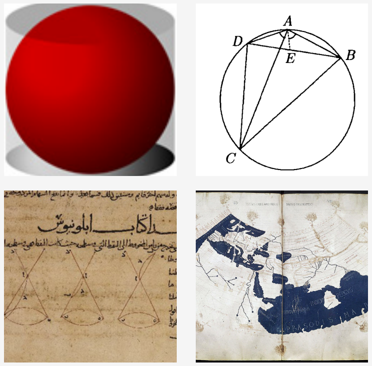
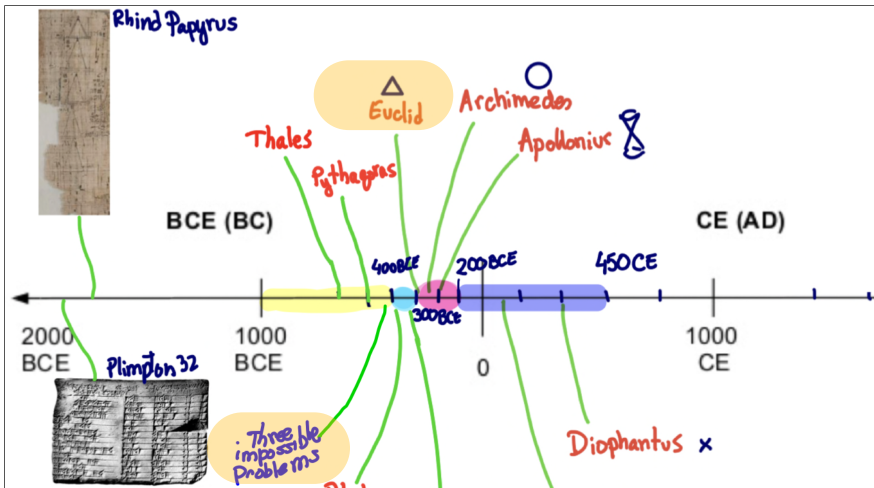
- Apollonius and conic sections
- Ptolemy and his table of chords.
- Archimedes
- Eratosthenes and the measurement of the Earth
- Diophantus and Algebra



MAT 336 Hellenic Mathematics After Euclid

17	Mar 31	Archimedes on the quadrature of the parabola **Vincent**	Ptolemy on Trigonometric tables **Sylvester**
18	Apr 2	The rule of false position **Rosalie**	Diophantus **Destiny**

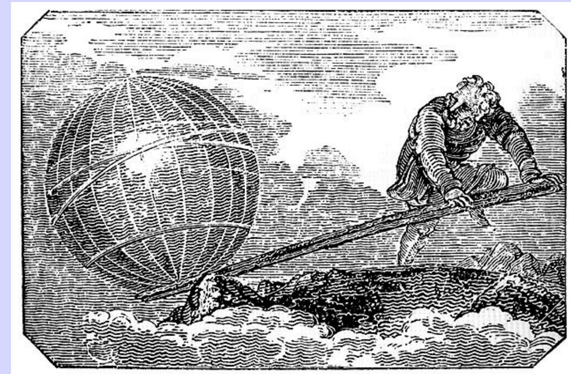


CE – Common Era
BCE – Before Common Era
BC – Before Christ
AD - “Anno Domini” Latin (In the Year of the Lord)

Birds are fed by their parents in their infancy. When the time comes to feed themselves, there can be some confusion when the food does not go into their mouth by itself.
https://twitter.com/fasc1nate/status/1582504199015002112?s=48&t=6rkCZIkmqBqzNYL WOLd4Q

Archimedes

Archimedes (~250 BCE)

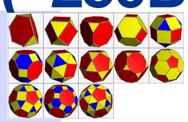


GIVE ME A PLACE TO STAND AND I WILL MOVE THE EARTH

Archimedes (~250BCE)

Derived and computed approximations to:

- Area of the circle
- Surface area and volume of the sphere
- Area of an ellipse.
- Area under a parabola

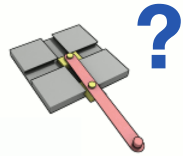


Ideas of calculus

- Infinitesimals
- Method of exhaustion

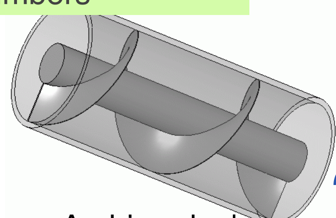
Dealt with infinity!!!

