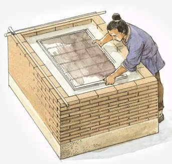
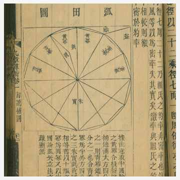
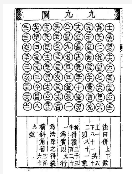
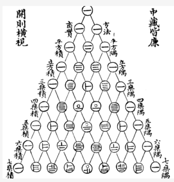
Ancient and Medieval Chinese Mathematics

1. Overview: China, India and the Islamic World
2. Counting rods and rod numerals
3. The Nine Chapters of the Mathematical Art (ex. Systems of linear equations)
4. Lui Hui on the volume of the pyramid- Areas and Volumes, scissor congruence, Hilbert 3rd problem
5. Liu Hui on the volume of the sphere
6. Zu Chingzhi and Zu Geng on the volume of the sphere.
7. Approximation of π
8. Gougu Theorem
9. Timeline summary

China, India, and the Islamic World

China, India, and Islamic World

Difficulties Studying Math History

- Large (in space and time).
- Not homogeneous (for instance, different languages)
- Western centered approach
  - tendency to see them as alien or exotic
  - accounts from historians or writers with a variety of agendas.
- Lack of documents (destroyed by climate, wars, fires, and people)

China, India, and Islamic World

Some reasons for doing mathematics

- Astronomy - including computations of the calendar, astrology and cosmology.)
- Religious: Calculating the direction of Mecca for the Islamic world, constructing altars in India.
- Measuring time.
- Land surveying
- Estimating areas and volumes
- Taxation and division of states.
- Teaching numeracy to an elite.
- Math for the sake of it.

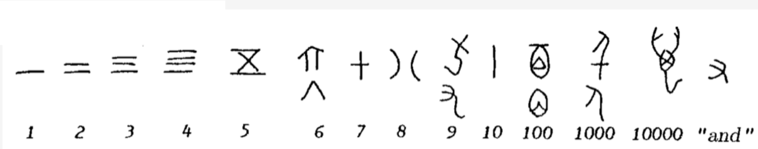
Who did math professionally in Antiquity? Priests? Scribes? Wealthy people? Bureaucrats? Somebody else? Who? (Answer the question for one or more of the following societies)

- In Egypt?
- In Babylonia?
- In the Hellenic world?
- In China?

Who did math professionally in Antiquity? Priests? Scribes? Wealthy people? Bureaucrats? Somebody else? Who? (Answer the question for one or more of the following societies)

- Egypt
  - Scribes and priest-scribes (bureaucratic administrators)
  - **Greek point of view:** “Thus the mathematical sciences originated in the neighborhood of Egypt, because there the priestly class was allowed leisure.” Aristotle (Metaphysics):
- Babylonia
  - Scribes and scholar-scribes in temple, palace, and scribal-school contexts
- Hellenic world
  - Scholars, teachers, philosophers, astronomers, engineers, surveyors, and other technical specialists
- China
  - State officials and technical specialists in the imperial bureaucracy, especially in astronomical and calendrical offices.

Oracle bone (~1200 BCE)
Numbers between 1 to 30,000.
Oldest extant Chinese numerals



From A history of Chinese mathematics Jean-Claude Martzloff copy

Inscribed tortoise carapace (“oracle bone”), Anyang period, late Shang dynasty, c. 1300–1050 B.C.E., tortoise shell, China, 6.5 high x 10.8 x 2.3 cm - Smithsonian Institution, Washington, D.C.

Rod numerals and counting boards

Complete the table using the hints (row a = 72, row b = 26). What are the numbers in rows c, d, and e?

Rod numerals were used (approximately) between 500 BCE and 1500 CE.

𝍯 𝍡	72	a
𝍪 𝍥	26	b
𝍬 𝍡		c
𝍣𝍩𝍡		d
𝍣 𝍡		e

Counting rods and counting boards

1	2	3	4	5	6	7	8	9
𝍠	𝍡	𝍢	𝍣	𝍤	𝍥	𝍦	𝍧	𝍨
				or				
𝍩	𝍪	𝍫	𝍬	𝍭	𝍮	𝍯	𝍰	𝍱

Rod numerals were used in China for over 2000 years. This number system was:
a) Additive b) Positional
c) Multiplicative d) Ciphered

𝍯 𝍡	72	a
𝍪 𝍥	26	b
𝍬 𝍡	42	c
𝍣𝍩𝍡	412	d
𝍣 𝍡	402	e

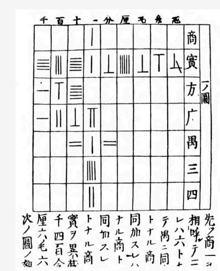
Rod number system

- 475-221 B.C. A **base ten positional number system** was in common use in China
- Calculations were done using rods made from bone or bamboo, on a **counting board**.
- The numerals from 1 to 9 were represented by rods.
- Rods representing odd powers of ten were rotated 900 for clarity in distinguishing the powers.
- A zero was represented by a blank square, called a **kong**.

𝍯 𝍡	72	a
𝍪 𝍥	26	b
𝍬 𝍡	42	c
𝍣𝍩𝍡	412	d
𝍣 𝍡	402	e

Counting boards

- **400 BCE,**
- Polished wood grids with square cells
- **Positional** number system: Columns = units, tens, hundreds (blank = zero)
- **Advanced set of algorithms** for
  - multiplication, division,
  - computation of square and cubic roots
  - Solving linear equations
  - Solution of higher degrees with multiples unknowns.
- "Some" use of **negative numbers**

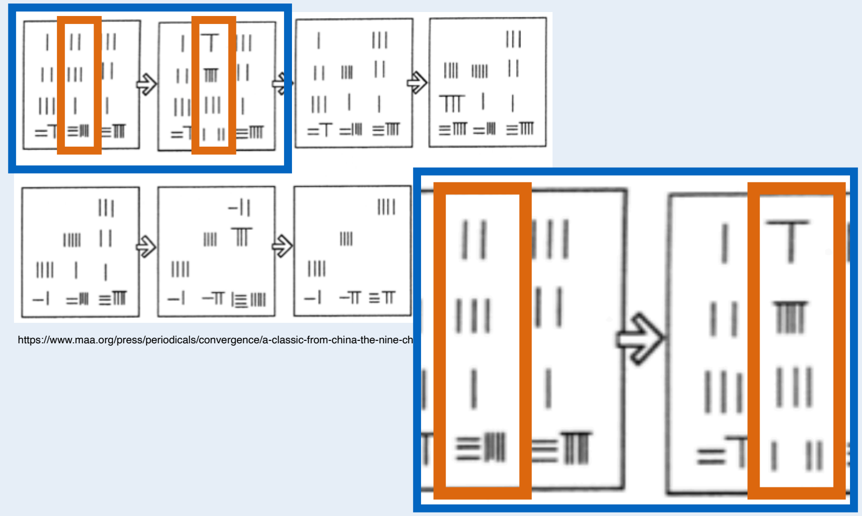


Counting Board (Sangi in Japanese). Source: Smith & Mikami 1914
https://kartsci.org/kocomu/computer-history/history-abacus-ancient-computing/

Counting rods and counting boards

The Chinese counting board consisted on a grid of square cells for storing and manipulating rows and columns of numbers. This same grid concept later appeared in the West under different names. Which of these terms describe such grids? (choose all the appropriate): lattice, matrix, magic square, spreadsheet?

