

$$\int \cos^5(3x) dx = \int \cos^4(3x) \cdot \cos(3x) dx$$

$$= \int [\cos^2(3x)]^2 \cdot \cos(3x) dx = \int [1 - \sin^2(3x)]^2 \cos(3x) dx$$

u-substitution

$$u = \sin(3x)$$

$$du = 3 \cos(3x) dx$$

$$\frac{1}{3} du = \cos(3x) dx$$

$$= \int [1 - u^2]^2 \frac{1}{3} du = \frac{1}{3} \int 1 - 2u^2 + u^4 du = \frac{1}{3} \left[u - \frac{2u^3}{3} + \frac{u^5}{5} \right] + C$$

$$= \frac{\sin(3x)}{3} - \frac{2 \sin^3(3x)}{9} + \frac{\sin^5(3x)}{15} + C$$