- Approximation of π
- Created a number system to deal with **arbitrary** large numbers



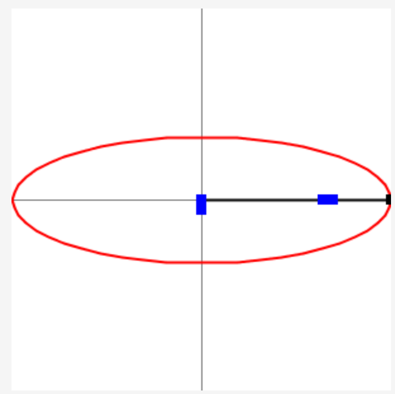
?

- Applied mathematics to physics
- Explanation of principle of the lever
- Innovative machines

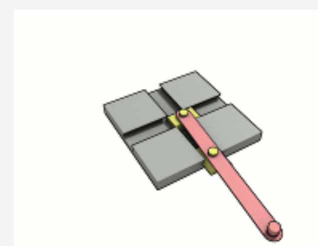


Archimedes' screw ?

Trammel of Archimedes



Animation of the trammel of Archimedes by Alastair Rae for Wikimedia Commons



Archimedes' screw

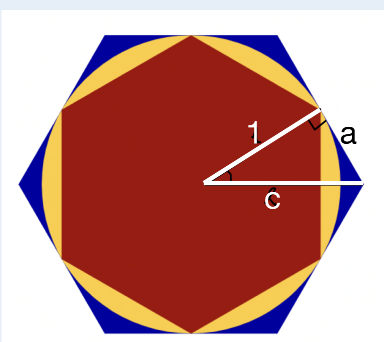
Stunteltje, (looped gif by Jahobr)

Archimedes and π

Archimedes' Method for Approximating π

A circle has **radius 1**.
Two regular hexagons are drawn:

- 🔴 **Inscribed hexagon:** vertices on the circle
- 🔵 **Circumscribed hexagon:** sides tangent to the circle

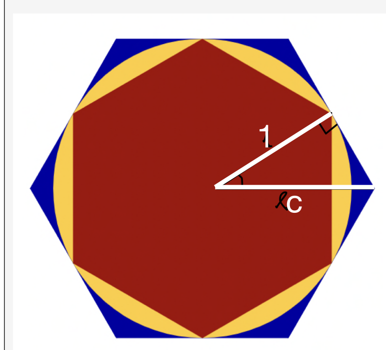


1. What is the perimeter of the inscribed hexagon?
2. What is the perimeter of the circumscribed hexagon? (Hint: What is the relation between a and c?)

Archimedes' Method for Approximating π

A circle has **radius 1**.
Two regular hexagons are drawn:

- 🔴 **Inscribed hexagon:** vertices on the circle
- 🔵 **Circumscribed hexagon:** sides tangent to the circle



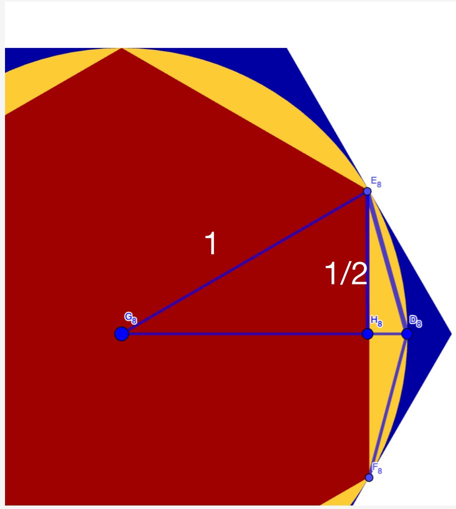
1. What is the perimeter of the inscribed hexagon?
2. What is the perimeter of the circumscribed hexagon? (Hint: study the angles in the three triangles in the figure below)

$$(c/2)^2+1^2=c^2$$
$$c^2=4/3$$

Archimedes ~250BCE and the method of exhaustion

Method of exhaustion, idea: Repeated subdivision can make a remainder smaller than any assigned magnitude

https://www.geogebra.org/m/ajamkye2



Archimedes’ Bounds for π (radius = 1)

Idea: Computed the perimeter of inscribed and circumscribed regular n-gons

n = 6 → 12 → 24 → 48 → 96

Each doubling narrows the bounds for π

Used 96-sided polygons for his final bounds.

