Math 331, Fall 2004: Problem Set 5. Due Oct18

1. Find all the solutions to the differential equation

$$\frac{dx}{dt}(t) = -2x(t), \quad t \in \mathbf{R}.$$

Among them, single out the one for which $x(0) = 3$. [*Hint: read the help page for* `dsolve`*, or just do it in your head. It is that easy.*]

2. Consider the first order system

$$\begin{cases} x' &= -2x + 3y \\ y' &= -3x \end{cases}$$

- a Find the fixed points.
- b Determine the 'shape" of the solutions around the fixed point (that is, study whether it is a saddle, a source, a spiral source, a sink, a spiral sink or a center)
- c Determine whether

$$\begin{cases} x(t) &= e^{-t}\cos(2\sqrt{2}t) + 2\sqrt{2}e^{-t}\sin(2\sqrt{2}t)) \\ y(t) &= 3e^{-t}\cos(2\sqrt{2}t) \end{cases}$$

a solution of the system. Justify your answer.

3. (no `Maple`)Consider a linear system $X' = AX$, where A is a real 2x2 matrix. Show that if $V(t) = (v_1(t), v_2(t))$ and $U(t) = (u_1(t), u_2(t))$ are two solutions and r and s are two real numbers, then $W(t) = rV(t) + sU(t)$ is also a solution.

4. Find all the fixed points of the system

$$\begin{cases} \dot{x} &= x^2 + y, \\ \dot{y} &= x(y^2 - 1), \end{cases}$$

a fixed point being a solution for which *both* $x(t)$ *and* $y(t)$ stay constant. For each of these points, describe the behavior of the solutions that have initial conditions nearby. You can use `Maple` to figure out what happens for nearby points, or you can use more mathematical methods.