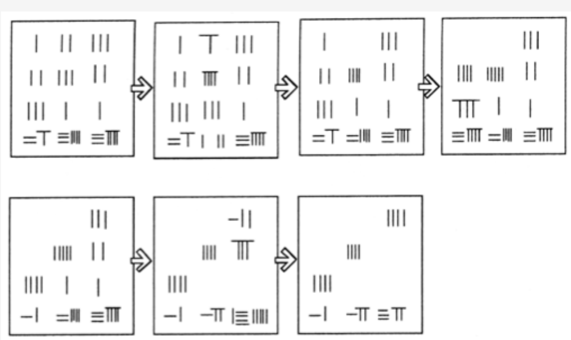
What is the relation between the two highlighted columns?



https://www.maa.org/press/periodicals/convergence/a-classic-from-china-the-nine-ch

"Matrices" in the Nine Chapters

The illustration shows how the above problem **would** be solved on a traditional Chinese counting board.

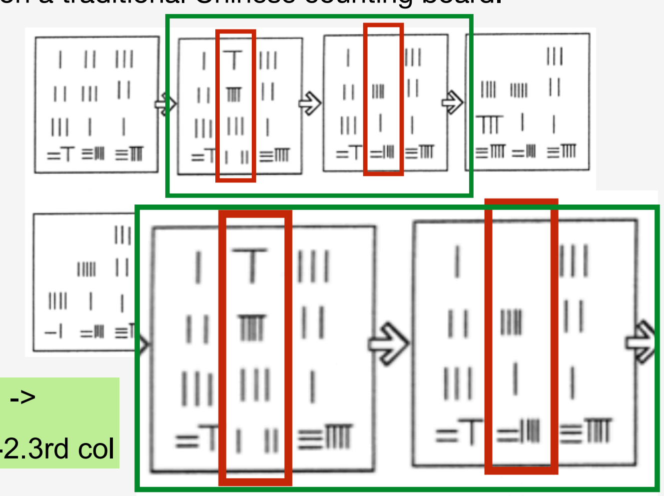


https://www.maa.org/press/periodicals/convergence/a-classic-from-china-the-nine-chapters-matrices

"Matrices" in the Nine Chapters

"Matrices" already appeared in the Nine chapters

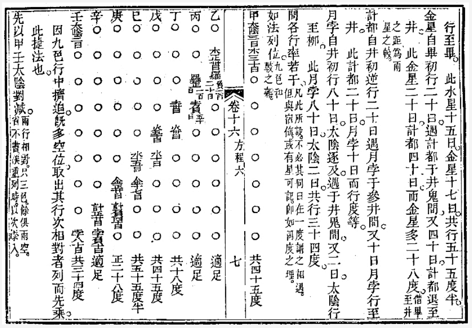
The following illustration shows how the above problem **would** be solved on a traditional Chinese counting board.



2nd col. ->

2nd col-2.3rd col

A 9x9 linear system ~1700

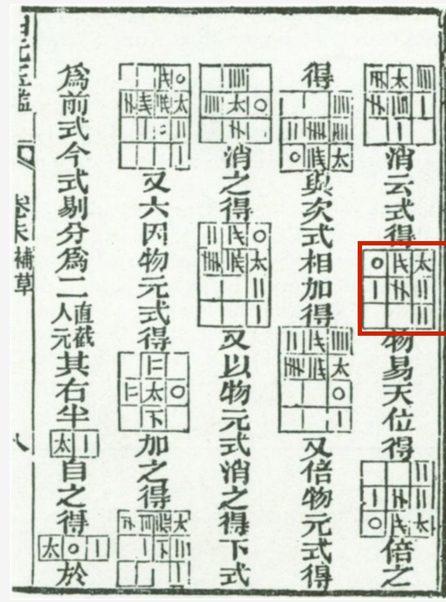


$$\begin{bmatrix} 3 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 45 \\ 0 & 30 & -120 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 30 & 30 & 0 & 0 & 0 & 0 & 18 \\ 0 & 0 & 0 & 0 & 50 & 10 & 0 & 0 & 0 & 45 \\ 0 & 0 & 0 & 0 & 0 & 15 & 17 & 0 & 0 & \frac{111}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 20 & -40 & 0 & 28 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20 & -10 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 80 & 34 \end{bmatrix}$$

Source: http://rhart.org/algebra/

A *fangcheng* problem with 9 conditions in 9 unknowns of the form of the generalized "well problem," from Mei Wending's 梅文鼎 (1633–1721) *On* Fangcheng (*Fangcheng lun* 方程論, c. 1674)

Polynomials Equations - 1303



A page from a 19th century edition of the *Ssu Yuan Yii Chien of* Chu Shih-Chieh **(1303 a.D.)**

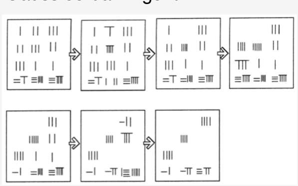
0	−120	·
1	−2	2
		2

$xy^2 - 120y - 2xy + 2x^2 + 2x$

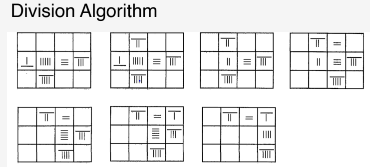
Source: On ancient Chinese mathematics by D. Struik, The Mathematics Teacher , Vol. 56, No. 6 (OCTOBER 1963), pp. 424-432

Examples of Counting Board Uses

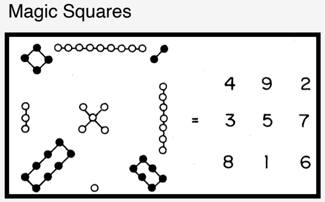
Gauss-Jordan Algorithm



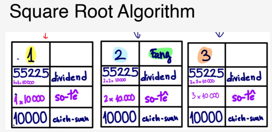
Division Algorithm



Magic Squares



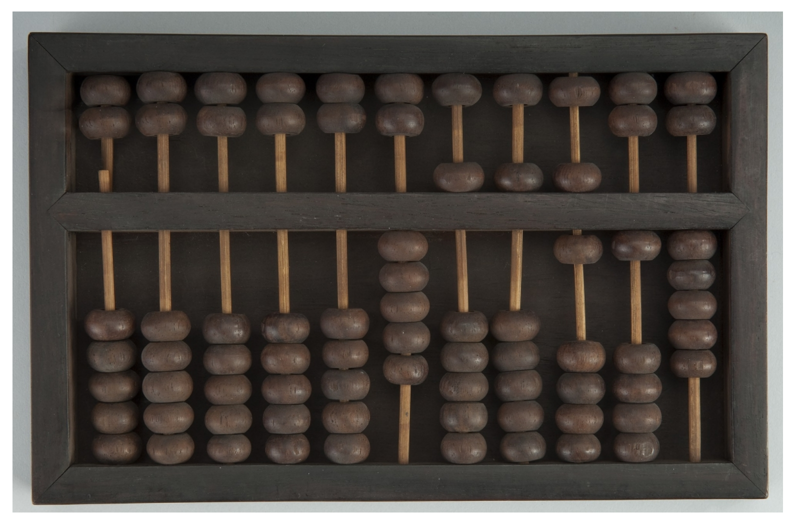
Square Root Algorithm



The Chinese counting board is a good example of how a technological invention can influence how science develops, and even how people think.

Randy K. Schwartz- *A Classic from China: The Nine Chapters -*
https://www.maa.org/press/periodicals/convergence/a-classic-from-china-the-nine-chapters-numbers-and-units

Modern Chinese Abacus



https://old.maa.org/press/periodicals/convergence/mathematical-treasure-modern-chinese-abacus

Nine Chapters of the Mathematical Art

Book of Numbers and Computations

The Book of Numbers and Computations - Suan Shu Shu

1. **235 BCE** (discovered in **1983** in a 186 BCE tomb).
2. Anonymous
3. Earliest known Chinese mathematical text.
4. **original source** (not a copy)
5. Possibly from a variety of sources.
6. 68 problems, order unknown
7. About 200 bamboo strips.
8. **practical mathematics** for administrators: fractions, proportions, wastage, and solid volumes
   - Calculations with fractions and proportions,
   - Rule of False Position
   - Volumes of various solid shapes.
   - tax calculations, grain conversions.