Archimedes Proposition: The ratio of the circumference of any circle to its diameter as less than 3 1/7 but greater than 3 10/71.
In symbols

$$3\frac{10}{71} < \frac{\text{circumference}}{\text{diameter}} < 3\frac{1}{7}$$

Educated guess: Why did Archimedes stop at 96 sides? Why not 192=(2.96) or 384=(4.96)?

Archimedes approximation of π
Recall the alphabetic number system

units	α	β	γ	δ	ε	ϛ	ζ	η	θ
	1	2	3	4	5	6	7	8	9
tens	ι	κ	λ	μ	ν	ξ	ο	π	ϟ
	10	20	30	40	50	60	70	80	90
hundreds	ρ	σ	τ	υ	φ	χ	ψ	ω	ϡ
	100	200	300	400	500	600	700	800	900
thousands	͵α	͵β	͵γ	͵δ	͵ε	͵ϛ	͵ζ	͵η	͵θ
	1000	2000	3000	4000	5000	6000	7000	8000	9000

Source: Wikimedia

Archimedes' Bounds for π (radius = 1)

n	Inscribed	Circumscribed
6	3	$2\cdot 3^{1/2}$
12	$(2 - 3^{1/2})^{1/2} \times 12$	$2\cdot 3^{1/2}/(1 + 3^{1/2}) \times 12$
24	$(2 - (2 + 3^{1/2})^{1/2})^{1/2} \times 24$	$2\cdot(2 - 3^{1/2})^{1/2}\cdot(2\cdot 3^{1/2}/(1 + 3^{1/2})) / ((2 - 3^{1/2})^{1/2} + 2\cdot 3^{1/2}/(1 + 3^{1/2})) \times 24$
48	$(2 - (2 + (2 + 3^{1/2})^{1/2})^{1/2})^{1/2} \times 48$	$2\cdot a_{24}\cdot b_{24} / (a_{24} + b_{24}) \times 48$
96	3 10/71	3 1/7

Archimedes' Proposition: The ratio of the circumference of any circle to its diameter as less than 3 1/7 but greater than 3 10/71.

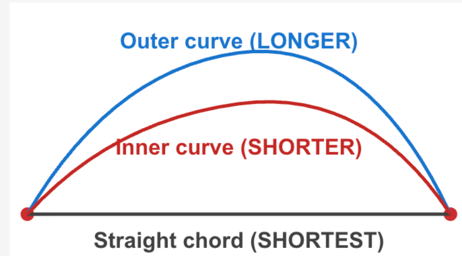
$$3\frac{10}{71} < \frac{\text{circumference}}{\text{diameter}} < 3\frac{1}{7}$$

What did Archimedes mean by the *length of a curve?*

Archimedes Ordering axioms (modern version).

Axiom 1: Of all lines which have the same extremities, the straight line is the least.

Axiom 2: Of other lines in a plane with the same endpoints and concave in the same direction, that which is **closer to the straight line** (or lies entirely between **the other line** and the **straight line**) is the shorter.



Archimedes treated the ***length of a curve*** as a **magnitude**—something you can compare (greater/equal/less) but not compute.

So how did Archimedes computed π?

