Math 331, Fall 2004: Problems 8-9

NOTE 1: *Each exercise is worth 10 points and can be turned in at any time before its "expiration date". At the end of the semester, I will expect you to have turned in at least half of the exercises assigned. If you do more, I will pick your best grades. If you do less, the missing grades will be counted as zeros. Altogether, these will count the same as one project.*

NOTE 2: *Maple is not necessary to the next two exercises, although you can use is as a word processor.*

11. (*expires 10/5*) Following Section 4 of the notes, prove that if we describe the circle of center (a, b) and radius r using the parameters (a, b, k), with $k = a^2 + b^2 - r^2$, rather than the more natural parameters (a, b, r), then the error function $H(a, b, k) = E(a, b, \sqrt{a^2 + b^2 - k})$ is quadratic in a, b and k. What does this imply about the number of critical points?

12. (*expires 10/5*) With reference to Problem #11, show that, for $r > 0$, the transformation $(a, b, r) \mapsto (a, b, k)$ is a valid change of variables, that is, it is one-to-one. This should help you prove that $E(a, b, r)$ has only one "physical" critical point, which is a minimum, and is mapped, through the transformation, into the unique critical point of $H(a, b, k)$.