Review exercises for midterm I

1. Find the limit
    a. $\lim_{x \to 2^-} \dfrac{x+3}{x-2}$
    b. $\lim_{x \to -\infty} e^{x^3}$
    c. $\lim_{x \to \infty} e^{x^3}$

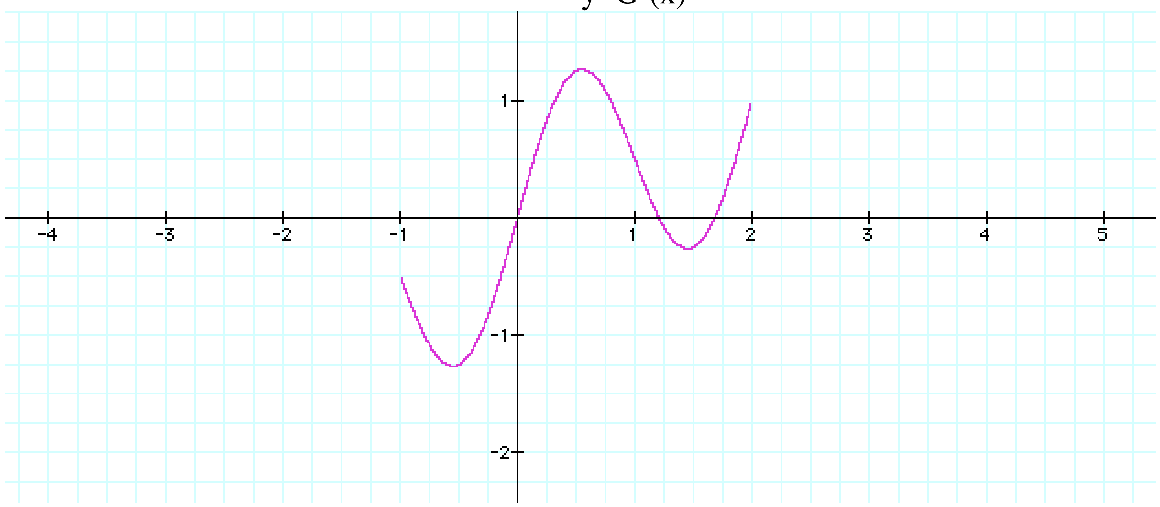
2. If an arrow is shot upward on the moon with a velocity of 70m/s, its height (in meters) after t seconds is given by $H(t)=70t-0.83t^2$.
    a. Find the velocity of the arrow after one second.
    b. Find the velocity of the arrow when t=b.
    c. When will the arrow hit the moon?
    d. With what velocity will the arrow hit the moon?
3. Sketch the graph of a function g for which g(0)=1, g'(0)=2, g'(1)=0 and g'(2)=-2.
4. Each limit represents the derivative of some function f at some number a. State f and in each case and find the limit.
    a. $\lim_{h \to 0} \dfrac{(2+h)^3 - 8}{h}$
    b. $\lim_{h \to 0} \dfrac{\tan(h+\pi)}{h}$
    c. $\lim_{h \to 0} \dfrac{\cos(\pi+h)+1}{h}$
    d. $\lim_{h \to 0} \dfrac{e^{2+h} - e^2}{h}$
5. Decide about each of the following statements whether is true or false.
    a. If a function is differentiable at a point c, then it is continous at c.
    b. There is a function F on the interval (0,6) with the following properties
        i. F' exists and is continuous
        ii. F is decreasing on (0,1)
        iii. F is increasing on (5,6)
        iv. F'(x) is different from 0 for all x in (0,6).
    c. If a function is continuous at -2 then it is differentiable at –2
    d. If f and g are differentiable functions then (f-g)'=f'-g'
6. Find the derivative using the definition of derivative. State the domain of the function and the domain of the derivative.

