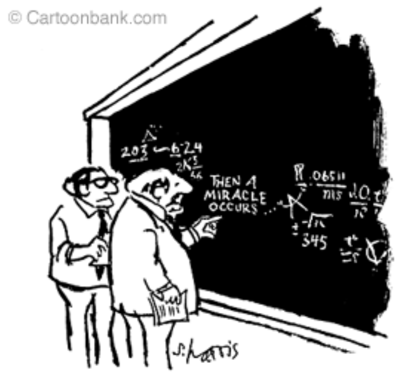
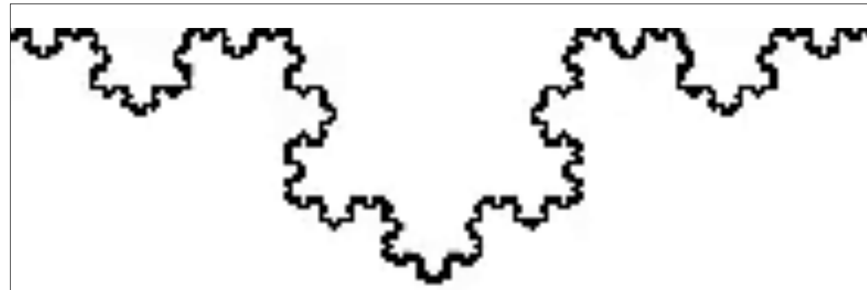
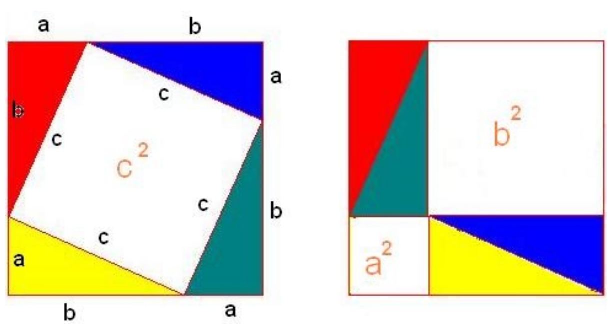
MAT 132



"I think you should be more explicit here in step two."



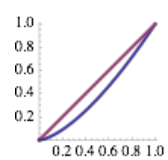
Pythagorean Theorem (and one of its beautiful proofs)

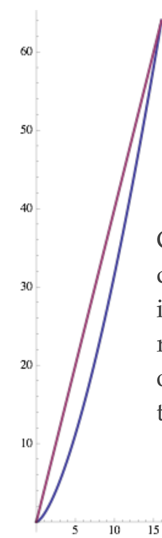


$$c^2 = a^2 + b^2$$

Consider the curve $f(x)=x^{3/2}$, x in [0,1]. Give a rough estimation of the length of the graph of f.

Estimation $\sqrt{2} \sim 1.14$
Length 1.11145…





Consider the curve $f(x)=x^{3/2}$, x in [0,16]. Give a rough estimation of the length of the graph of f.

Estimation 16 $.17^{1/2}$… ~65.9697
Length 411.033

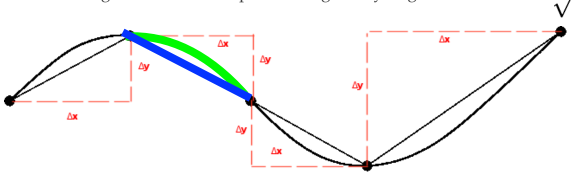
Arc length

We have a curve given by the equation

y=g(x)

We can approximate the length of the blue arc, by the length of the green segment.

The length of the free segment can be computed using the Pythagorean theorem. $\sqrt{(\Delta x)^2+(\Delta y)^2}$



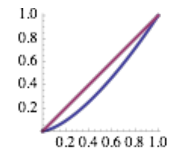
We repeat this for every arc, and we obtain an approximation of the length of the whole curve. $\sum_{i=1}^{n}\sqrt{(\Delta x_i)^2+(\Delta y_i)^2}$

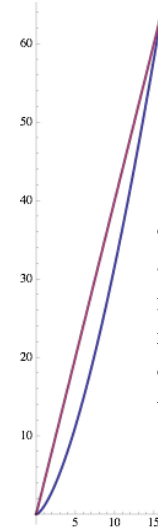
If the function g and its derivative are continuous, when the lengths Δx of each interval goes to zero, the above sum converges to $\int_a^b \sqrt{(1+(g'(x))^2}\ dx$

where a and b are the extremes of the interval where x "lives"

Consider the curve $f(x)=x^{3/2}$, x in [0,1]. Give a rough estimation of the length of the graph of f.

