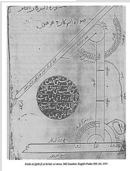
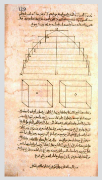
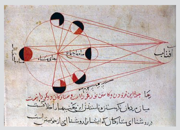
# MAT 336: Mathematics in the Islamic Golden Age

Wikipedia: Engraving of Abū Sahl al-Qūhī's perfect compass to draw conic sections.







- Islamic Golden Age
- al-Khwarizm
- Omar Khayyam
- Perfect compass

# Heading

| | | | | |
|---|---|---|---|---|
| 23 | Apr 21 | Euler and the bridges of Königsberg **Liya** | Euler on Number Theory **Jonathan** |
| 24 | Apr 23 | Maria Agnesi **Pat** | The Bernoullis and the origin of probability theory **Alexis** |

# A (too) brief history of the Islamic Empire

## Rough Chronology

- 622 CE (622): Religion of Islam is founded in Arabian peninsula.
- Unites diverse groups of people
- 750: Bagdad was established as capital by the **Abbasid** dynasty and became center of Islamic knowledge.
- **700 to 1300: Islamic Golden age.**
- 766: House of Wisdom (Bayt al-Hikma)
- Arabic became the language of science.

https://www.britannica.com/place/Caliphate/The-Abbasid-caliphate

About the denomination "Islamic"

# Why Did Islamic Scholarship Flourish?

- Abbasid dynasty encouraged scholarship (some scholars suggest that the motivation was to establish its own cultural hegemony)
- House of Wisdom.
- Translation movement: Scholarly manuscripts from all around the world were collected and translated.
- Confluence of many cultures.
- Freedom of scholarship
- Religious tolerance and strong interest in knowledge. ("Seek knowledge from the cradle to the grave")
- Paper (from China) although no printing press. (woodblock printing though)

# One Thousand and One Nights

- **The story:** Scheherazade marries a sultan who kills each wife after one night. She survives by telling a story each night, always stopping at dawn at a cliffhanger. 1001 nights later, she has her life and his respect.
- **The book**: Compiled in Arabic from Persian and Indian sources, 8th-14th centuries.

- **Sinbad the Sailor**
- **Ali Baba and the Forty Thieves**
- **The Fisherman and the Genie**
- **Aladdin and the Magic Lamp**

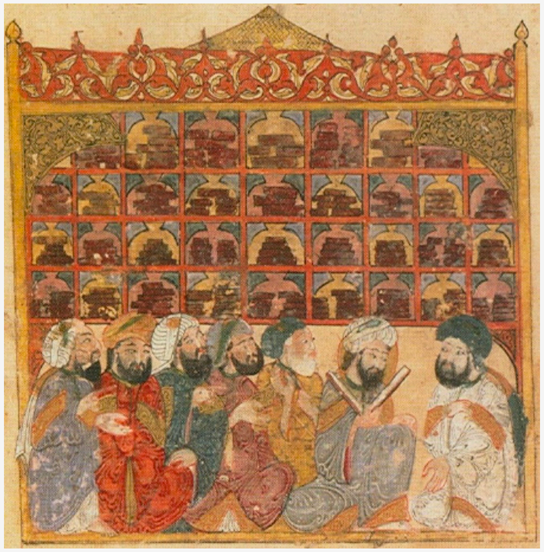
**Legacy and impact**: from Borges to Prince of Persia, from Magic: The Gathering to Disney, stories we know but don't realize where they came from.

Scheherazade and the sultan by Iranian painter Sani al Mulk (1849–1856)

By Sani ol-Molk (1814-1866) - http://www.ir-tmca.com/Exhibition/negargari/work1.htm [1], Public Domain, https://commons.wikimedia.org/w/index.php?curid=2631435

# The House of Wisdom

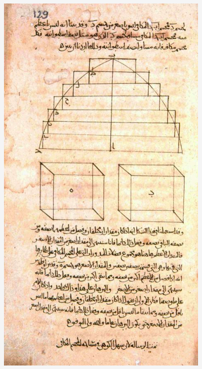
Baghdad's center of translation and scholarship — where manuscripts from Greece, Persia, and India were collected, translated, and built upon. (The exact nature of the institution is still debated.)



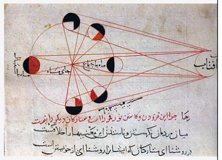
Scholars in library of the House of Wisdom. Book "Maqamat al-Hariri". Painter: Yahya al-Wasiti. Baghdad 1237 AD. At French Nat'l Library, Codex Parisinus Arabus 5847, fol. 5v. Online at: https://gallica.bnf.fr/ark:/12148/btv1b8422965p/f20

# Mathematical Achievements

- Remarkable mathematical developments
- **Translation movement**: Mathematical manuscripts from all around the world were collected and **translated**. Islamic scholars **amalgamated knowledge** from Egypt, Babylonia, Greece and India.
- Close relations between theory and practice.
- Systematization of decimal number system, including fractions.
- Started to understand and systematize relations between algebra and geometry.
- Advances in number theory and in geometry (plane, 3D and spherical)
- Combinatorics were brought from India.



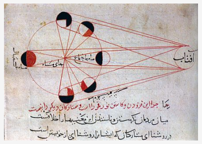
Geometrical shapes from Suhayl al-Quhi's book Fī istihraci mesaha al-muhassama al-maqafī or Risala-i abu Sahl [Suleymaniya library, Ayasofya-4832]



Wikipedia: al-Biruni's explanation of the phases of the moon

# Some Islamic Mathematicians

- **al-Khwārizmī** ~800 - many **influential** math books - **Algebra**
  - first ***systematic*** solution of linear and quadratic equations.
  - introduced the decimal positional number system
  - systematized material he learned from various sources:
    - base 10 number system from India.
    - solving quadratic equations from Babylonia (using a geometric complete the square method)
    - Hebrew mathematics - astrolabe
    - elaborated Ptolemy geometry model.
    - **al-Biruni** astronomer and mathematician. ~1000
- **Omar al-Khayyam** ~1100 - geometry (fifth postulate), classification and solution of equations.