Archimedes and large numbers

Archimedes The sand reckoner (~250BCE)

Created a number system to deal with **arbitrary** large numbers

$$1,373,943\tfrac{33}{64} = \overset{\rho\lambda\zeta}{M}{}_{,}\gamma ϡ\mu\gamma' \;\; \lambda\gamma' \;\; \xi\delta''.$$

Archimedes computed the **number of grains of sand that would fill a sphere whose diameter was equal to the distance from earth to the fixes stars.**

To work with very large numbers, he imagined a "doubled class" of numbers of eight numerals (instead of the four the the Greek ciphered system)

1. 1 to 99,999,999
2. 100,000,000 to 10^{16}-1
3. etc

units	α	β	γ	δ	ε	ϛ	ζ	η	θ
	1	2	3	4	5	6	7	8	9
tens	ι	κ	λ	μ	ν	ξ	ο	π	ϟ
	10	20	30	40	50	60	70	80	90
hundreds	ρ	σ	τ	υ	φ	χ	ψ	ω	ϡ
	100	200	300	400	500	600	700	800	900
thousands	͵α	͵β	͵γ	͵δ	͵ε	͵ϛ	͵ζ	͵η	͵θ
	1000	2000	3000	4000	5000	6000	7000	8000	9000

Wikimedia

Some of the numbers to which I have given a name [...] surpass not only the number of grains of sand that could fill the Earth [...] but even the number of grains of sand that could fill the universe itself.
Archimedes, Sand-Reckoner

The Archimedes' Palimpsest

The Archimedes' Palimpsest

- **10th century:** Scribe in Constantinople copied Archimedes texts
- **13th century:** Christian monks needed vellum for a prayer book — washed off Archimedes, wrote prayers over it
- Named **Palimpsest** (manuscript with text written over earlier, erased text)
- **Discovered 1906** in a monastery in Constantinople. **reappeared in the 1970's**,(in the hands of a French family that had bought it in Istanbu in the 1920s) **resurfaced** at Christie's auction **1998** (badly damaged)
- Anonymous collector bought for **$2m** → deposited at **Baltimore's Walters Art Museum**
- **Importance:** Revealed *The Method* — Archimedes' techniques for discovering geometric results using physical reasoning before formal proof.

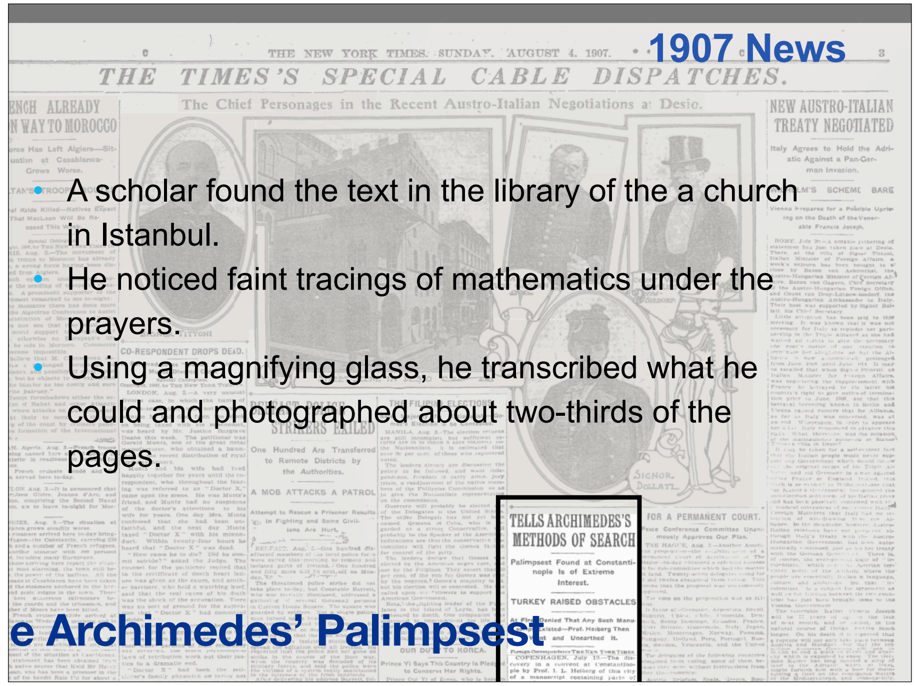
Vellum = writing material made from treated calfskin; expensive and reusable

The Method uses physical reasoning — imagining geometric shapes balanced on a lever and comparing cross-sections — to discover results before proving them rigorously.