Estimation √2 ~ 1.14
Length 1.11145…



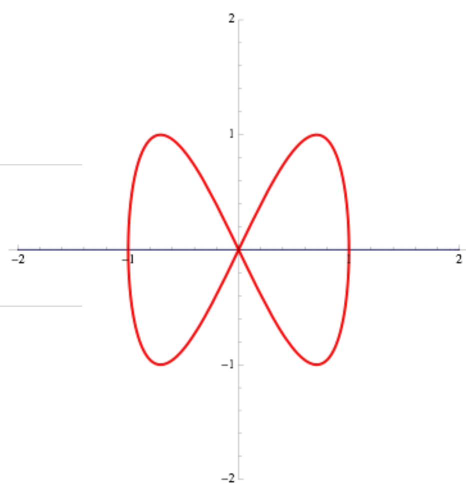


Consider the curve $f(x)=x^{3/2}$, x in [0,16]. Give a rough estimation of the length of the graph of f.

Estimation 16 .17$^{1/2}$… ~65.9697
Length 411.033

EXAMPLE

Sketch the curve
x=sin(t) ,
y= sin(2t) , t in [0,2π]



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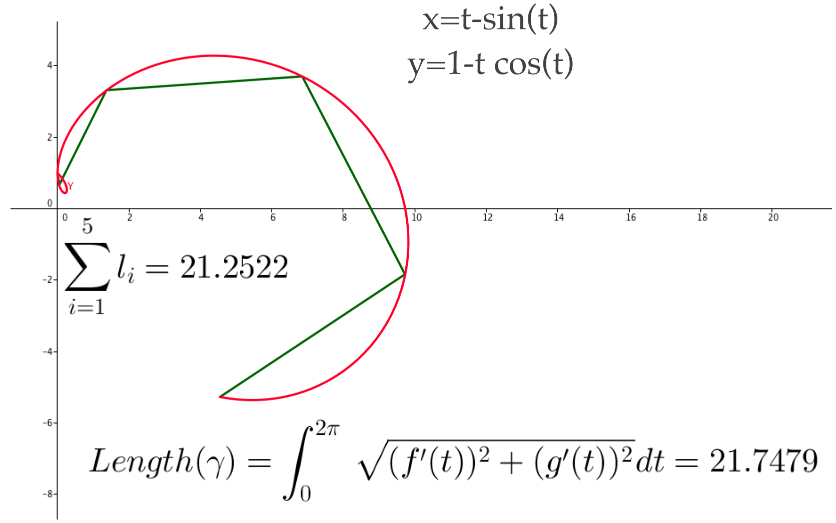
x=t-sin(t)
y=1-t cos(t)



$$\sum_{i=1}^{5} l_i = 21.2522$$

$$Length(\gamma)=\int_0^{2\pi}\sqrt{(f'(t))^2+(g'(t))^2}dt=21.7479$$

Arc length

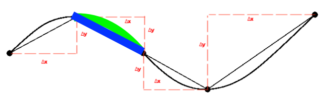
We have a curve given by parametric equations

x=f(t) , y=g(t)

We can approximate the length of the blue arc, by the length of the green segment.

The length of the free segment can be computed using the Pythagorean theorem.

$$\sqrt{(\Delta x)^2 + (\Delta y)^2}$$



We repeat this for every arc, and we obtain an approximation of the length of the whole curve.

$$\sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$$

$$\Delta x = f(t_{i+1}) - f(t_i) \approx f'(\hat{t}_i)\Delta t$$

$$\Delta y = g(t_{i+1}) - g(t_i) \approx g'(\hat{t}_i)\Delta t$$

If the function g and its derivative are continuous, when the lengths Δx of each interval goes to zero, the above sum converges to

$$\int_c^d \sqrt{((f'(t))^2 + (g'(t))^2} \; dt$$

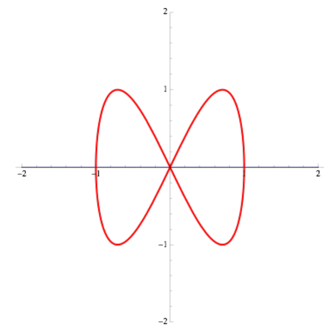
where a and b are the extremes of the interval where x "lives"

EXAMPLE

Set up (but do not evaluate) the length of the curve defined by parametric equations

x=sin(t) ,

y= sin(2t) , t in [0,25]

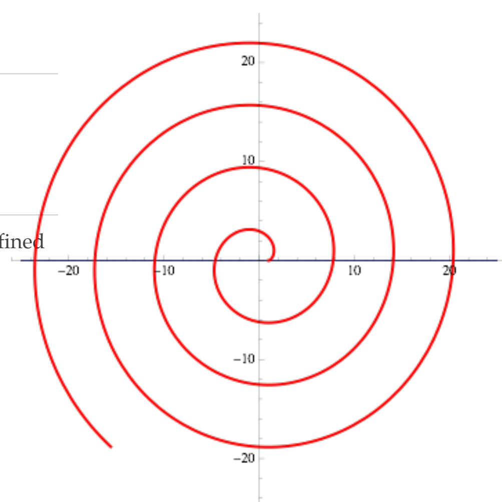


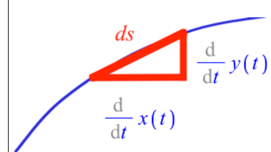
EXAMPLE

Compute the length of the curve defined by parametric equations

x=cos(t) + t.sin(t) ,

y= sin(t) - t.cos(t) , t in [0,25]