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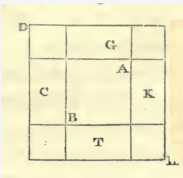
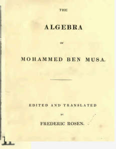


Educated guess: Name authors of books which were translated into Arabic during the Islamic Golden age. (Just write one, if you know more than one, tell the name to a classmate)
Hint: we studied most of them in class

## Some of the math works translated

- **Euclid**: Elements, the Data, the Optics, the Phaenomena, and On Divisions
- **Archimedes**: Sphere and Cylinder and Measurement of the Circle
- **Apollonius**: all
- **Diophantus**: Arithmetica
- **Menelaus**: Sphaerica
- **Ptolemy**: Almagest.
- **Diocles**' treatise on mirrors,
- Theodosius's Spherics,
- Pappus's work on mechanics,
- **Ptolemy's** Planisphaerium, and
- Hypsicles' treatises on regular polyhedra (the so-called Books XIV and XV of Euclid's Elements)

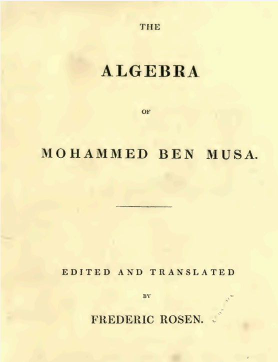
Socrates and his students" As imagined by a 13th century Turkish manuscript. Greek knowledge didn't just pass through Islamic scholarship, it was absorbed, debated, and reimagined.

Socrates and his Students, illustration from 'Kitab Mukhtar al-Hikam wa-Mahasin al-Kilam' by Al-Mubashir, Turkish School, (13th c) Photo by Bridgeman

https://aeon.co/ideas/arabic-translators-did-far-more-than-just-preserve-greek-philosophy

# Al-Khwārizmī

# Al-Khwārizmī ~800 CE Bagdag


THE

ALGEBRA

OF

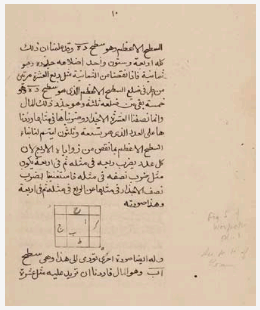
MOHAMMED BEN MUSA.

EDITED AND TRANSLATED

BY

FREDERIC ROSEN.

https://www.wilbourhall.org/pdfs/The_Algebra_of_Mohammed_Ben_Musa2.pdf



This is a page from al-Khwārizmī's algebra text, Kitāb al-jabr wa'l-muqābala, written in about 825, the first extant algebra text, by Muḥammad ibn Mūsā al-Khwārizmī. https://www.maa.org/press/periodicals/convergence/mathematical-treasures-al-Khwārizmī s-algebra

Statue of al-Khwārizmī in front of the Faculty of Mathematics of Amirkabir University of Technology in Tehran, IRAN

By M. Tomczak - http://www.es.flinders.edu.au/~mattom/science+society/lectures/illustrations/lecture16/alkhwarizmi.html, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=27754448

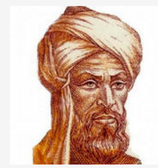
Sometimes called "the father of algebra" (same as Diophantus). (My opinion is that Algebra has many, many parents)

(e.g. Al-Khwārizmī acknowledged that he derived ideas from Indian mathematician Brahmagupta)

13

## al-Khwārizmī ~ 800CE

- **CLASSIFICATION:** six standard forms for linear or quadratic equations
- **ABSTRACTION:** solutions using **algebraic methods and geometrical diagrams -**
- **NEGATIVE NUMBERS**
  - thinking linked to geometry, thus he rejected negative results.
  - in laws inheritance, used negatives as **debts**.



## al-Khwārizmī ~ 800

Wrote The Book of Restoration and Balancing (Kitāb al-jabr wa'l-muqābala)

- **al-jabr:** "restoration," "completion," or "rejoining" (the operation of moving a term from one side of **=** to the other to eliminate a **minus** sign)
- **al-muqābala:** "balancing," "comparison," or "setting opposite" (canceling equal terms on both sides of **=**)

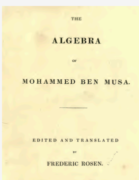
https://en.wikisource.org/wiki/The_Compendious_Book_on_Calculation_by_Completion_and_Balancing/Compendium_on_calculating_by_completion_and_reduction

## 1. What reason does al-Khwārizmī give for writing this book? 2.What stands out to you most about this reason?

That fondness for science, ... that affability and condescension which God shows to the learned, that promptitude with which he protects and supports them in the elucidation of obscurities and in the removal of difficulties, has encouraged me to compose a short work on calculating by al-jabr and al-muqabala , confining it to what is easiest and most useful in arithmetic.

> WHEN I considered what people generally want in calculating, I found that it always is a number.

Muhammad ibn Musa al-Khwārizmī




https://en.wikisource.org/wiki/The_Compendious_Book_on_Calculation_by_Completion_and_Balancing/Compendium_on_calculating_by_completion_and_reduction

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## What mathematical system is al-Khwārizmī describing here?

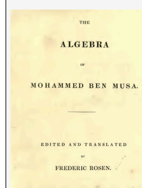
## Positional number system in base 10

Muhammad ibn Musa al-Khwārizmī

When I consider what people generally want in calculating, I found that it always is a number.

I also observed that every number is composed of units, and that any number may be divided into units.

**Moreover, I found that every number, which may be expressed from one up to ten, surpasses the preceding by one unit: afterwards the ten is doubled or tripled, just as before the units were: thus arise twenty, thirty, etc., until a hundred; then the hundred is doubled and tripled in the same manner as the units and the tens, up to a thousand; then the thousand can be thus repeated at any complex number; and so forth to the utmost limit of numeration.**

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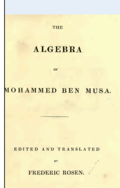
I observed that the numbers which are required in calculating by completion and reduction are of three kinds, namely, roots, squares, and simple numbers relative to neither root nor square.

