mathematical models

- The goal is not to produce an identical copy of the real object but give a representation of some aspect of the object.
- We can make a model by simplifying assumptions and combining aspects that may or may not belong together.
- Once the model is build, one should compare predictions of the model with data.

Modeling via differential equations

Quantities

- independent variable (almost always time in our course)
- dependent variable (function of the independent variable)
- parameters (quantities which do not change with time. They can be adjusted).

1. State assumptions. This assumptions should describe relationships between quantities.
2. Describe variables and parameters used in the model.
3. Use assumptions in 1. to derive equations involving variables and parameters in 2. Key words are for instance: "rate of change of..." or "reate of increase of...", "velocity", "acceleration", "proportional to"

An elementary model of population growth

Assumption: The number of individuals in a population grows at a rate proportional to the size of this population.

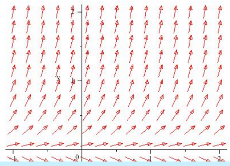
dP/dt = k P

- t
- P
- k
- Example of a differential equation.
- It is a first order (only first derivatives)
- It is an ordinary differential equation (no partial derivatives)

An elementary model of population growth

Assumption: The number of individuals in a population grows at a rate proportional to the size of this population.

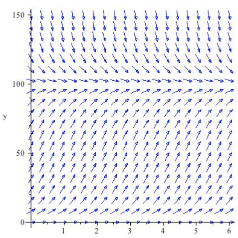
dP/dt = k P



Each arrow represents the slope of the tangent line of the solution passing through that point

Another model of population growth

Assumption: The number of individuals in a population grows at a rate proportional to the size of this population when the number of individuals is small, but decreases when surpasses a certain number.



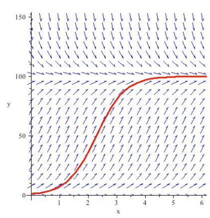
k=2 and M=100

dP/dt = k P(1-P/M)

This equation "fits" the above assumption but it is not the only equation with that property. (The assumption does not give the decreasing rate)

There are two constant solutions of this equations, P(t)=0 and P(t)=M for all t.

Equilibrium solutions



The curves represent solutions of the equation dP/dt = 2 P(1-P/100) What are the initial conditions for each of them?