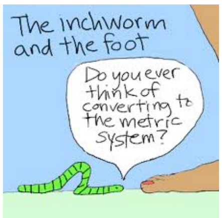
MAT 132

6.6 Work



When a force moves an object, we say *the force does work*. If the force F is constant, the work done is given by the equation W = F. d, where d is the distance moved.

work = force x distance

W (work) gives a measure of the "effort" of the force F in moving an object a certain distance.

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Example: A constant force of F pounds acts in the direction of motion of an object moving to the right along the x axis from point A to point B.

If B-A is the number of feet in the distance the object moves, and W is the number of foot-pound of work done by the force, then W = F (B-A)

- W gives a measure of the "effort" in moving an object from a point a to a point b.

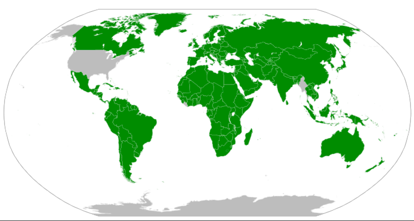
work = force x distance

Example: Find the work done by lifting a 70-lb weight to a height of 3 ft.

W = F (b-a)

Units

	IS Metric System	US Customary system
Time	seconds	seconds
Distance	meters	feet
Force	Newton	Pounds
Work	Joules	ft - lb



Countries which adopted the metric system (from Wikipedia)

Suppose that an object moves along the x-axis, in the positive direction, from x=a to x=b.

At each point x in [a,b], the force f(x) acts on the object. (Note that the force is not *necessarily* constant).

If $\Delta x = (b-a)/n$ and $x_i = a + i\Delta x$, the work done on the interval $[x_{i-1}, x_i]$ can be approximated by $W_i = f(x_i^*)\Delta x$, where x_i^* is in $[x_{i-1}, x_i]$

Thus, the work on [a,b], $W \approx \sum_{i=1}^{n} f(x_i^*)\Delta x$

If an object moves along the x axis from x=a to x=b, and the force at each point is f(x) then the work is

$$W=\int_a^b f(x)dx$$

EXAMPLE:

- A particle is moved along the x-axis by a force that measures $10/(1+x)^2$ pounds at a point x feet from the origin.
- Find the work done in moving the particle from the origin to a distance of 9 ft (in the positive direction).

EXAMPLE

- A spring can by compressed by 4 in. from its natural length of 12in. when a force of 6 lb. is applied. How much work is done in compressing the spring this distance?

Hooke's Law

- The force required to hold a spring stretched x meters beyond its natural length is f(x) = k x where k is a constant that depends on the spring.

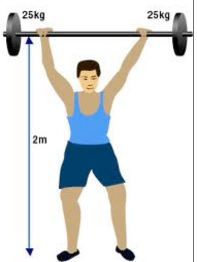
EXAMPLE

- A chain lying on the ground is 10m long and its mass is 80 kg. How much work is required to raise one end of the chain to a height of 6m?

Computing work with "big" objects.

- Find a "wise" location of the x-axis and the origin.
- Divide the object in n pieces. These pieces should be such that one can approximate the work for each of them, with the formula W=F.d.
- Add the work of each part. This yields a Riemann sum, and therefore, an integral.

Note that not all the problems in this section are direct application of the formula $W=\int_a^b f(x)dx$



EXAMPLE

- An aquarium 2m long, 1m wide and 1m deep is full of water. Find the work needed to pump half of the water out of the aquarium. (Use the fact that the density of water is $1000kg/m^3$).

Guidelines

- W = F. d . F is measured in pounds or kilograms. Many times, F is the weight
- If you are given the mass, you need to compute the weight by the formula F=ma, where a is the gravity ($9.8 m/s^2$)
- If you are given the volume and density, then you compute mass = density x volume, and with the mass, compute F.

EXAMPLE

- A 1600lb elevator is suspended by a 200-ft cable that weighs 10lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30ft?

EXAMPLE

- A spring has a natural length of 20cm. If a 25-N force is required to keep it stretched to a length of 30cm, how much work is required to stretch it from 20cm to 25cm.?

Newton's second Law of Motion

- Consider an object that moves in a straight line, with position function s(t) and mass m. Then the force F on the object (in the same direction).

$$F = m.a(t), \text{ where } a = \frac{d^2 s}{dt^2}$$

Find the volume of the solid obtained by revolting the
$x^2 + (y - 1)^2 = 1$ about the x-axis
What happens if we revolt the circle about the y-axis?