A root is any quantity which is to be multiplied by itself, consisting of units, or numbers ascending, or fractions descending.

A square is the whole amount of the root multiplied by itself.

A simple number is any number which may be pronounced without reference to root or square.

A number belonging to one of these three classes may be equal to a number of another class; you may say, for instance, " squares are equal to roots," or " squares are equal to numbers," or " roots are equal to numbers.




**1. According to al-Khwārizmī, what are the three kinds of numbers? Use his terms. 2. In modern notation, write an example of an equation where 'squares are equal to roots'.**

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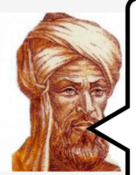

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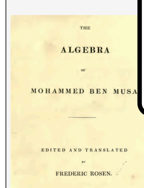



root → unknown, x
square → its square, $x^2$
simple number → constant term

Al-Khwārizmī classified terms of a linear and quadratic equations as follows:

A. Square (of the unknown, $x^2$ for us)
B. Root (the unknown, x)
C. Number (the constant term)

**Classification**

Al-Khwārizmī used

- Only positive coefficients
- Equations with at least one positive solution.
- Zero as numeral but not as a number.

**Write down as many different types of equations as you can find with the above rules. (For instance: a $x^2$ =c is one of these types)**

## Types of Equations According to al-Khwārizmī (Linear and Quadratic)

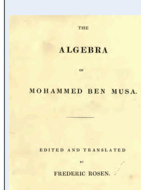
1. Squares equal to roots: $ax^2=bx$
2. Squares equal to numbers: $ax^2=c$
3. Squares equal to roots and numbers: $ax^2=bx+c$
4. Squares and roots equal to numbers: $ax^2+bx=c$
5. Squares and numbers equal to roots: $ax^2+c=bx$
6. Roots equal to numbers: $bx=c$

Table 1-1

| Equation form | Translation of equation | al-Khwarizmi examples modernized |
|---|---|---|
| $x^2 = c$ | Square equals number | $x^2 = 9$ |
| $x^2 = bx$ | Square equals roots | $x^2 = 10x$ |
| $x^2 = bx + c$ | Square equals roots and number | $x^2 = 10x + 21$ |
| $x^2 + c = bx$ | Square and number equal roots | $x^2 + 21 = 10x$ |
| $x^2 + bx = c$ | Square and roots equal number | $x^2 + 10x = 39$ |
| $bx + c = x^2$ | Roots and number equal square | $3x + 4 = x^2$ |

Of the case in which squares are equal to roots, this is an example. "A square is equal to five roots of the same;" the root of the square is five, and the square is twenty-five, which is equal to five times its root.

**Express "A square is equal to five roots of the same" as an equation" in modern notation.**




root → unknown, x
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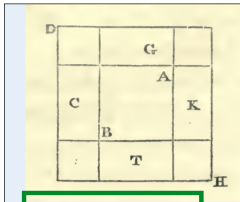
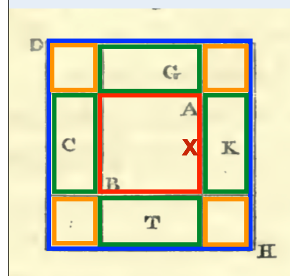
# Questionnaires Lect 1

First :
https://app.wooclap.com/POQCUQ/questionnaires/691a2422b9acd49f18fa45be

Second
https://app.wooclap.com/POQCUQ/questionnaires/691a27f699b654259acc83a2




**Demonstration of the Case: "a Square and ten Roots are equal to thirty-nine Dirhems."**

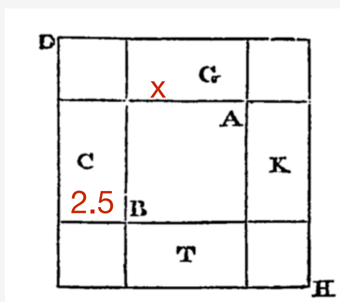
The figure that explains this is a square whose sides are unknown. It represents the square whose root you wish to find. This is the figure AB, where each side represents a root. If you multiply one side by any number, that product represents the number of roots added to the square. Since in this problem the roots are combined with the square, we take one-fourth of ten—that is, two and a half—and add it to each of the four sides of the figure.

Questionnaires
https://app.wooclap.com/ICLEXS/questionnaires/69e7a38b2dc72ec5a23f0afe

https://app.wooclap.com/ICLEXS/questionnaires/69e7a467b7bbb6d0f6b1bae1

## al-Khwārizmī- Case a Square and ten Roots are equal to thirty-nine dirhams.

The figure that explains this is a square whose sides are unknown. It represents the square whose root you wish to find. This is the figure AB, where **each side represents a root**. If you multiply one side by any number, that product represents the number of roots added to the square. **Since in this problem the roots are combined with the square, we take one-fourth of ten—that is, two and a half—and add it to each of the four sides of the figure.**



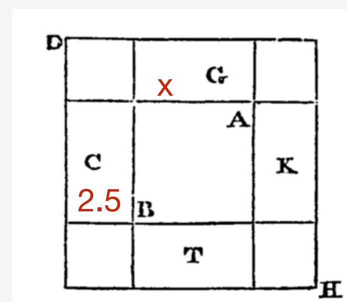

Find the area of **one** of the four attached rectangles (C, G, T, or K) in terms of x.

# al-Khwārizmī cont.

With the original square AB, we attach four rectangles, each having a side of the square as its length and two and a half as its width. These are rectangles C, G, T, and K. We now have a figure with unknown but equal sides, but at each of the four corners a small square of two and a half by two and a half is missing. To fill this gap and complete the large square, we must add four times the square of two and a half—that is, twenty-five.

Find the area of one of the four corner little squares.