算數書

The Book of Numbers and Computations - Suan Shu Shu

A fox, raccoon, and hound go through customs, and (together) pay tax of 111 qian. The hound says to the raccoon, and the raccoon says to the fox: since your fur is worth twice as much as mine, then the tax you pay should be twice as much! How much should each one pay?

Find an equation to solve this problem.

Bamboo strips - The Book of Numbers and Computations Suan Shu Shu

The Book of Numbers and Computations - Suan Shu Shu

Statement of the problem

A fox, raccoon, and hound go through customs, and (together) pay tax of 111 qian. The hound says to the raccoon, and the raccoon says to the fox: since your fur is worth twice as much as mine, then the tax you pay should be twice as much! How much should each one pay?

The result says: the hound pays 15 6/7 qian, the raccoon pays 31 5/7 qian, and the fox pays 63 and 3/7 qian.

The method says: let each one double the other; adding them together (1 + 2 + 4), 7 is the divisor; taking the tax, multiplying by each (share) is the dividend; dividing the dividend by the divisor gives each one's (share).

The Book of Numbers and Computations - Suan Shu Shu

(The tax on) 3 (square) bu of millet is 1 dou; (on) 4 (square) bu of wheat is 1 dou; (and on) 5 (square) bu of small beans is 1 dou. If the combined tax (on all of them together) is 1 shi (capacity), then how much is the tax (on each one)?

The result says: the tax on millet is 4/12 dou; the tax on wheat is 3 9/47 dou; (and) the tax on beans is 2 26/47 dou.

The meaning of “Put down” here is “Put down on the counting board

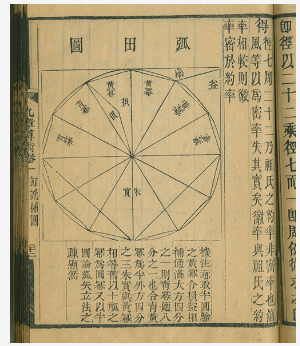
The method says: put down (**on the counting board** the amount of) millet 3 bu, wheat 4 bu, and beans 5 bu; let the product of the millet and wheat be the dividend for the beans; the product of the beans and the millet be the dividend for the wheat; (and the product of the wheat and the beans be the dividend for the millet); for each of the different (amounts) put down (on the counting board) one shi multiplied by each (of the amounts for beans, wheat, and millet) as the dividends; (taking) 47 as the divisor gives the result in dou.

Fundamental source. The Nine Chapters of the Mathematical Arts

Compilation

1. **~200 BCE**
2. Earliest copy made in 1213 CE.
3. From a variety of sources (including Book of Numbers)
4. Practical handbook of mathematics to provide methods for everyday problems of engineering, surveying, trade, and taxation.
5. 246 problems
   a. statement
   b. numerical answer
   c. method to calculate the solution.
6. Organized in 9 chapters by topics (often by nature of the problem).
7. Often sophisticated and difficult in its treatment of algorithms.
8. Later commentators made **important contributions**

Method for approximating π from a sixteenth-century version of **Liu Hui**’s third-century edition and commentary on the Nine Chapters.



Source: Mathematical Treasures - *Jiuzhang suanshu*, MAA Convergence)

The Nine Chapters on the Mathematical Art, around 200BC.

There are three types of grain (millet): best quality, medium quality, and worst quality.

- Three bundles of the best quality, two bundles of medium quality, and one bundle of the worst quality produce 39 units of grain.
- Two bundles of the best quality, three bundles of medium quality, and one bundle of the worst quality produce 34 units of grain.
- One bundle of the best quality, two bundles of medium quality, and three bundles of the worst quality produce 26 units of grain.

Question: How much grain does one bundle of each type produce?

3x + 2y + 1z = 39
2x + 3y + 1z = 34
12 + 2y + 3z = 26

How would you solve this problem?

Table 1

	Suan Shu Shu	Nine Chapters on the Mathematical Art
Date (approx.)	3rd entury BCE	1st century BCE
Number	68 problems	246 problems
Medium	190-200 bamboo strips	Text on silk/book form (later)
Organization / content	No chapters, loosely grouped by topic, order uncertain. Practical problems: fractions, proportions, taxation, volumes, grain conversion	Divided into 9 organized chapters by topic: arithmetic, algebra, geometry, proportions, root extraction, volumes, linear systems, right triangles
Level	Basic/proto-algebraic; lacks advanced matrix or geometric algorithms	More advanced: includes matrix algebra and Pythagorean theorem applications
Influence	Practical handbook for officials, possibly source for later texts	**Enormous influence**, basis for Chinese mathematical tradition
Known Authors	Anonymous (scribal names appear in places)	Traditionally attributed to Zhang Cang, Geng Shouchang; major commentary by Liu Hui

The Nine Chapters on the Mathematical Art, around 200BC.

Liu Hui Commentary (~200 CE): Things from different groups are put together in wholes that are mixed up. For each type, in a row, there are quantities, and one expresses their products/dividends [that is, their constant term] globally. One does it in such a way that each column is made of lu. If there are two things, one measures [each] twice, if there are three things; one measures [each] thrice; all types of things] are measured [as many times as] the number of things. One brings rows together to make columns, this is why one calls this [procedure] 'measures in square. To the right and to the left of a column, no [other column] exists that is the same. Moreover, one ensures that they are formulated only if there is something on which to ground them. **This is a universal procedure. However, with abstract formulations it would be difficult to understand them. This is why the [authors] linked it on purpose to millet to eliminate the obstacle.** Further, one lays down the middle and the left columns like the right one.

Chinese mathematicians were clearly concerned about justifying their methods and establishing the validity of their results. Their proofs were not axiomatic proofs, but they were proofs nevertheless, and they were clearly able to establish the truth of correctness of the solutions they proffered. Joseph Dauben

Liu Hui
(~250)
劉徽

Liu Hui (~250 CE)

- Outstanding and original mathematician with a deep understanding of difficult concepts.
- Familiar with the literary and historical classics of China.
- Never claimed results of which he was not fully confident. He wrote:-

  Let us leave the problem to whoever can tell the truth.
- Cared about the conditions of people and about the economy of the country

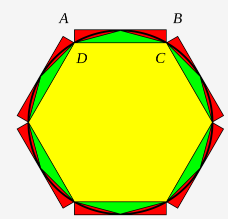


Diagram of Liu Hui's π inequality
Source: derivative work: Pbroks13 (talk)
Gisling - Wikimedia Commons

Worked on:
- Volumes of plane and solid figures.
- Solution of linear equation with two unknowns.
- Algorithm to compute π.

https://mathshistory.st-andrews.ac.uk/Biographies/Liu_Hui/

Liu Hui Introduction to his commentary of the Nine Chapters

I read the Nine Chapters as a boy, and studied it in full detail when I was older. [I] observed the division between the dual natures of Yin and Yang [the positive and negative aspects] which sum up the fundamentals of mathematics. **Thorough investigation shows the truth therein, which allows me to collect my ideas and take the liberty of commenting on it.** Things are known to belong to various classifications. Just as the branches of a tree are to its trunk, so are a multitude of things to an archetype. Therefore I have tried to explain the whole theory as concisely as possible, with spatial forms shown in diagrams, so that the reader should have a reasonably good all-around understanding of it.