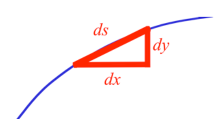




Curve defined by parametric equations (x(t),y(t)) t in [a,b].
The length of the curve is

$$\int_a^b \sqrt{\left(\frac{d}{dt}x(t)\right)^2 + \left(\frac{d}{dt}y(t)\right)^2}\, dt$$

(if the curve is traversed only once)

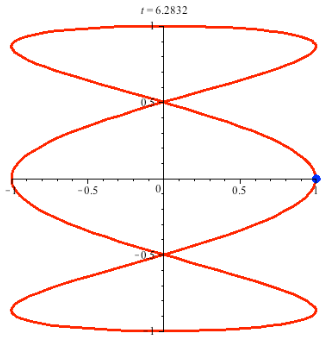


Curve is a the graph of a function y=f(x) x in [a,b]
The length of the graph of f is

$$\int_a^b \sqrt{\left(\frac{d}{dx}f(x)\right)^2 + 1}\, dx$$

Note: the formula for the graph is a special case of the formula for parametric equations

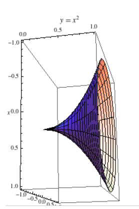
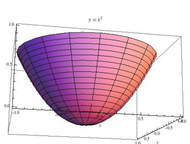
Example: Estimate the length of the curve given by the function F(t)=(cos(3t), sin(t)),t in [0,2π] using a partition of [0, 2π] into 4 intervals. What intervals would you choose to get a better estimation?

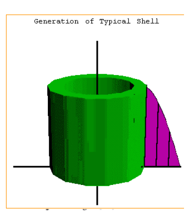


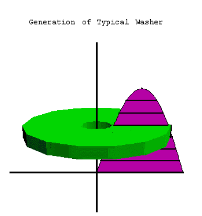
Peano curve animation from Wikipedia

Review of Volumes

Generate solids rotating a curve about vertical or horizontal axes. The obtained solids are called solids of revolution.





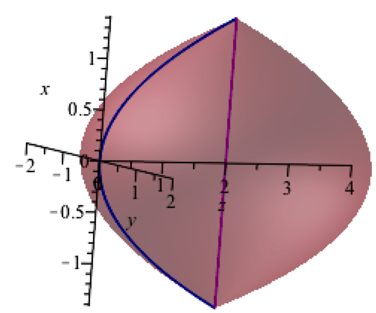
$$V = \int_a^b 2\pi x f(x)\, dx$$

$$\pi \int_a^b (g(y)^2 - f(y)^2) dy$$

Find a volume of solids of revolution by the washer method or the cylindrical shell method.

Example

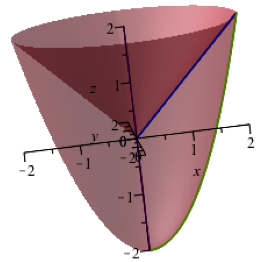
The closed region bounded by the graph of $x=y^2$ and the vertical line x=2 is revolved about the line x=2. Calculate the volume of the solid of revolution.



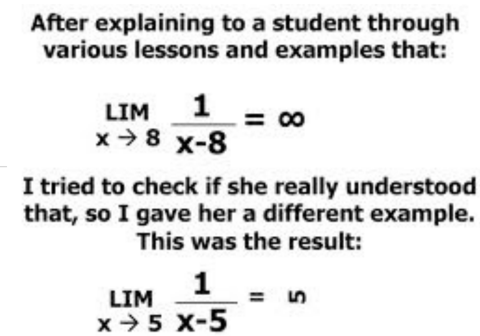
Example

Denote by R the region bounded by the y-axis the graph of f(x)=x and the graph of $g(x)=x^2-2$, on the right of the y-axis.

Calculate the volume of the solid of revolution obtained by revolving R about the y-axis.

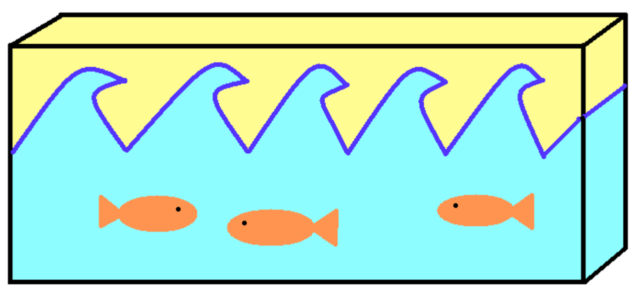


MAT 132



Consider the following picture:

- How high would the water level be if the waves all settled?



