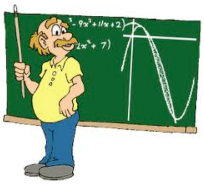
MAT 132

6.1 Areas between curves



Find the area of the region bounded by the curves

- $y=x^2$ and $y=-x^2+4$.
- $y^2=2x-2$ and $y=x-5$.
- The x-axis and the curve given by parametric equations $x=1+e^t$ and $y=t-t^2$

- Find intersection points. These points will determine the limits of integration .
- Sketch a figure.
- Compute the definite integral.

Sometimes you will need to "rotate" the figure π/2.

Find the area of the region bounded by the curves

$\frac{16}{3}\sqrt{2}$

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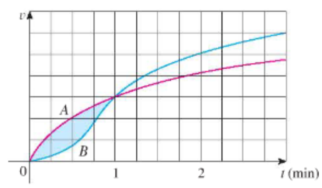
- The x-axis and the curve given by parametric equations $x=1+e^t$ and $y=t-t^2$ $3-e$

$-e^t(3-3t+t^2)$

Suppose that f and g are two continuous functions and that for all x in [a,b], f(x)≤g(x).

The area bounded by the curves y=f(x) and y=g(x) is $\int_a^b f(x)\,dx$

26. Two cars, A and B, start side by side and accelerate from rest. The figure shows the graphs of their velocity functions.
(a) Which car is ahead after one minute? Explain.
(b) What is the meaning of the area of the shaded region?
(c) Which car is ahead after two minutes? Explain.
(d) Estimate the time at which the cars are again side by side.



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Sometimes you will need to "rotate" the figure π/2 (considering x as a function of y)

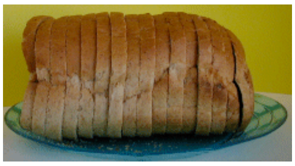
29. If the birth rate of a population is $b(t) = 2200e^{0.024t}$ people per year and the death rate is $d(t) = 1460e^{0.018t}$ people per year, find the area between these curves for $0 \leqslant t \leqslant 10$. What does this area represent?

33. Use the parametric equations of an ellipse, $x = a\cos\theta$, $y = b\sin\theta$, $0 \leqslant \theta \leqslant 2\pi$, to find the area that it encloses.

Compare the result of 33 with area of a circle.

6.2 Volumes

- To estimate the volume of the loaf of bread, we slice it, find the volume of each slice and add up all those volumes.
- The volume of each slices is approximately, the area of the slice multiplied by the height (thickness).



height 

What can be do to get a better estimation?

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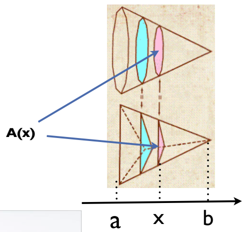
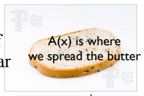


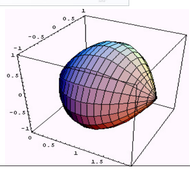
We want to compute the volume of a solid that lies between $x=a$ and $x=b$.

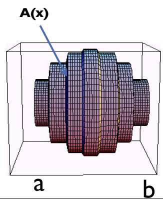
Denote the cross-sectional area of the solid in the plane perpendicular to the x-axis by $A(x)$,



If A is a continuous function, then the **volume** of S is

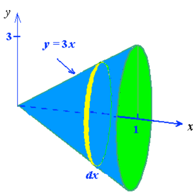
$$\int_a^b A(x)\,dx$$





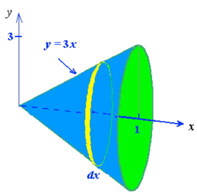
Make sure you understand in which direction you slice.
This direction is perpendicular to "x"

Find the volume of the cone below.



- A right piramide 30 ft. high has a square base measuring 40 ft. on a side. Find its volume.

Find the volume of the cone below.



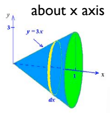
Answer 3π

16,000 ft^3

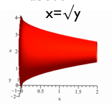
- Rotate y=f(x), x in [a,b] about a horizontal line.
  - (Ex 1) Rotate y=f(x) about the x axis
  - Note: The x-axis is the line y=0.
  - Rotate y=f(x) about y=A.

$\pi\left(\int_a^b Radius(x)^2\,dx\right)$

about x axis



about x =2
x=√y



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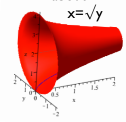
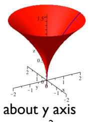
x=√y



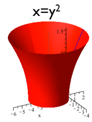
- Rotate x=g(y), y in [c,d] about a vertical line.
  - Rotate the curve x=g(y), y in [c,d] about the y axis
  - Note: The y-axis is the line x=0.
  - Rotate x=g(y) about x=B.

$\pi\left(\int_c^d Radius(y)^2\,dy\right)$



about y axis
$x=y^2$

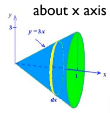
about x=-3
$x=y^2$



- Rotate y=f(x), x in [a,b] about a horizontal line.
  - (Ex 1) Rotate y=f(x) about the x axis
  - Note: The x-axis is the line y=0.
  - Rotate y=f(x) about y=A.

$\pi\left(\int_a^b Radius(x)^2\,dx\right)$

about x axis



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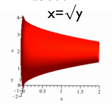

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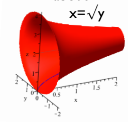

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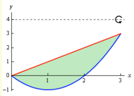
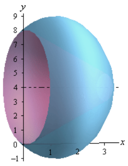
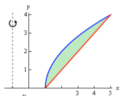
$\pi\left(\int_a^b Radius(x)^2\,dx\right)$

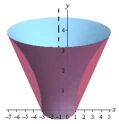




- Rotate x=g(y), y in [c,d] about a vertical line.
  - Rotate the curve x=g(y), y in [c,d] about the y axis
  - Note: The y-axis is the line x=0.
  - Rotate x=g(y) about x=B.

$\pi\left(\int_c^d Radius(y)^2\,dy\right)$





- **(Slicing) If A(x) denotes the cross-sectional area (in the plane perpendicular to x) of a solid, the volume of the solid is** $\pi\left(\int_a^b A(x)\,dx\right)$
- Solid of revolution (cross section a disk)
  - (Ex 1)Rotate the curve y=f(x), x in [a,b] about the x axis $\pi\left(\int_a^b f(x)^2\,dx\right)$
  - Rotate the curve x=g(y), y in [c,d] about the y axis $\pi\left(\int_c^d g(y)^2\,dy\right)$
- Solid of revolution (cross section a washer) $\pi\left(\int_a^b \left(outerR(x)^2 - innerR(x)^2\right)dx\right)$
-

1. Find the area of the solid of revolution generated if the region bounded by one arch of the curve y = sin(x) is revolved about the x axis. $\frac{1}{2}\pi^2$
2. Find the volume of the solid obtained by rotating the function $y = x^3$, x in [0,2] about the y axis. $\frac{3}{5}\pi\,2^{5/9}$
3. The region bounded by $y=x^2$ and $x=y^2$ is rotated about the x-axis. Find the volume of the solid generated. $\frac{3}{10}\pi$
4. The region bounded by the parabola $x=y^2$ is rotated about the line x=2. Find the volume of the resulting solid of revolution. $\frac{64}{15}\pi\sqrt{2}$
5. Find the volume of a solid with circular base of radius 1 and such that parallel cross sections perpendicular to the base are square. $A(x) = 4 - 4x^2$ Correct!