MAT131-REVIEW Midterm 1

Instructions: The exam will consist in six questions (like the ones bellow). You will have 90 minutes to answer all six questions. You will not be allowed to use any books or notes, but you may use your calculator.

(1) You are given the following information about the function f(x)

- (i) The domain of f(x) is the interval [-3,4]
- (ii) The function f(x) is continuous in the intervals (-3,1) and (1,4) and it is not continuous at x=1.
- (iii) f(2)=37

Which, if any, of the following limits exists? Which limits, if any, can you find using this information. Justify your answer.

- (a) $\lim_{x\to 2} f(x)$
- (b) $\lim_{x\to 2^+} f(x)$
- (c) $\lim_{x\to 4^-} f(x)$
- (d) $\lim_{x\to 1+} f(x)$

(2) Let c be a real number and let f(x) be the function

$$f(x) = \begin{cases} x^{20} + 4x & \text{if } x < -1. \\ x + c & \text{if } x \geq -1. \end{cases}$$

- (a) Find $\lim_{x\to 0} f(x)$
- (b) Find $\lim_{x\to -1^+} f(x)$
- (c) Find $\lim_{x\to -1^-} f(x)$
- (d) Find $f(-1)$
- (e) Does there exist any value of c which makes the function f(x) continuous?

(3) Let f(x) be given by

$$f(x) = \begin{cases} \sqrt{x+5} & \text{if } x \geq -5. \\ x^2 - 25 & \text{if } -10 \leq x \leq -5. \\ \frac{1}{x+10} & \text{if } x < -10 \end{cases}$$

- (a) Find right and left hand limits of this function at x=0, x=-5 and x=-10.
- (b) Does $\lim_{x\to 0} f(x)$ exist?
- (c) Does $\lim_{x\to -5} f(x)$ exist?

(d) Does $\lim_{x\to -10} f(x)$ exist?

(4) Evaluate the following limits if possible. If the limit does not exist, explain why it doesn't.

(a) $\lim_{x\to \frac{\pi}{2}} e^{\cos x}$

(b) $\lim_{x\to 1} \frac{x^3-x^2}{x^2-1}$

(c) $\lim_{x\to -2} \frac{2x^3-x^2+3x}{x^2-4}$

(d) $\lim_{t\to 0} \frac{(t+4)^{\frac{1}{2}}-2}{t}$

(e) Evaluate the following limits if possible. If the limit does not exist, explain why it doesn't.

(f) $\lim_{x\to 0} \sin(\frac{1}{x})$

(g) $\lim_{x\to 3}(\frac{1}{x-3} - \frac{6}{x^2-9})$

(h) $\lim_{x\to 7} \frac{x-7}{|x-7|}$

(i) $\lim_{t\to 37} \frac{t}{t-37}$

(j) $\lim_{x\to -1} \frac{x^4-x^3+x^2}{x^3-x}$

(5) State the intervals where each of the following functions are continuous

(a) $f(x) = |x+3|$

(b) $f(x) = \frac{|x-4|}{x-4}$

(c) $\ln(x^2 - 3x)$

(d) $\cos(x)$

(e) $\frac{x^2+x+2}{x^2-9}$

(6) Sketch the graph of a function with the following properties:

(i) $f(x)$ is one-to-one.

(ii) The domain of f is the inverval (-3,7)

(iii) The range of f is the interval (-4,6)

(iv) f(x) is continuous everywhere except at x=3 and x=4,

(v) f(x) is continuous from the left at x=4 and it is continuous from the right at x=3.

(vi) $f^{-1}(4) = 5$

(7) Prove that the following equations have at least one real solution. Find an interval of length at most 2 that contains a solution.

(a) $\log(x) + x = 3$

(b) $x^4 - x + 3 = 0$

(c) $\sin(\tan x) = 0.5$

(8) If $\lim_{x\to 2}(f(x) + g(x)) = 4$ and $\lim_{x\to 2}(f(x) - g(x)) = -3$, find

(a) $\lim_{x\to 2} f(x)$

(b) $\lim_{x\to 2} g(x)$

(c) $\lim_{x\to 2} 3f(x) - g(x)$

(d) $\lim_{x\to 2} f(x)g(x)$

(9) Find all values of c such that f is continuous on all the real numbers.

$$f(x) = \begin{cases} x^2 - 5 & \text{if } x \geq c. \\ x - 5 & \text{if } x < c. \end{cases}$$

(10) Sketch the graph of a function f that satisfies the following conditions

(i) $\lim_{x\to 2^-} f(x) = -1$, $\lim_{x\to 2^+} f(x) = 2$

(ii) $f(1)$ is not 2.

(iii) $\lim_{x\to 3} f(x) = 4$

(iv) f is increasing an defined everywhere.

(11) An aeroplane is flying from New York to Paris. At t hours, it has covered d(t) miles.

(a) Give a formula for the average velocity during the second hour of flight.

(b) State what is the instantaneous velocity at t=2 in terms of the graph of $y = d(t)$

(12) (Hard) Let $[[x]]$ denote the largest integer that is less than or equal to x (For example, $[[2.7]] = 2$ and $[[3]] = 3$).

(a) Find $\lim_{x\to 4^-} f(x)$

(b) Find $\lim_{x\to 4^+} f(x)$

(c) Sketch a graph of f.

(d) For which values of x, $[[x]]$ is not continuous?

(13) For which values of a does $\lim_{x\to 1} \frac{x^2-ax-1}{x-1}$ exists?

(14) For $f(x) = \frac{\sqrt{x}-\sqrt{1-x}}{2x-1}$

(a) Find its domain

(b) Calculate $\lim_{x\to\frac{1}{2}} f(x)$.

(15) For

$$f(x) = \begin{cases} x & \text{if } x > 0. \\ \cos x & \text{if } x \leq 0. \end{cases}$$

(a) Sketch the graph of $f(x)$

(b) State the intervals where $f(x)$ is continious.

(16) The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why. Justify all your steps.

(a) $\lim_{x\to 1}(2f(x) - 3g(x))$

(b) $\lim_{x\to -1} f(g(x))$

(c) $\lim_{x\to 0} \frac{f(x)}{g(x)}$

(d) $\lim_{x\to 0} \frac{f(x)}{x+3}$

(e) $\lim_{x\to 2} g(x)$

(f) $\lim_{x\to 0}[f(x)]^2$