Add the areas of the four corner squares to the areas of AB, C, G, T and K to get the area of the large square DH.

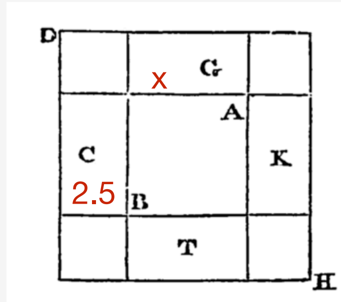


## al-Khwārizmī cont.

We know from the problem that the original square, together with the four rectangles around it (representing the ten roots), equals thirty-nine. If we add twenty-five (the four corner squares that complete the large figure DH), the total is sixty-four. One side of this large square is its root: eight.

Note that the original square AB plus the four rectangles equals 39. What is the total area of the completed large square DH?

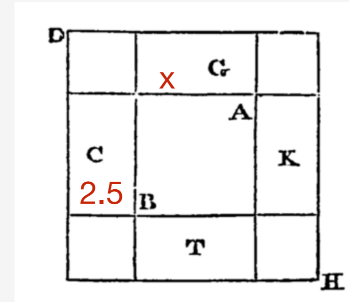
What is the side length of the large square DH?




## al-Khwārizmī cont.

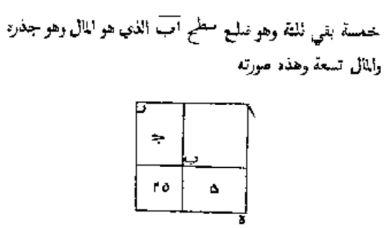
If we subtract twice two and a half (that is, five) from eight—removing the added lengths from both ends of the side of the large square DH—the remainder is three. This is the root of the original square—that is, the side of figure AB. Note that we halved the number of roots and added the square of this half to thirty-nine in order to complete the four corners of the large figure. This is because one-fourth of any number, squared and then multiplied by four, equals the square of half that number. Therefore, we squared only half of the roots rather than squaring one-fourth and then multiplying by four.

What is x, the side length of the original square AB?

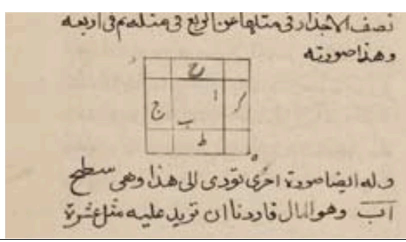



## Each of the solutions below, al-Khwārizmī completed the square.

Solution 1




Solution 2



**Why negative numbers did not appear as solutions?**

# About algebra and meaning

One way to put the dichotomy in a more philosophical or literary framework is to say that algebra is to the geometer what you might call the Faustian offer. As you know, Faust in Goethe's story was offered whatever he wanted (in his case the love of a beautiful woman), by the devil, in return for selling his soul. Algebra is the offer made by the devil to the mathematician. The devil says: I will give you this powerful machine, it will answer any question you like. All you need to do is give me your soul: give up geometry and you will have this marvelous machine. (Nowadays you can think of it as a computer!)

**Sir Michael Atiyah**

What do you think Atiyah means by " algebra is to the geometer what you might call the Faustian offer"

Of course we like to have things both ways; we would probably cheat on the devil, pretend we are selling our soul, and not give it away. Nevertheless, the danger to our soul is there, because when you pass over into algebraic calculation, essentially you stop thinking; you stop thinking geometrically, you stop thinking about the meaning.

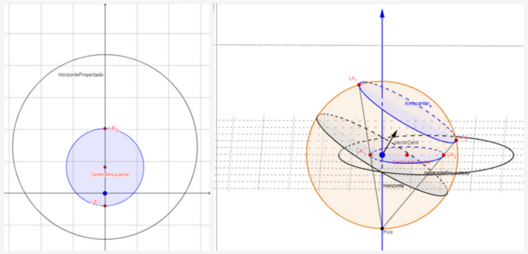
I am a bit hard on the algebraists here, but fundamentally the purpose of algebra always was to produce a formula which one could put into a machine, turn a handle and get the answer. You took something that had a meaning; you converted it into a formula, and you got out the answer. In that process you do not need to think any more about what the different stages in the algebra correspond to in the geometry. You lose the insights, and this can be important at different stages. You must not give up the insight altogether! You might want to come back to it later on. That is what I mean by the Faustian offer. I am sure it is provocative.

Do you agree? Why or why not?

**Sir Michael Atiyah**

# Astrolabe

## Astrolabe



Vgonzalez630 - Sitio web: http://www.geogebra.org

- Handheld astronomical instrument.
- Stereographic projection of the celestial sphere on a plane.

**Uses**

- Solve problems geometrically (no calculation)
- From Sun/star position one can find time, latitude, direction of Mecca (qibla).

**Parts**

- Plate (depends on latitude)
- Rotating star map (rete)
- Rule

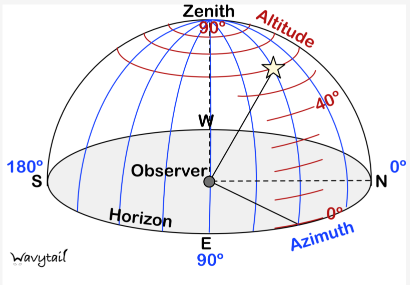
The moving parts simulate the sky's rotation for any date, time, and latitude.