The Archimedes' Palimpsest contains

- The Equilibrium of Planes
- Spiral Lines
- The Measurement of the Circle → π
- Sphere and Cylinder → Today
- On Floating Bodies → Eureka
- The Method of Mechanical Theorems → Today
- The Stomachion → Puzzle

1907 News



- A scholar found the text in the library of the a church in Istanbul.
- He noticed faint tracings of mathematics under the prayers.
- Using a magnifying glass, he transcribed what he could and photographed about two-thirds of the pages.

e Archimedes' Palimpsest

1907 NY Times

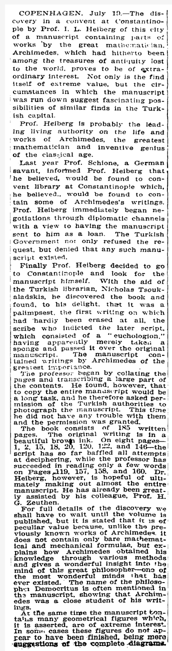
TELLS ARCHIMEDES'S METHODS OF SEARCH

Palimpsest Found at Constantinople Is of Extreme Interest.

TURKEY RAISED OBSTACLES

At First Denied That Any Such Manuscript Existed—Prof. Heiberg Then Went and Unearthed It.

Foreign Correspondence THE NEW YORK TIMES.



COPENHAGEN, July 19.—The discovery in a convent at Constantinople by Prof. I. L. Heiberg of this city of a manuscript containing parts of works by the great mathematician, Archimedes, which had hitherto been among the treasures of antiquity lost to the world, proves to be of extraordinary interest. Not only is the find itself of extreme value, but the circumstances in which the manuscript was run down suggest fascinating possibilities of similar finds in the Turkish capital.

Prof. Heiberg is probably the leading living authority on the life and works of Archimedes, the greatest mathematician and inventive genius of the classical age.

Last year Prof. Schione, a German savant, informed Prof. Heiberg that he believed, would be found to convent library at Constantinople which, he believed., would be found to contain some of Archimedes's writings. Prof. Heiberg immediately began negotiations through diplomatic channels with a view to having the manuscript sent to him as a loan. The Turkish Government not only refused the request, but denied that any such manuscript existed.

Finally Prof. Heiberg decided to go to Constantinople and look for the manuscript himself. With the aid of the Turkish librarian, Nicholas Tsoukaladskis, he discovered the book and found, to his delight, that it was a palimpsest, the first writing on which had hardly been erased at all, the scribe who indicted the later script, which consisted of a "euchologion," having apparently merely taken a sponge and passed it over the original manuscript. The manuscript contained writings by Archimedes of the greatest importance.

The professor began by collating the pages and transcribing a large part of the contents. He found, however, that to copy the entire manuscript would be a long task, and he therefore asked permission of the Turkish authorities to photograph the manuscript. This time he did not have any trouble with them and the permission was granted.

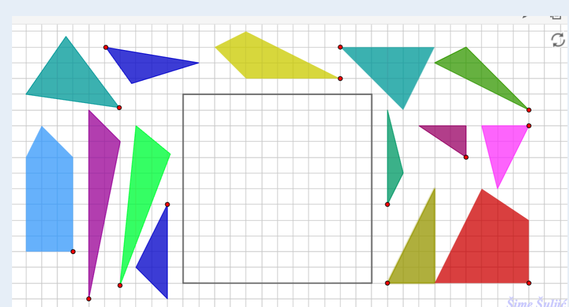
The book consists of 185 written pages. The original writing is in a beautiful brown ink. On eight pages—1, 2, 15, 18, 20, 120, 122, and 146—the script has so far baffled all attempts at deciphering, while the professor has succeeded in reading only a few words on Pages 119, 157, 158, and 160. Dr. Heiberg, however, is hopeful of ultimately making out almost the entire manuscript. He has already been greatly assisted by his colleague, Prof. H. G. Zeuthen.

For full details of the discovery we shall have to wait until the volume is published, but it is stated that it is of peculiar value because, unlike the previously known works of Archimedes, it does not contain only bare mathematical and mechanical formulae, but explains how Archimedes obtained his knowledge through various methods and gives a wonderful insight into the mind of this great philosopher—one of the most wonderful minds that has ever existed. The name of the philosopher Democritus is often mentioned in the manuscript, showing that Archimedes was a close student of his writings.