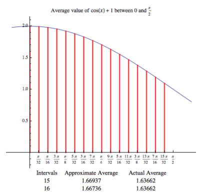
The homework grades of a student are 6, 6, 7, 8, 10. Find the average homework score.

average = sum of grades/ number of hw

The temperature of a room is 70 degrees Fahrenheit at 10AM, 72 degrees Fahrenheit at 11:05AM and 74 at 11:30AM. Use these data to estimate the average temperature.

What if we want to make a more accurate estimation of the average temperature?

If the temperature is given by a function f, f(x)=temperature at time x, x in [a,b].
We want to estimate the average value of f.
Divide [a, b] into n equal intervals.



$$\Delta x = (b-a)/n$$

x_i is a number the i-th interval

We estimate for the average value:

$$f_{average} \approx \frac{f(x_1) + f(x_2) + \ldots + f(x_n)}{n}$$

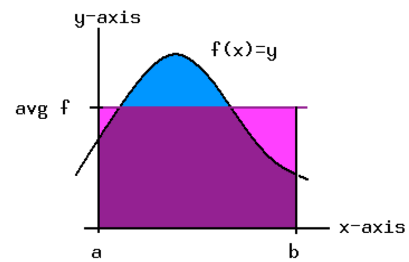
$$f_{average} \approx \frac{f(x_1) + f(x_2) + \ldots + f(x_n)}{n}$$

$$= \frac{\Delta x}{b-a}[f(x_1) + f(x_2) + \ldots f(x_n)]$$

Since

$$\Delta x = (b-a)/n$$

Taking limits $\frac{1}{b-a}\int_a^b f(x)dx$

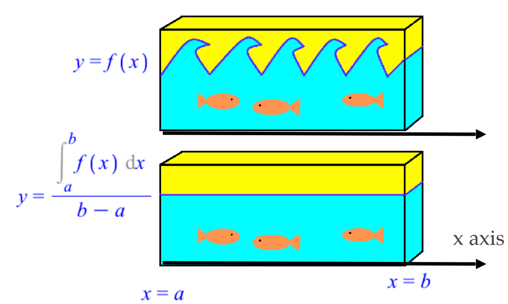


Example

- If f(x) = x^2, find the average value of f on the interval [1, 3] and interpret the result geometrically.

- http://demonstrations.wolfram.com/DistanceAndAverageVelocityForPiecewiseTrajectory/

How high would the water level be if the waves all settled?



Example

- The temperature of a room is 70 degrees Fahrenheit at 10AM, 72 degrees Fahrenheit at 11:05AM and 74 at 11:30AM. Use these data to estimate the average temperature.

- The equation below gives the temperature T(t) of a room after t minutes.

$$T(t) = \frac{8}{14625}t^2 - \frac{14}{2925}t + 70$$

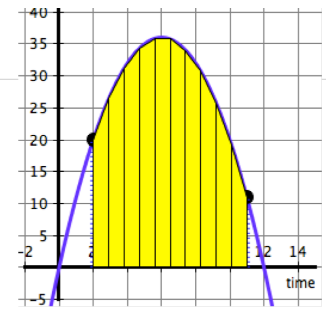
- What is the average temperature during the first 90 minutes?
- What is the average temperature during the first 30 seconds?

Example

The speed of an object is given by the equation $v(t) = 12t - t^2$ where v is in meters/sec and t is in seconds.

Determine what is the total distance traveled and the average speed of the object between t = 2 s and t = 11 s.

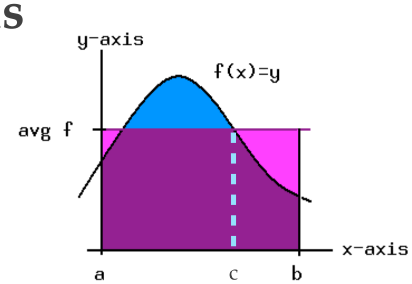
To determine the average value, we find a horizontal line such that the area under this horizontal line is equal to the area under the curve between two specified values of t.



The mean value theorem for integrals

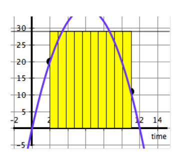
If f is continuous on [a, b], then there exists a number c in [a, b] such that

$$\int_a^b f(x)\,dx = f(c)(b-a)$$



Example: If $f(x)=12x-x^2$, ind the value of c such that

$$\int_2^{11} f(x)\ dx = f(c)(11-2)$$



Example

Find the average value of the function $f(x)= 20 - x^2$ in the interval [-2, 4].
Also, find all the values of x at which the average occurs.
Give the geometric interpretation of the results.