Liu Hui Introduction to his commentary of the Nine Chapters

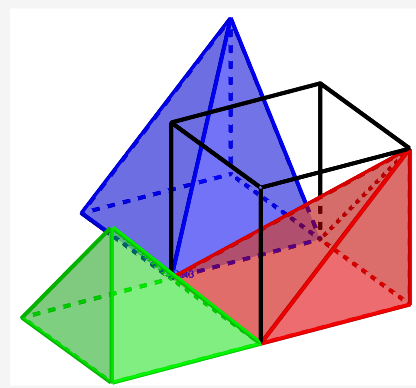
Some of the material in the Nine Chapters predates the great book-burning and burial-alive of scholars of 213 B.C., ordered by emperor ShihHuang-ti of the Qin dynasty. Indeed, Liu Hui writes in the preface of his commentary:

In the past, the tyrant Qin burnt written documents, which led to the destruction of classical knowledge ... Because of the state of deterioration of the ancient texts, Zhang Cang and his team produced a new version ...filling in what was missing

Liu Hui and the First Golden Age of Chinese Mathematics, Philip D. Straffin, Jr.,Mathematics Magazine, Jun., 1998, Vol. 71, No. 3 (Jun., 1998)

Liu Hui on the volume of the pyramid

Cube dissected into Yangmas - Square faces marked



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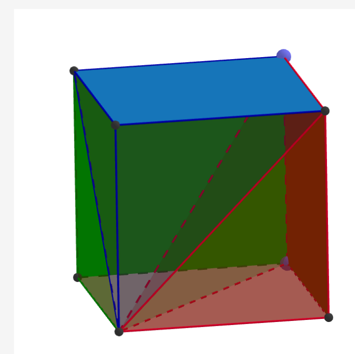

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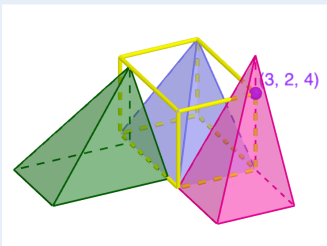
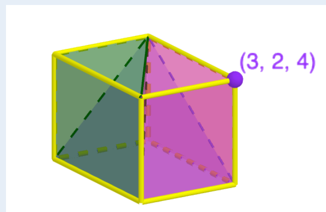
Liu Hui on the volume of yangma (rectangular pyramid)

A ***yangma*** is a
- pyramid with rectangular base and
- one of its lateral edges perpendicular to the base

Liu Hui (among many other topics) studied the volume of the Yangma.

Are these three yangmas congruent? Educated guess: Do they have the same volume?





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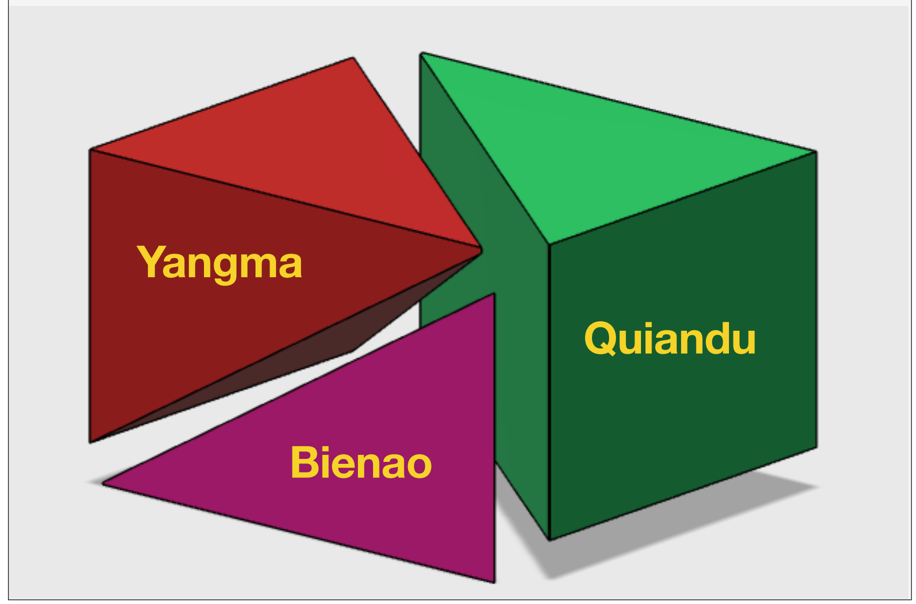
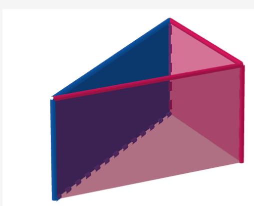
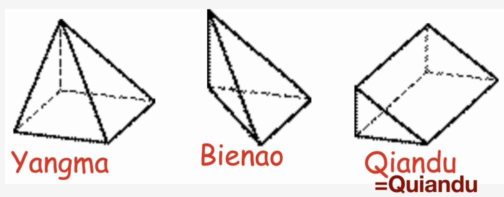
Liu Hui dissection of the rectangular prism



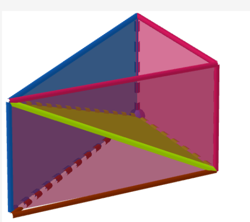
Liu Hui Components for the dissection of the rectangular prism

Quiandu

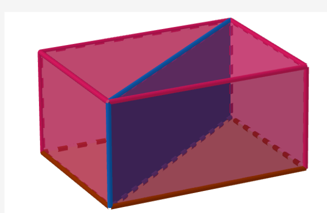




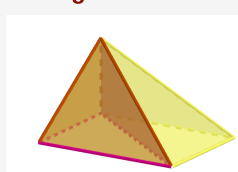
Quiandu = Yangma+ Bienao



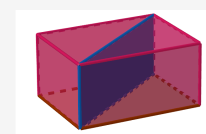
Rectangular Prism = 2Quiandu



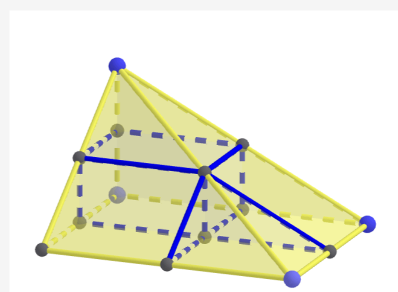
Yangma=2Bienao



Midpoints -Yangma and Bienao
What are the parts?



Rectangular Prism = 2Quiandu



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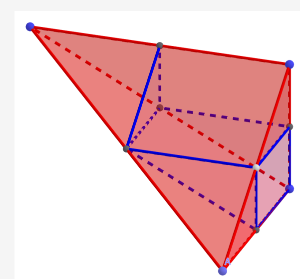