**Astrolabe made by Abd al-Karim al-Misri/al-Madrabi, brass with silver and copper inlay, AH633 (1236AD), probably Mayyafariqin (Turkey)**

https://www.britishmuseum.org/collection/object/W_1855-0709-1

# Astrolabe Latitud ~ 41

Basic diagram to show how the lines on the plate relate to the sky above you:
This star has an Azimuth of 45° and an altitude of 60°

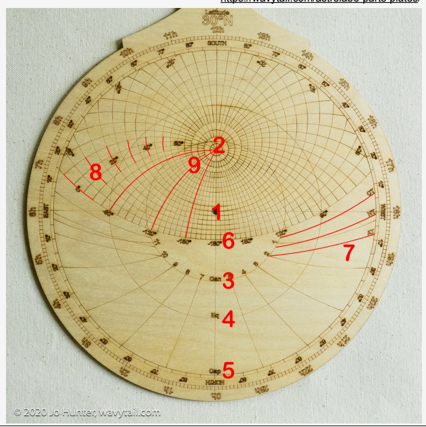



**Words of Arabic origin**

- **Azimuth** → from *al-sumūt*, "the directions"
- **Almucantar** → from *al-muqanṭarāt*, "circles of altitude"
- **Nadir** → from *naẓīr*, "opposite"
- **Zenith** → from *samt al-ra's*, via Latin corruption

1. North Pole
2. Zenith (the point in the sky directly above your head)
3. Tropic of Cancer
4. Equator
5. Tropic of Capricorn
6. Horizon
7. Civil, Nautical and Astronomical Twilight Arcs
8. Altitude circles go around the zenith, from 0° at the horizon to 90° at the Zenith
9. Azimuth circles converge at the zenith, from 180° facing South to 0° facing North

https://wavytail.com/astrolabe-parts-plates/




# Astrolabe Uses

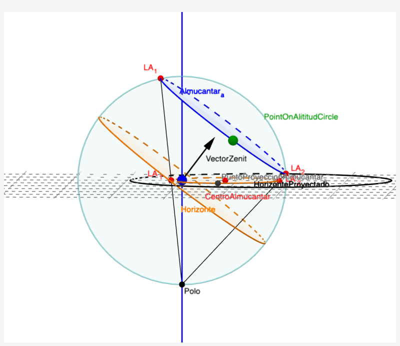
- Time: Finding local time from the Sun or stars.
- Latitude: Estimate or verify latitude from celestial observations
- Prayer times: Determine times of prayer
- Qibla: Find the direction of Mecca
- Astrology: Often used to cast horoscopes
- Surveying: Measure heights and distances
- Teaching: Teaching and calculating problems in spherical astronomy.

The astrolabe works because stereographic projection sends circles on the sphere to circles (or lines) on the plane.

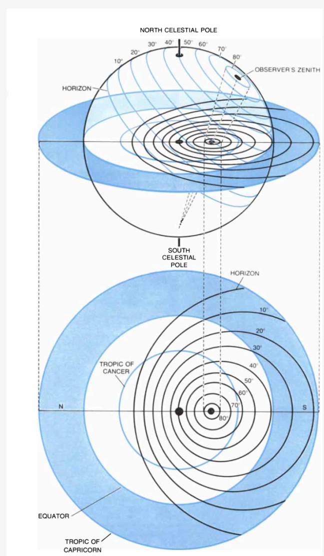
https://archive.org/details/ilmetauqeet_gmail_659/page/n168/mode/thumb

# Steregoraphic projection

- Explained by Ptolemy (Alexandria ~200)
- Preserves angles and circles




https://www.geogebra.org/classic/kya3qthv



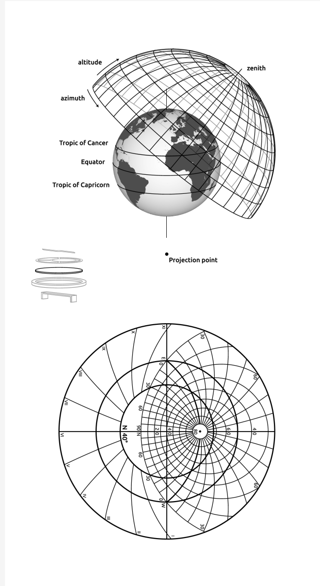

https://www.jstor.org/stable/10.2307/24949987

| | |
|---|---|
| ~150 BCE | **Hipparchus** associated with early stereographic projection methods |
| ~2nd c. CE | **Ptolemy** formalizes the mathematics (*Planisphaerium*) |
| ~4th–5th c. | **Theon of Alexandria** writes an early astrolabe treatise |
| ~8th c. | Arabic translations; adoption in the Islamic world |
| ~9th–10th c. | Abbasid development; earliest surviving instruments |
| ~10th c. | **al-Sufi** and others refine use and design |
| ~10th–11th c. | **al-Biruni** systematizes theory and applications |
| ~11th–12th c. | Transmission to Europe via al-Andalus |
| ~13th–15th c. | European flourishing; **Chaucer's** treatise (~1391) |
| ~16th–17th c. | Gradual decline; replaced by specialized instruments and mechanical clocks |

# Astrolabe

Animation showing how celestial and geographic coordinates are mapped onto an astrolabe's base plate (tympan) through a stereographic projection.

Hypothetical tympan (40 degrees North Latitude) of a 16th century European planispheric astrolabe.




By This image has been created during "DensityDesign Integrated Course Final Synthesis Studio" at Politecnico di Milano, organized by DensityDesign Research Lab in 2016. Credits goes to Michele Invernizzi - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=45222752

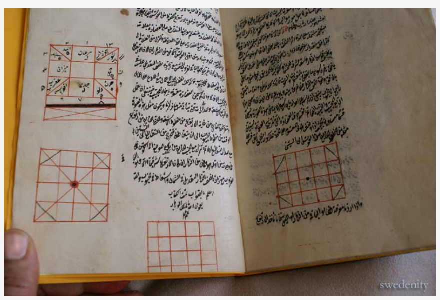
# Hebrew Astrolabe

Astrolabe inscribed with words in Hebrew and Arabic

https://www.bbc.co.uk/sounds/play/b00st9z8

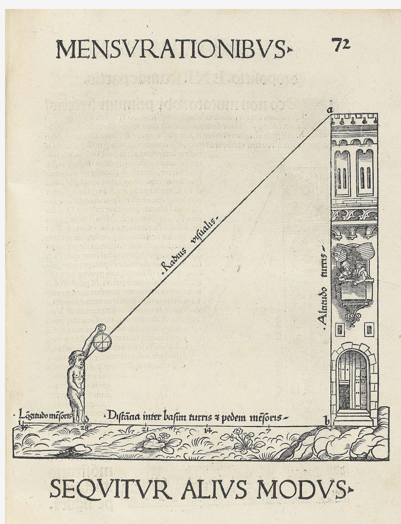
From A History of the World in 100 Objects

