The Logistic Equation

t = time
P (or $P(t)$) = the population size at time t.

For small populations, the rate of growth is proportional to its size. That is $dP/dt \approx k P$, for some constant k

k=the *relative growth* coefficient.

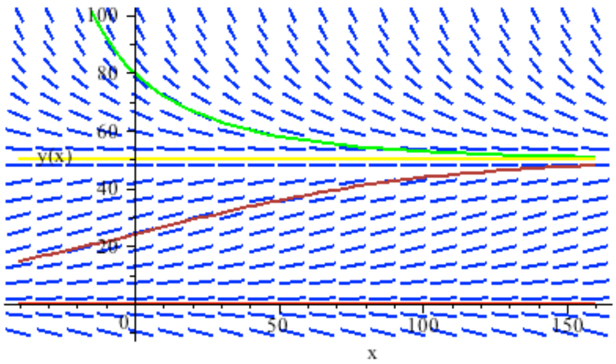
If the population is too large , the population decreases. In that case, the rate of growth (dP/dt) is negative.

M = *carrying capacity*, the amount that when exceeded will result in the population decreasing.

$dP/dt = k P (1- P/M)$
logistic differential equation

Solve the logistic differential equation in general (for all values of k and M).

Suppose a species of fish in lake is modeled by a logistic population model with relative growth rate of k = 0.02 per year and carrying capacity of K = 50.
a.Write the differential equation describing the logistic population model for this problem.
b.Draw a vector field for this problem.
c.Determine the equilibrium solutions for this model.
d.If 25 fishes are introduced in the lake, estimate the time it will take to have 8000 fish in the lake.



9. One model for the spread of a rumor is that the rate of spread is proportional to the product of the fraction y of the population who have heard the rumor and the fraction who have not heard the rumor.
(a) Write a differential equation that is satisfied by y.
(b) Solve the differential equation.
(c) A small town has 1000 inhabitants. At 8 AM, 80 people have heard a rumor. By noon half the town has heard it. At what time will 90% of the population have heard the rumor?

6. The table gives the number of yeast cells in a new laboratory culture.

Time (hours)	Yeast cells	Time (hours)	Yeast cells
0	18	10	509
2	39	12	597
4	80	14	640
6	171	16	664
8	336	18	672

(a) Plot the data and use the plot to estimate the carrying capacity for the yeast population.
(b) Use the data to estimate the initial relative growth rate.
(c) Find both an exponential model and a logistic model for these data.
(d) Compare the predicted values with the observed values, both in a table and with graphs. Comment on how well your models fit the data.
(e) Use your logistic model to estimate the number of yeast cells after 7 hours.