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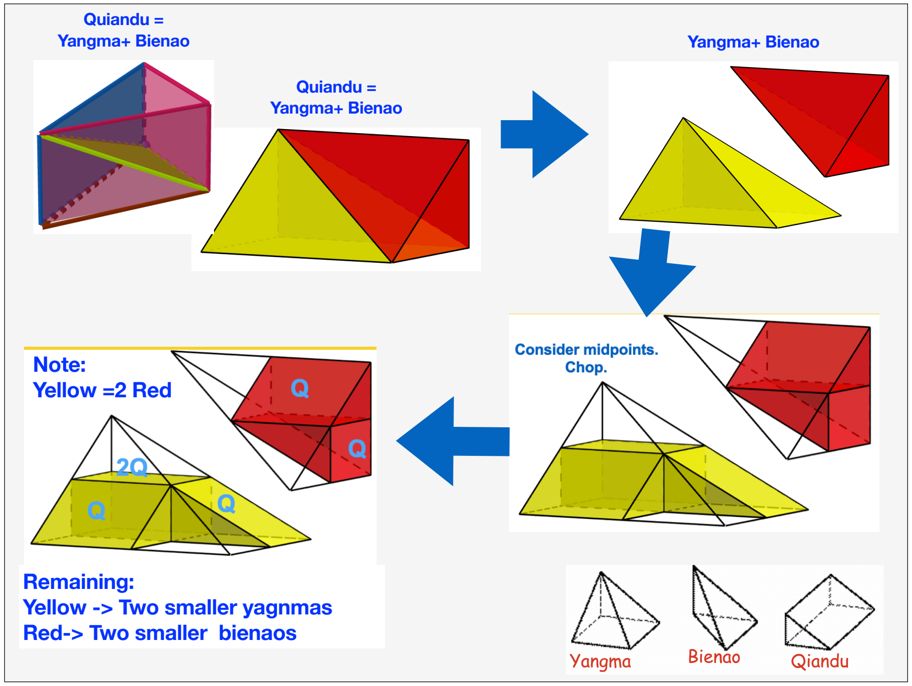
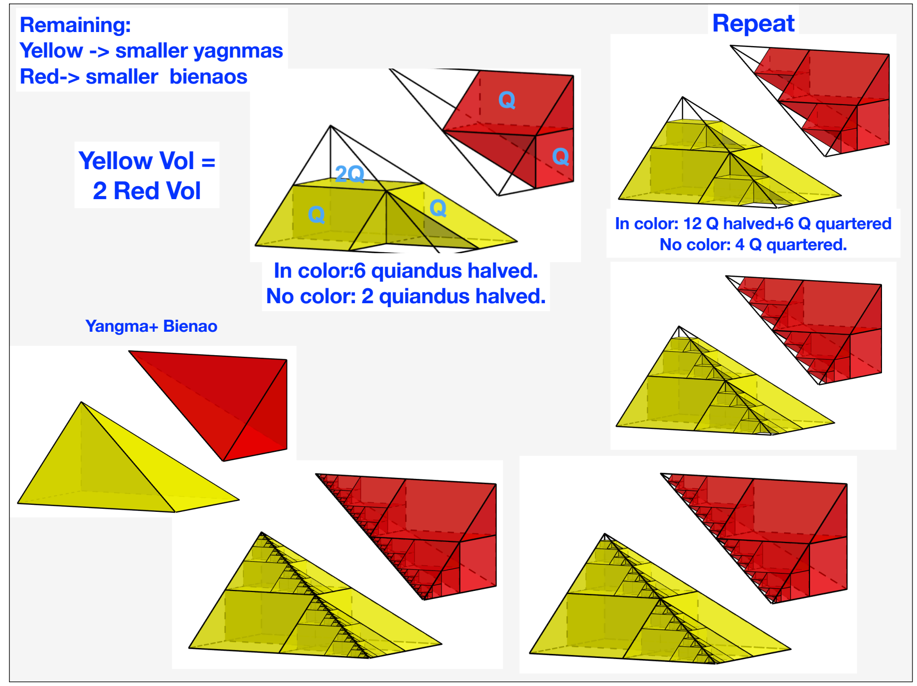
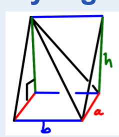
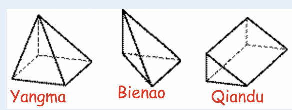




Liu Hui on the volume of yangma (rectangular pyramid)

Liu Hui showed that the volume of a yangma is **(1/3)a.b.h** , where **a** and **b** are the sides of the rectangular base and **h** is the height.

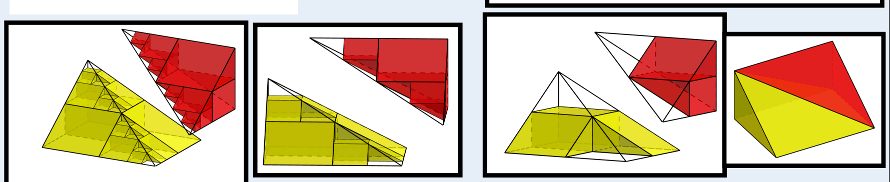




This proof is from ~250 CE.What does this proof suggest to you about early Chinese mathematical thinking?

https://www.geogebra.org/m/vwvuznur

Start: Red Volume =(1/2)Yellow volume. The "leftover" (with no color) gets smaller and smaller, so red and yellow fill more and more.
Limit: Red Volume =(1/2)Yellow volume.
Therefore, Red=(1/3) of qiandu and Yellow=(2/3) of qiandu



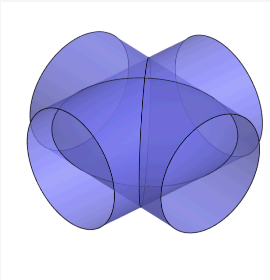
Vol(Quiandu)=(1/2)Vol(Rect. prism)=(1/2)a.b.h=(3/2)Vol(Yangma)
Then **Vol(Yangma)=(1/3)a.b.h**

http://donwagner.dk/Pyramid/Pyramid-4.html

Liu Hui on the volume of the sphere

Liu Hui on the Volume of a Sphere

Box Lid - Bicylinder - Mouhefanggai - 牟合方蓋



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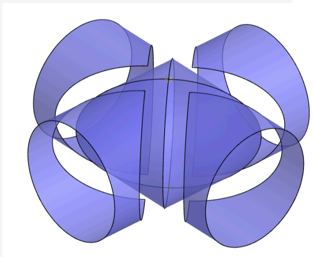
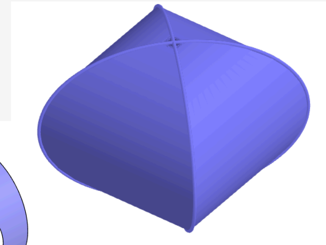




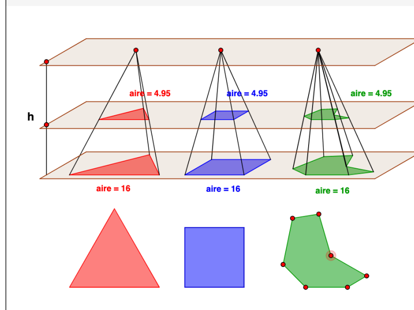
Liu -200 CE- discovered the Mouhefanggai (牟合方蓋面面觀) -the solid formed by two square umbrellas

Zu Geng's (~450 CE) statement

疊棋成立積
緣冪勢既同
則積不容異

If blocks are piled up to form volumes,
And corresponding areas are equal,
Then the volumes cannot be unequal

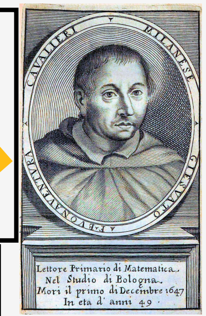
Source: http://donwagner.dk/SPHERE/SPHERE.html



Source: http://donwagner.dk/SPHERE/SPHERE.html

This is called now the "**Cavalieri principle**" ~1600

Is it fair?

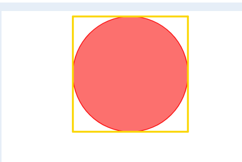


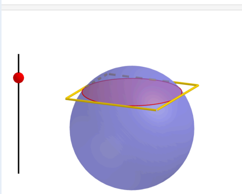
Trattato della sfera e prattiche per vso di essa, Roma, 1682

Liu Hui (~200 CE) used this principle before Zu Geng's statement

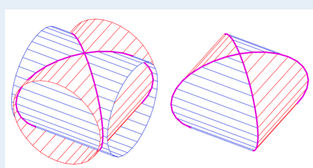
Recalll Archimedes!

