MAT 132

6.1 Areas between curves

Find the area between the curves \( f(x) = \frac{x}{2} \) and \( g(x) = -x \) on the interval \([1,4]\). The theorem states that if \( f(x) \geq g(x) \) for all \( x \) in \([a,b]\), then the area \( A \) of the region bounded by the curves \( y = f(x) \) and \( y = g(x) \) and the lines \( x = a \) and \( x = b \) is

\[
A = \int_{a}^{b} f(x) - g(x) \, dx
\]

Recipe to find the area bounded by curves.

1. Find intersection points. In most cases, these points will determine the limits of integration.
2. Sketch a figure.
3. Compute the definite integral.
4. Sometimes you will need to “rotate” the figure \( \pi/2 \) (considering \( x \) as a function of \( y \)).

Examples:

1. Find the area enclosed by the x-axis and the curve given by parametric equations \( x = 1 + e^t \) and \( y = t - t^2 \).
2. Find the area of the asteroid of equation \( x = \cos^3(t), y = \sin^3(t), t \) in \([0,2\pi]\).
3. Find the area of the ellipse of equation \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) using parametric equations.

26. Two cars, A and B, start side by side and accelerate from rest. The figure shows the graphs of their velocity functions.
(a) Which car is ahead after one minute? Explain.
(b) What is the meaning of the area of the shaded region?
(c) Which car is ahead after two minutes? Explain.
(d) Estimate the time at which the cars are again side by side.

28. If the birth rate of a population is \( b(t) = 2200e^{0.06t} \) people per year and the death rate is \( d(t) = 1460e^{0.04t} \) people per year, find the area between these curves for \( 0 \leq t \leq 10 \). What does this area represent?