a. $f(x) = x + \sqrt{x}$

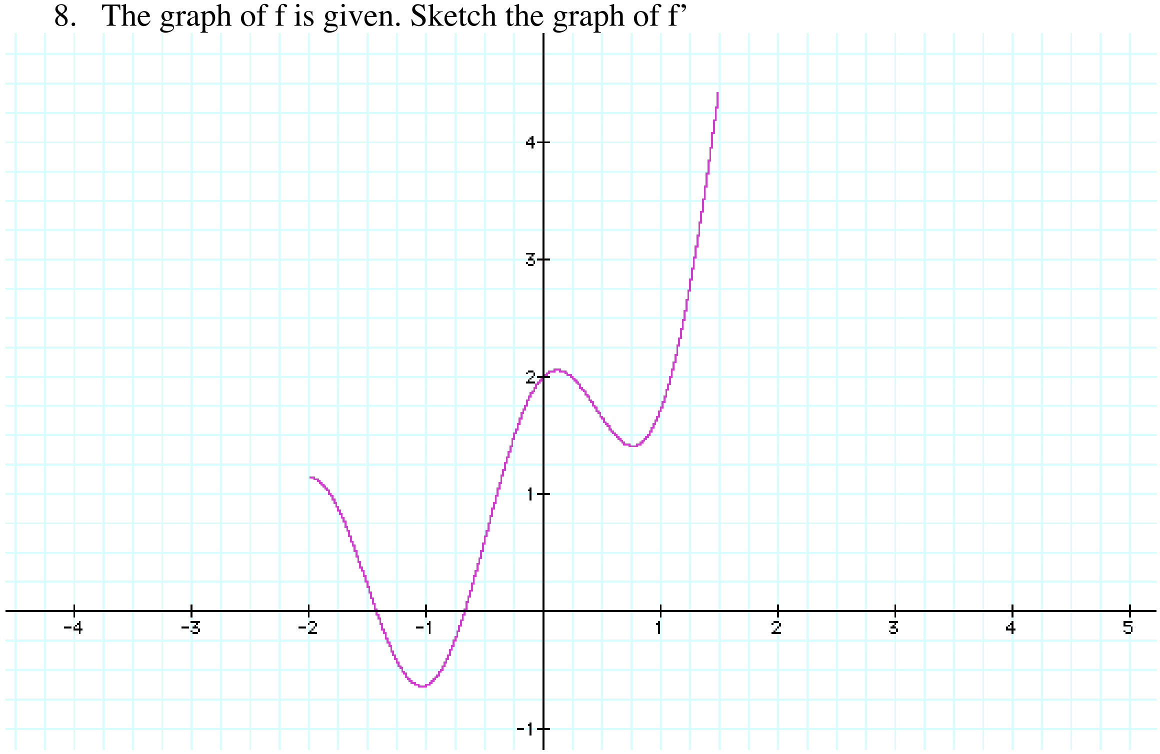
b. $g(t) = \dfrac{2t}{t+3}$

7. The graph of the derivative of a function G is shown.
   a. On what intervals G is increasing or decreasing?
   b. At which values of x does G have a local maximum or minimun?
   c. If it is know that G(1)=3, sketch a possible graph of G.

y=G'(x)




8. The graph of f is given. Sketch the graph of f'



9 . Sketch the graph of a function that satisfies all the given conditions.

a. $\lim_{x \to 4} f(x) = \square$

b. $f''(x) < 0 \ if \ x \neq 4$

c. $f''(0) = 0$

d. $f'(x) > 0 \ if \ x < 0 \ \ or \ \ x > 4$

10. Differentiate the functions

a. $f(x) = \sin(2x)e^{x^3}$

b. $g(x) = \dfrac{\cos x}{3x^4 - 2x^3}$

c. $h(x) = \ln(\sqrt{x+3})$

d. $r(x) = \dfrac{\cos x}{3x^4 - 2x^3}$

e. $s(x) = \cos^{-1}(x^2)$

f. $n(x) = e^x \ln x$

11. Differentiate the function.

a. $f(x) = (\cos^{-1}(x))$

b. $u(x) = (\ln x)^x$

c. $v(x) = \dfrac{\sin^2 x \tan^4 x}{(x+1)^2}$

d. $w(x) = \sqrt{\dfrac{x^2+1}{x^2-1}}$

e. $f(x) = e^{e^x}$

f. $g(x) = \dfrac{x}{\sin x + \cos x}$

g. $h(x) = x^{\sin x}$

11. If is a differentiable function, find an expression for the derivative of each of the following functions.

a. $y=x^3f(x)^{22}$
b. $y=x^2/f(x)$
c. $y=(1+\sin x\ f(x))/(x+3)$
d. $y=\ln|f(x)|$

12. Find an equation to the tangent line to the curve $y=3/(1+e^{-2x})$ at the point (0, 3/2)

13. On what interval s is the curve $y=x^3-3x+7$

a. increasing?
b. .concave upward?

14. Find an equation to the tangent line to the curve $y^2=x^3(2-x)$ at the point (1,1)

15. Find dy/dx if a) $\cos(x-y)=xe^x$ b) $x^3+x^2+4y^2=6$

16. Find the linearization of $f(x)=\sqrt[3]{4x+1}$ at a=0. State the corresponding linear approximation and use to give an approximate value for $\sqrt[3]{1.03}$

17. Find the points on th ellipse $x^2+2y^2=1$ where the tangent line has slope 1.

18. Find the tangent to the curve $x^2y+xy^2=3x$ at the point (2,1).

19 Let f be a function such that f(1)=2$ and $f'(x)=\sqrt{x^2+3}$. Use a linear approximation to estimate the value of f(0.99).