M = volume of the Mouhefanggai 牟合方蓋
S =volume of the sphere (S)
Find M/S





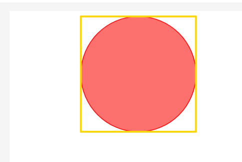
https://www.geogebra.org/m/pfj8sd75

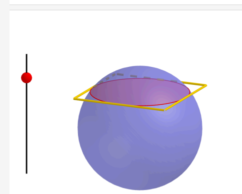


By Ag2gaeh - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=63549042

Yellow -> Mouhefanggai (box lid)

M = volume of the Mouhefanggai 牟合方蓋
S = volume of the sphere
Find M/S





https://www.geogebra.org/m/pfj8sd75

By "sections" principle (or Cavalieri)

Since the ratio of all slices is 4/π, the ratio of the volume is 4/π. That is,

$$M/S = 4/\pi$$

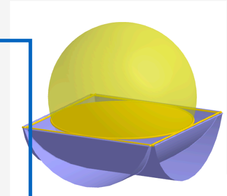
Yellow -> Mouhefanggai (box lid)

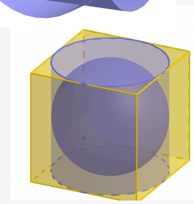
Summary: Liu Hui and the volume of the sphere

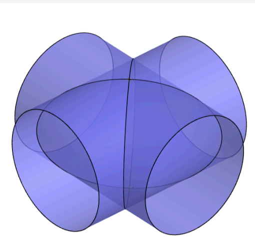
Liu -200 CE- showed that the relation

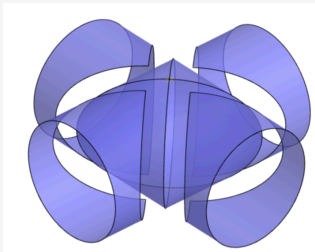
Vol(cylinder)/Vol(sphere)= $4/\pi$

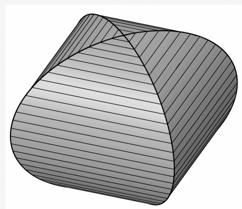
does not hold. The Mouhefanggai "squeezes" the sphere more than the cylinder.











Math Poem

Liu Hui concludes with the following
The geometer's frustration by Liu Hui

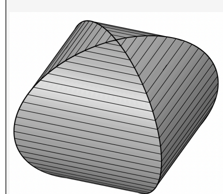
Look inside the cube
And outside the box-lid;
Though the diminution increases,
It doesn't quite fit.
The marriage preparations are complete;
But square and circle wrangle,
Thick and thin make treacherous plots,
They are incompatible.
I wish to give my humble reflections,
But fear that I will miss the correct principle;
I dare to let the doubtful points stand,
Waiting for one who can expound them.

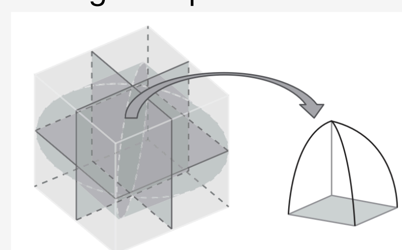
Source: http://donwagner.dk/SPHERE/SPHERE.html

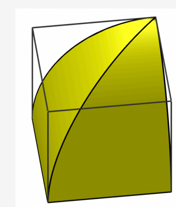
Zu Gengzhi (Zu Geng, 5th c.).on the Volume of a Sphere

Zu Geng on the Volume of a Sphere

Divide M(r) into eight congruent pieces.





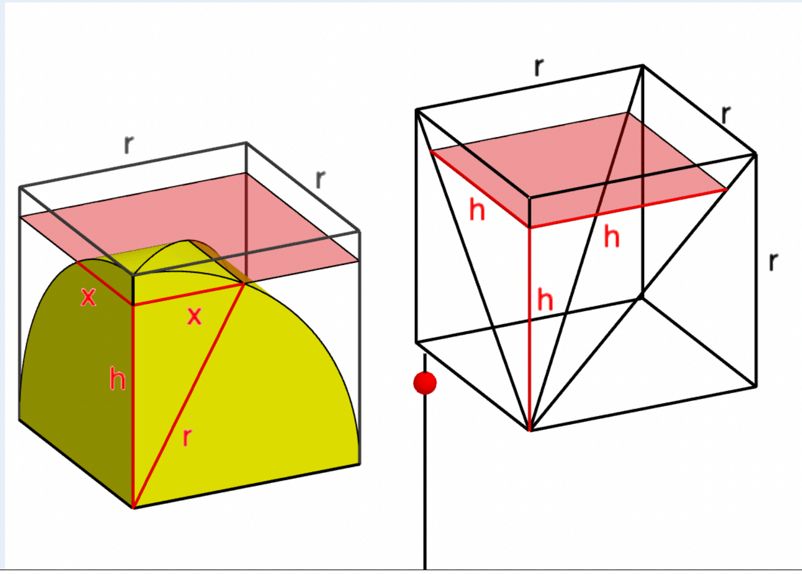


Each of the eight blocks is "like" a rounded *yangma* and "fits" in C(r).

https://www.geogebra.org/m/bprtgwh8#material/gevzbyfd
https://www.geogebra.org/m/bprtgwh8

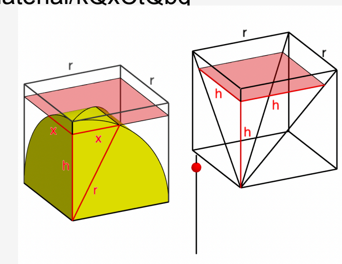
Compute red areas in terms of r and h.

https://www.geogebra.org/m/ygrrfnsw



Zu Geng on the Volume of a Sphere

https://www.geogebra.org/m/nkdqansv#material/kQxCtQbq



Cube(r) denotes cube of side r

Pyr(r) denotes slanted pyramid in cube of side r.

Sph(r) sphere of radius r

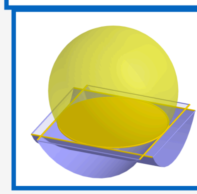
M(r) Mouhefangai around S(r)

Volume(M(r))/8 = Vol(Cube(r)) - Vol(Pyr(r)) = (2/3)Vol(Cube(r))

Volume(M(r)) = = (16/3)Vol(C(r))

Zu Geng on the Volume of a Sphere

$$\frac{\text{Volume(S(r))}}{\text{Volume(M(r))}} = \pi/4$$



Liu Hui

C(r) = cube of side r

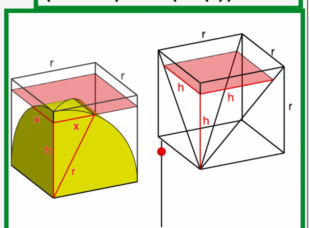
P(r) = slanted pyramid in cube of side r.

S(r) = sphere of radius r

M(r)= Mouhefangai enclosing S(r)

Volume(S(r)) = (π/4) Volume(M(r))= = (π/4) (16/3)Vol(C(r)) = (4/3)π r^3

Volume(M(r)) = (16/3)Vol(C(r))



Math Poems

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The marriage preparations are complete;
But square and circle wrangle,
Thick and thin make treacherous plots,
They are incompatible.
I wish to give my humble reflections,
But fear that I will miss the correct principle;
I dare to let the doubtful points stand,
Waiting for one who can expound them.

The geometers triumph
by Zu Gengzhi

The proportions are extremely precise
And my heart shines.
Zhang Heng copied the ancient,
Smiling on posterity;
Liu Hui followed the ancient,
Having no time to revise it.
Now what is so difficult about it?
One need only think.

Source: http://donwagner.dk/SPHERE/SPHERE.html

Answer one or more.

- 1. Rod Numerals and Place Value. What advantages did the rod numeral system offer for computation? Give a specific example.
- 2. Nine Chapters. The Nine Chapters solved systems of linear equations algorithmically. Why might this approach have developed in China?
- 3. Liu Hui's pyramid proof (~250 CE). What does this proof suggest about Chinese mathematical thinking? What is the meaning of the word proof in this case?
- 4. Chinese and Greek Mathematics. Based on what we covered, how would you characterize the differences between Chinese and Greek mathematical approaches?

