

MAT 132

6.1 Areas between curves

Find the area between the curves $f(x) = x/2$ and $g(x) = -x$ on the interval $[1,4]$

$f(x) = x/2$
 $g(x) = -x$
 $a = 1$
 $b = 4$
 $n = 6$ $k = 1$
 $d = 0.5$

Exact Area = 11.25
Approximate Area = 11.25 units.

Theorem

- Consider two continuous functions $f(x)$ and $g(x)$ both defined on the interval $[a,b]$.
- Suppose that $f(x) \geq g(x)$ for all x in $[a,b]$
- Then the area A of the region bounded by the curves $y=f(x)$ and $y=g(x)$ and the lines $x=a$ and $x=b$ is

$$A = \int_a^b f(x) - g(x) dx$$

Find the area of the region bounded by the curves in the following case:

- $y=x^2$ and $y=-x^2+4$. $\frac{16\sqrt{2}}{3}$
- $y^2=2x-2$ and $y=x-5$. $-\frac{8\sqrt{10}}{3}$

Recipe to find the area bounded by curves.
Find intersection points. In most cases, these points will determine the limits of integration.
Sketch a figure.
Compute the definite integral.

Sometimes you will need to "rotate" the figure $\pi/2$ (considering x as a function of y)

Area enclosed by parametric curves
Theorem: The area of the region bounded by the curve $x=f(t)$, $y=g(t)$, where t in $[\alpha,\beta]$ and the x -axis is

$$\int_{\alpha}^{\beta} g(t) f'(t) dt$$

Examples:

- Find the area enclosed by the x -axis and the curve given by parametric equations $x=1+e^t$ and $y=t-t^2$
- Find the area of the asteroïd of equation $x=\cos^3(t)$, $y=\sin^3(t)$, t in $[0,2\pi]$.
- Find the area of the ellipse of equation $(x/a)^2+(y/b)^2=1$ using parametric equations.

$3 - e$
 $\{-e^t(3-3t-t^2)\}$

$\frac{3\pi}{8}$

$a b \pi$

26. Two cars, A and B, start side by side and accelerate from rest. The figure shows the graphs of their velocity functions.

- Which car is ahead after one minute? Explain.
- What is the meaning of the area of the shaded region?
- Which car is ahead after two minutes? Explain.
- Estimate the time at which the cars are again side by side.

29. If the birth rate of a population is $b(t) = 2200e^{0.024t}$ people per year and the death rate is $d(t) = 1460e^{0.018t}$ people per year, find the area between these curves for $0 \leq t \leq 10$. What does this area represent?