At the same time the manuscript contains many geometrical figures which, it is asserted, are of extreme interest. In some cases these figures do not appear to have been finished, being mere suggestions of the complete diagrams.

Wild guess: How many ways to form a square do you think there are?

https://www.geogebra.org/material/copy/id/pteravjV



The Stomachion: Archimedes' Forgotten Work

A puzzle well known in Antiquity: 14 irregular pieces → how many ways to form a square?

Archimedes presents systematic ideas of counting all these ways

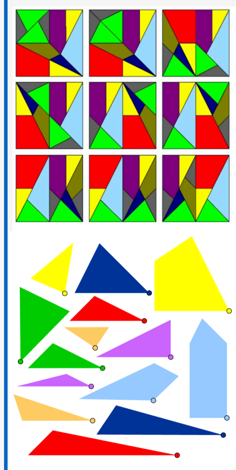
2006 answer to Archimedes' question (interpreted): 17,152 configurations (required systematic counting of *all* possibilities)

Why ignored? Only references until the discovery of the Palimpsest.

Why important? A treatise on **combinatorics** — a field that didn't develop until the rise of **computer science**

Far ahead of its time

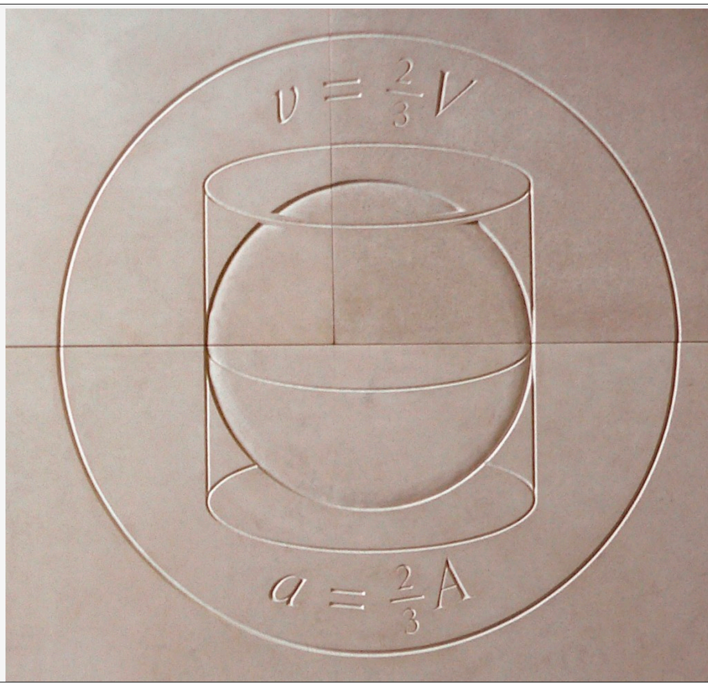
Challenge: How many ways can *you* arrange these pieces into a square?



https://www.geogebra.org/m/d9thyaxv

https://pi.math.cornell.edu/~mec/GeometricDissections/1.2%20Archimedes%20Stomachion.html

Archimedes' favorite result



1. The horizontal green plane cuts the **sphere** of radius **r** at height **h**, forming a **circle**. Write the area of this circle in terms of **r** and **h**.

2. The same plane cuts the cylinder with a cone removed, forming an annulus (ring). Write the area of this annulus in terms of r and h.

3. Compare your two expressions.

What relationship do you find between the **areas** of the sphere's cross-section and the ring's cross-section?
What does this imply about the **volumes** of the sphere and the cylinder–cone solid?