Computation of π by Liu Hui and Zu Chongzhi

Computation of π by Liu Hui and Zu Chongzhi

Liu Hui approximation of π (**~200 CE**)

- Observed that the ratio of the circumference of an inscribed hexagon to the diameter of the circle was three, so π must be greater than 3.
- Using Gougu (Pythagorean) theorem , computed the **perimeter** of regular polygons of 6,12, 24, ..., **1536**. (3.2, 3.2^2, 3.2^3,… 3.2^9) sides to approximate the circumference.

Mathematician Zu Chongzhi (**~500CE**)

- same method as Liu Hui
- used a regular polygons of 6,12, 24, ..., **3072**. (3.2, 3.2^2, 3.2^3,…, 3.2^{10}) sides

Page from a sixteenth century edition of the Nine Chapters on the Mathematical Art.

https://www.maa.org/press/periodicals/convergence/mathematical-treasures-jiuzhang-suanshu

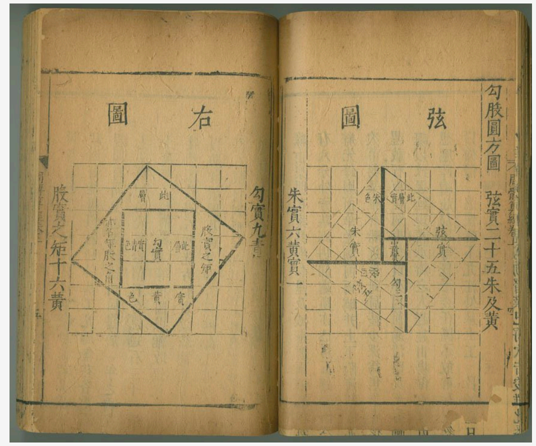
- Calculated with counting rods
- Duplicating the number of sides
- Gougu (Pythagorean) theorem

Date (approx.)	Author / Source	Method	Polygon (sides = $2^n \cdot 3$)	Sphere / Cone / Cylinder (treatment)
1st c. BCE	Nine Chapters on the Mathematical Art	Rule of thumb “3” (implicit π)	—	Same π used in circle area; cylinder and cone volumes use base area with π=3; no separate sphere treatment here
ca. 250 CE	Liu Hui (Commentary on Nine Chapters)	Inscribed polygon; double sides; compute perimeters; compare C/D; then apply same π to area/volume	6 → 12 → 24 → 48 → 96 → 192 → 384 → 768 → 1536 ($2^9 \cdot 3$)	Sphere volume via cross-section comparison with cylinder+cone (“box-lid”); cone and cylinder justified; same π across circle/sphere/cone/cylinder
ca. 480 CE	Zu Chongzhi (independent works; Zhui Shu lost)	Further polygon refinement; selects superior rationals (e.g., 22/7, 355/113)	Likely ≥ $2^{10} \cdot 3$ (exact max not securely recorded)	Uses same π framework for spherical volume; the explicit Cavalieri-type statement is due to Zu Geng(zhì) (his son)
7th c. CE → later	Later Chinese practical texts (engineering/ astronomy)	Rule of thumb √10 (convenience)	—	Same constant used across circle area and circular volumes for fieldwork (not a proof tradition)

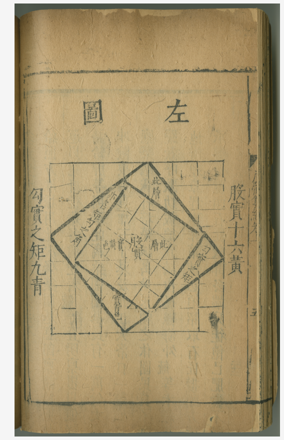
Gougu theorem

Pythagorean or Gou-Gu Theorem in

Zhoubi suanjing - Arithmetical Classic of the Gnomon and the Circular Paths of Heaven - book on astronomy and mathematics ~ 100 BCE.



https://maa.org/press/periodicals/convergence/mathematical-treasures-zhoubi-suanjing



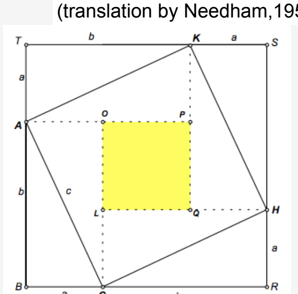
https://maa.org/press/periodicals/convergence/mathematical-treasures-zhoubi-suanjing

Pythagorean or *Gou-Gu* Theorem in the Zhoubi suanjing (The arithmetic classic of the gnomon and the circular paths of heaven

Let us cut a rectangle (diagonally), and make the width 3 (units) wide, and the length 4 (units) long. The diagonal between the (two) corners will then be 5 (units) long. Now, after drawing a square on this diagonal, circumscribe it by half-rectangles like that which has been left outside, so as to form a (square) plate. Thus the (four) outer half-rectangles, of width 3, length 4 and diagonal 5, together make two rectangles (of area 24); then (when this is subtracted from the square plate of area 49) the remainder is of area 25. This (process) is called "piling up the rectangles." Ji ju

1st century BC

(translation by Needham,1959)



https://cd1.edb.hkedcity.net/cd/maths/en/ref_res/material/MSS_e/Exemp21.pdf

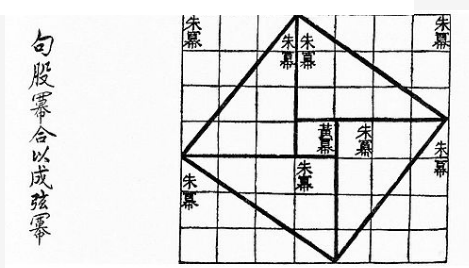


Diagram added by Zhao Shuang to the Zhoubi Suanjing that can be used to prove the Pythagorean Theorem

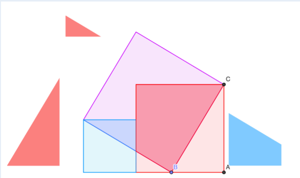
https://en.wikipedia.org/wiki/Zhoubi_Suanjing#Background_behind_Pythagorean_derivation

Chapter 9 of the Nine Chapters: Gou-gu 勾股

- States that on a right triangle, the square on the hypotenuse equals the sum of the squares on the other two sides (the Pythagorean theorem).
- Gives many practical surveying problems using that rule.
- Liu Hui gives a verbal dissection argument, explaining how rearranging areas demonstrates the equality of the squares.

Reconstruction of Liu Hui visual proof of Pythagorean Theorem

Note: The diagram for Liu Hui's proof is lost.



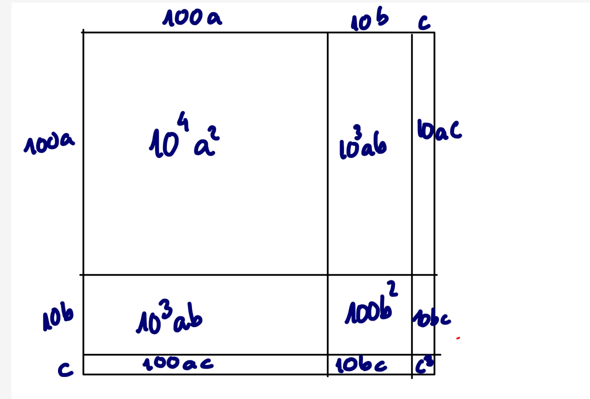
https://www.geogebra.org/m/zzkst6ep

https://www.geogebra.org/m/zzkst6ep

In Liu Hui's visual proof, the colored pieces can be rearranged to cover the regions outside the overlapping area—either outside the large square or outside the two smaller squares

Why does the fact that the pieces can be rearranged this way prove the theorem — and not just illustrate it? Does your answer change if you can do this for any right triangle?