**A treatise explaining the importance of the astrolabe by Nasir al-Din al-Tusi, Persian scientist**



By Danieliness at the English-language Wikipedia, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=11449840

**16th-century woodcut of the measurement of a building's height with an astrolabe**




By Johannes Stöffler - http://christies.com/img/LotImages/2017/CKS/2017_CKS_14298_0349_000(stoffler_johannes_elucidatio_fabricae_usuque_astrolabii_ex_secunda_aut051338).jpgfromhttp://christies.com/lot/lot-stoffler-johannes-1452-1531-elucidatio-fabricae-usuque-astrolabii-6069643, Public Domain, https://commons.wikimedia.org/w/index.php?curid=108928273

# Astrolabe Material

- https://youtu.be/N8oWGwcdFmA?si=HeM7cEMAb_0upp3_&t=94 1.34 up to minute 2.40
- https://youtu.be/N8oWGwcdFmA?si=aMrSjjUAPye2K3Xq&t=294 4:55 up to minute 6:00
- https://www.ted.com/talks/tom_wujec_learn_to_use_the_13th_century_astrolabe
- https://www.britishmuseum.org/blog/seeing-stars-astrolabes-and-islamic-world
- https://www.geogebra.org/m/BUFWB5mD specially https://www.geogebra.org/m/BUFWB5mD#chapter/80821
- https://www.geogebra.org/m/HNFgEkzf

## Astrolabe in Istanbul

Painting of the Istanbul Observatory. It shows workers at the observatory of Taqi al-Din at Istanbul in 985 H (1577). Two observers are working with an astrolabe. A universal astrolabe of the saphea form is on the table in front of the man with the dividers and paper. The painting is from Shahinshah-nama (History of the King of Kings), an epic poem by ‘Ala ad-Din Mansur-Shirazi, written in honour of Sultan Murad III (reigned 1574-95 [AH 982-1003]).

https://muslimheritage.com/using-an-astrolabe/

# Omar Khayyam ~1000 CE

*How swiftly does this caravan of life pass;*
*Seek thou the moment that with joy does lapse.*
*Saghi, why lament tomorrow’s misfortunes today?*
*Bring forth the chalice, for the night shall pass.*

*From the Rubáiyát of Omar Khayyam*
*Translation by Joobin Bekhrad*

## Omar Khayyam May 18, 2019 Google Doodle celebrating his 971st Birthday ~1000 CE

Khayyam was Persian, but wrote scientific works in Arabic, the scholarly language of his era. He wrote poetry in Persian.

Symbols (according to chatGPT)

- **Sun:** his calendar reform was one of history's most accurate solar calendars
- **Glowing Earth:** Astronomical observations. Yellow lines = stylized Arabic script.
- **Sphere with crosshairs:** Spherical geometry and astronomical instruments.
- **Golden curve:** Mathematical and astronomical contributions
- **Book with Arabic script:** Treatises on algebra, geometry, and Euclid
- **Grapes:** Wine imagery from the Rubā‘iyāt
- **Persian landscape:** Nishapur, his birthplace

## From Omar Khayyam’s Rubā‘iyāt

A moment’s innocence, a breath of grace,
A cup of wine, and friends in this green place—
This is the whole of life I know, my friend;
The rest is shadows passing in a race.

The grape that can with logic absolute
The Two-and-Seventy jarring sects confute;
The subtle alchemist that in a trice
Life’s leaden metal into gold transmute.

**"Confute"** = disprove or refute

You must realize that this treatise will be understood only by those who have mastered Euclid's Elements and Data as well as the two [first] books of Apollonius's Conics. He who lacks knowledge of one of these treatises will not reach an understanding of this book.

Omar Khayyam - Treatise on Algebra

**1.What does this paragraph reveal about how mathematical knowledge was organized and transmitted at the time?**

**2. How does this compare to modern expectations for reading a math text? (Prerequisites, background, structure.)**

**What does this paragraph reveal about how mathematical knowledge was organized and transmitted at the time?**

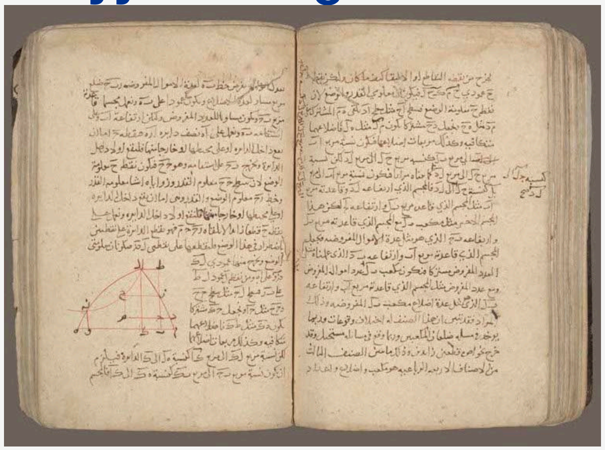
- Knowledge transmission was text-based and hierarchical: you couldn't skip foundational steps or learn in isolation.
- Scholars were expected to have direct familiarity with specific foundational texts—not just general concepts.
- This suggests a relatively small, well-defined set of essential mathematical works that mathematically trained scholars were expected to know.
- al-Khwārizmī naming three specific works shows a shared intellectual framework and a common set of reference texts.

**How does this compare to modern expectations for reading a math text? (Prerequisites, background, structure.)**

Today we still rely on prerequisites, but in practice:

- Modern textbooks are more self-contained.
- Authors usually review necessary background instead of expecting total mastery.
- Students learn through lectures, problem sets, and multiple sources, not a single canonical text.
- The structure is modular, not strictly linear: you can enter a subject at different points.