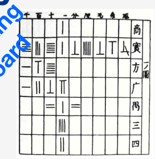
What is the square root of 65,536?



Timeline, summary

Rough Chronology Math in Ancient China

The Book of Numbers and Computations - Suan Shu Shu



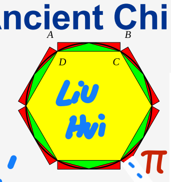
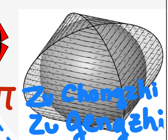


Diagram of Liu Hui's π inequality
Source: derivative work: Pbroks13 (talk) Gisling - Wikimedia Commons

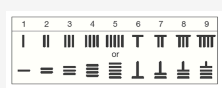


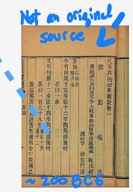
1000BCE

Numerals from oracle bones, from 11-12th century BCE (Numbers between 1 to 30,000)

〇	一	二	三	四	五	六	七	八	九
0	1	2	3	4	5	6	7	8	9
十	廿	卅	百	千	万	亿	兆		
10	20	30	100	1000	10000	10^8	10^{12}		

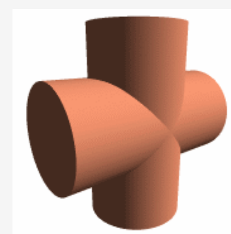
Rod Numerals



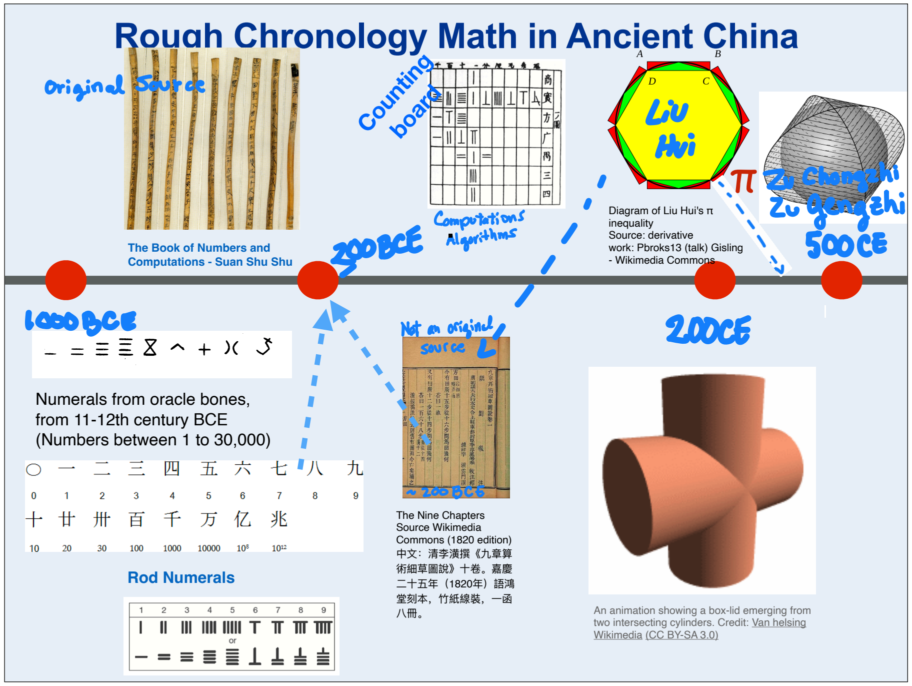


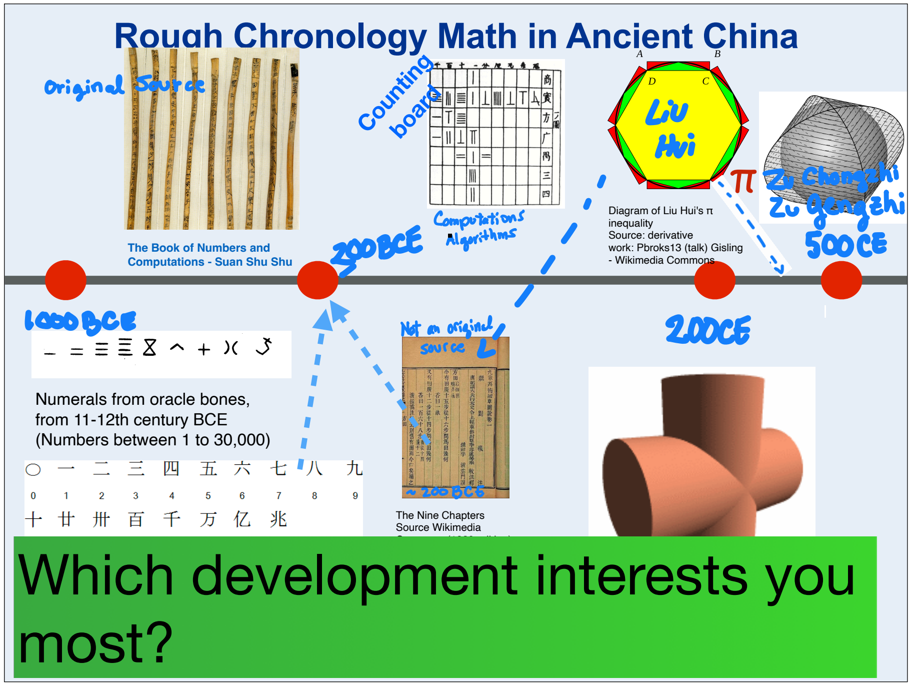
The Nine Chapters
Source Wikimedia Commons (1820 edition)
中文：清李潢撰《九章算術細草圖說》十卷。嘉慶二十五年（1820年）語鴻堂刻本，竹紙線裝，一函八冊。

200CE



An animation showing a box-lid emerging from two intersecting cylinders. Credit: Van helsing Wikimedia (CC BY-SA 3.0)


Rough Chronology Math in Ancient China
Original Source
The Book of Numbers and Computations - Suan Shu Shu
Counting board
Computations Algorithms
200 BCE
Liu Hui
π
Diagram of Liu Hui's π inequality
Source: derivative work: Pbroks13 (talk) Gisling - Wikimedia Commons
Zu Chongzhi
Zu Gengzhi
500 CE
1000 BCE
Numerals from oracle bones, from 11-12th century BCE (Numbers between 1 to 30,000)
〇 一 二 三 四 五 六 七 八 九
0 1 2 3 4 5 6 7 8 9
十 廿 卅 百 千 万 亿 兆
10 20 30 100 1000 10000
Rod Numerals
1 2 3 4 5 6 7 8 9
or
Not an original source
~200 BCE
The Nine Chapters Source Wikimedia Commons (1820 edition)
中文：清李潢撰《九章算術細草圖說》十卷。嘉慶二十五年（1820年）語鴻堂刻本，竹紙線裝，一函八冊。
200 CE
An animation showing a box-lid emerging from two intersecting cylinders. Credit: Van helsing Wikimedia (CC BY-SA 3.0)


Rough Chronology Math in Ancient China
Original Source
The Book of Numbers and Computations - Suan Shu Shu
Counting board
Computations Algorithms
200 BCE
Liu Hui
π
Diagram of Liu Hui's π inequality
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Zu Chongzhi
Zu Gengzhi
500 CE
1000 BCE
Numerals from oracle bones, from 11-12th century BCE (Numbers between 1 to 30,000)
〇 一 二 三 四 五 六 七 八 九
0 1 2 3 4 5 6 7 8 9
十 廿 卅 百 千 万 亿 兆
Not an original source
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The Nine Chapters Source Wikimedia
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