# Omar Khayyam's Algebra

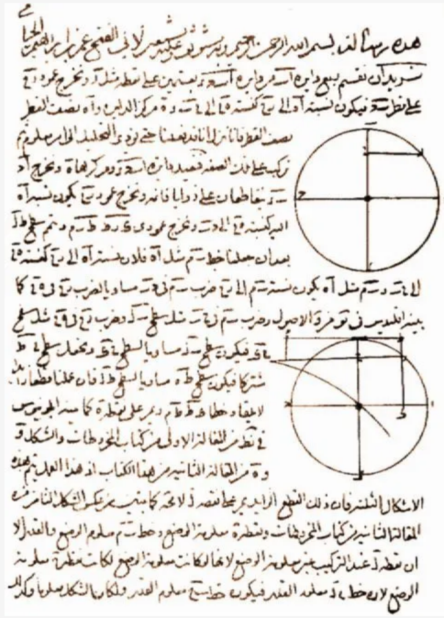


Maqalah fi al-jabr wa-al muqabalah - Omar Khayyam Algebra
https://dlc.library.columbia.edu/catalog/ldpd:113656

Solution of the various cases of the cubic equation by finding the intersections of appropriately chosen conic sections.

On this page, the case "a cube, sides and numbers are equal to squares," or, in modern notation, $x^3 + cx + d = bx^2$.

# Khayyam's cubic equation and intersection of conics manuscript



https://digitaleditions.sheridan.com/publication/?i=672591&article_id=3759503&view=articleBrowser
manuscript kept in Tehran University.

**For Khayyam, numbers could only be positive, which meant there were 19 different types of cubic equations with positive coefficients and solutions containing absolute numbers, sides, squares, and cubes. (Al-Khwārizmī similarly considered only positive coefficients for quadratics.)**

**List some of these types, ensuring the coefficient of the cubic term is 1.**

**Example: $x^3 + c\,x^2 = ax + b$**

There is some evidence suggesting that Khayyam recognized that certain cubic equations would have more than one positive real solution, while other cubic equations would have no positive real solutions

| Equation form | Translation of equation | Method of solution |
|---|---|---|
| $x^3 = c$ | Cube equals number | Cube root c |
| $x^3 = bx$ | Cube equals sides | Reduce to $x^2 = b$ |
| $x^3 = ax^2$ | Cube equals square | Reduce to $x = a$ |
| $x^3 + ax^2 = bx$ | Cube and square equal sides | Reduce to $x^2 + ax = b$ |
| $x^3 = ax^2 + bx$ | Cube equals square and sides | Reduce to $x^2 = ax + b$ |
| $x^3 + bx = ax^2$ | Cube and sides equal square | Reduce to $x^2 + b = ax$ |
| $x^3 + bx = c$ | Cube and sides equal number | Parabola and semicircle |
| $x^3 + c = bx$ | Cube and number equal sides | Parabola and hyperbola |
| $x^3 + c = ax^2$ | Cube and number equal square | Parabola and hyperbola |
| $x^3 + ax^2 = c$ | Cube and square equal number | Parabola and hyperbola |
| $x^3 = ax^2 + c$ | Cube equals square and number | Parabola and hyperbola |
| $x^3 = bx + c$ | Cube equals sides and number | Parabola and hyperbola |
| $x^3 + ax^2 = bx + c$ | Cube and square equal sides and number | Hyperbola and hyperbola |
| $x^3 + c = ax^2 + bx$ | Cube and number equal square and sides | Hyperbola and hyperbola |
| $x^3 = ax^2 + bx + c$ | Cube equals square, sides and number | Hyperbola and hyperbola |
| $x^3 + ax^2 + c = bx$ | Cube, square and number equal sides | Hyperbola and semicircle |
| $x^3 + ax^2 + bx = c$ | Cube, square, and sides equal number | Hyperbola and semicircle |
| $x^3 + bx = ax^2 + c$ | Cube and sides equal square and number | Hyperbola and circle |
| $x^3 + bx + c = ax^2$ | Cube, sides, and number equal square | Hyperbola and circle |

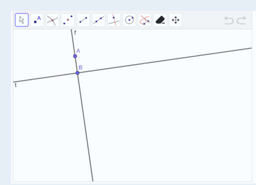
**Join the lesson at www.geogebra.org/classroom with the code:**

**ECJE EDPN**

Or you can also share the following link with your students:

www.geogebra.org/classroom/ecjeedpn

https://www.geogebra.org/classroom/ecjeedpn



https://www.geogebra.org/m/ua9j88ty

1. We have line **f** through **A** and **B**. and line **t** through **B**, perpendicular to **f** and point **C** on line **f**, below **B**.
2. Draw a circle with diameter **AC** (find midpoint with the midpoint tool).
3. Let **E** and **F** be the intersection points of this circle with line **t**.
4. Draw the line through **E** perpendicular to **t**.
5. Draw the line through **C** perpendicular to **f**.
6. Let **G** be their intersection.
7. Use the **locus tool**: select **G**, then **C**, to find the locus of G as C moves.
8. .Repeat step 9 with C on the other side of line **f**.
9. Answer the Wooclap question: what type of curve is the locus?

Draw line **f** through **A** and **B**.

Draw line **t** through **B**, perpendicular to **f**.

Place point **C** on line **f**, below **B**.

Draw a circle with diameter **AC** (find midpoint with the midpoint tool).

Let **E** and **F** be the intersection points of this circle with line **t**.

Draw the line through **E** perpendicular to **t**.

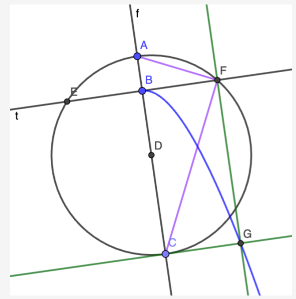
Draw the line through **C** perpendicular to **f**.

Let **G** be their intersection.

Use the **locus tool**: select **G**, then **C**, to find the locus of G as C moves.

Repeat step 9 with C on the other side of line **f**.

Answer the Wooclap questions.



AB/BF = BF/BC

Set

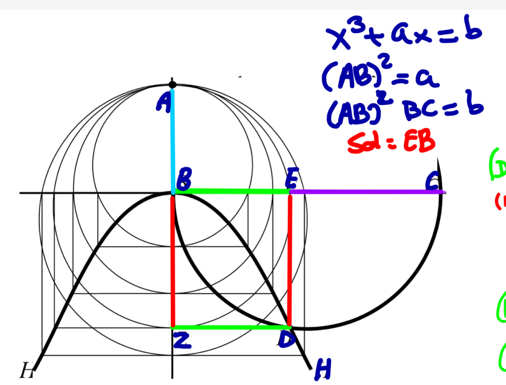
- $B = (0, 0)$
- $A = (0, a)$ ($a > 0$ fixed)
- $C = (0, y)$ ($y$ varies as C moves)
- $F = (x, 0)$ = intersection of the circle with diameter AC and the x-axis
- $G = (x, y)$ (same x as F, same y as C)

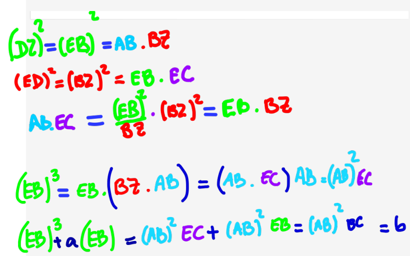
On the circle with diameter AC the right triangle at F gives

- $x^2 = BF^2 = AB \cdot BC = a \cdot y$
- Hence $x^2 = a y$ and $G = (x, (1/a) x^2)$

The locus of **G** is a **parabola** with vertex at **B** and axis the line **f**.

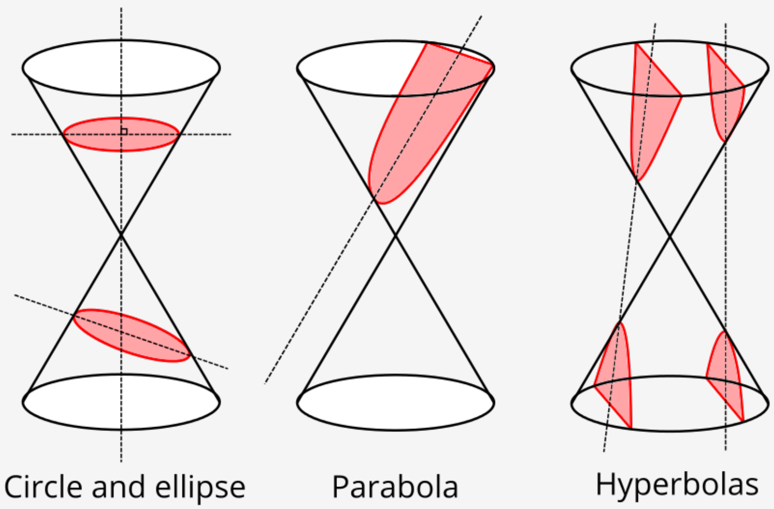
Intersecting this parabola with the circle built from the coefficients of the cubic yields a real root: the segment EB.



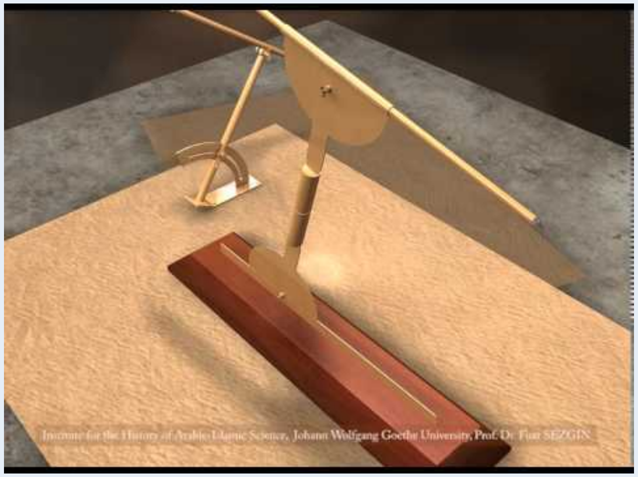
# Perfect compass

## Conic Sections



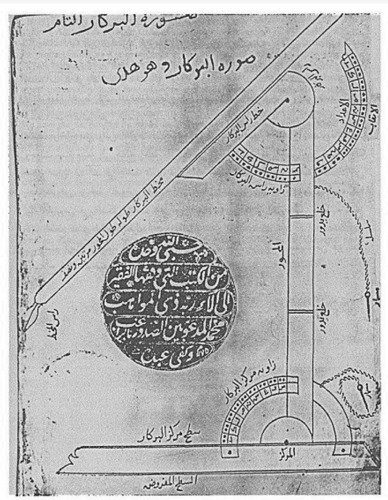



## Why was this compass capable of drawing conic sections?

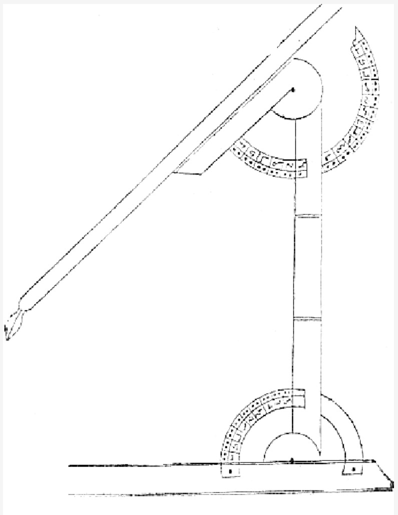


https://www.youtube.com/watch?v=lQp-W_Lt_gE

## Compass for Drawing Conic Sections



*Kitāb al-Qūhī fī al-birkār al-tāmm*, MS Istanbul, Raghib Pasha 569, fol. 235v.



## Galileo Compass