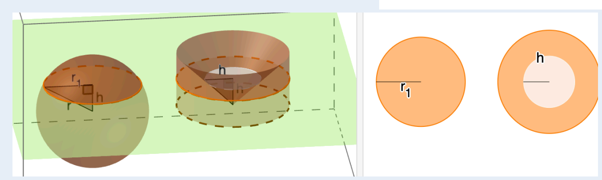
C = volume of cylinder (height **r**, radius base **r**)

S = volume of sphere (radius **r**)

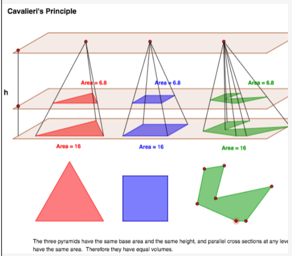
O = volume of cone (height **r**, radius base **r**) = C⁄3

https://www.geogebra.org/m/faxwvhsg

Archimedes (~250BCE)



Cavalieri's principle (although was known by Chinese mathematicians before Cavalieri stated it)



Two solids such that the section of one by each horizontal plane bears a fixed ratio to the section of the other by the same plane have volumes in that same ratio.

Archimedes (~250BCE) method to find the relation S = ⅔ C. (The proof is different)

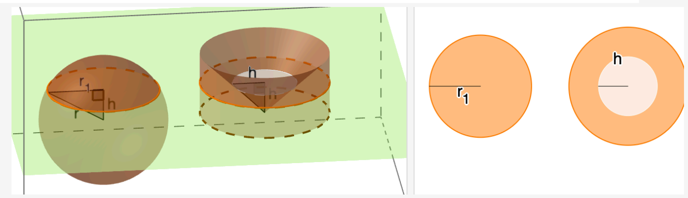
C = volume of cylinder (height **r**, radius base **r**)

S = volume of sphere (radius **r**)

O = volume of cone (height **r**, radius base **r**) = C⁄3

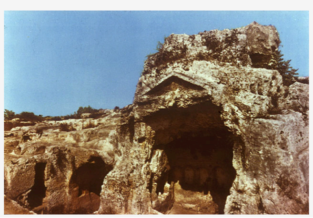
We saw that S = C - O. Then

S = C – O = ⅔ C.



And although he made many excellent discoveries, he is said to have asked his kinsmen and friends to place over the grave where he should be buried a cylinder enclosing a sphere, with an inscription giving the proportion by which the containing solid exceeds the contained.

Plutarch (AD 45-120), Parallel Lives: Marcellus



A tomb in Syracuse in the Necropolis of Grotticelli referred to affectionately (or deceptively) as "Archimedes' Tomb ", but known to be of Roman origin dating at least two centuries after the death of Archimedes. https://www.math.nyu.edu/~crorres/Archimedes/Tomb/TombIllus.html

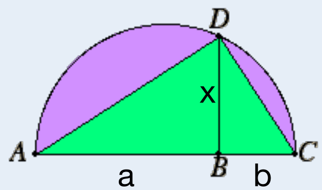
Mean Proportional
Conic Sections
Apollonius

Proposition VI.16 of Euclid's elements

To find a mean proportional to two given segments.

Prove that BD is the mean proportional between AB and BC. In other words, show that AB : BD = BD : BC.

A positive number x is the **mean proportional** of two (positive) numbers a and b if a/x=x/b.



Join the lesson at www.geogebra.org/classroom with the code:

JEUT QGMJ

Or you can also share the following link with your students:

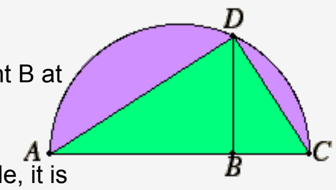
www.geogebra.org/classroom/jeutqgmj

https://www.geogebra.org/classroom/jeutqgmj

Proposition VI.16 of Euclid's elements

To find a mean proportional to two given straight lines.

- Let AB and BC be the two given straight lines.
- It is required to find a mean proportional to AB and BC.
- Place them in a straight line, and describe the semicircle ADC on AC. Draw BD from the point B at right angles to the straight line AC, and join AD and DC. (I.11)
- Since the angle ADC is an angle in a semicircle, it is right. (III.31)
- Since, in the right-angled triangle ADC, BD has been drawn from the right angle perpendicular to the base, therefore BD is a mean proportional between the segments of the base, AB and BC. (VI.8,Corollary)
- **Therefore a mean proportional BD has been found to the two given straight lines AB and BC**



A positive number x is the **mean proportional** of two (positive) numbers a and b if a/x=x/b.

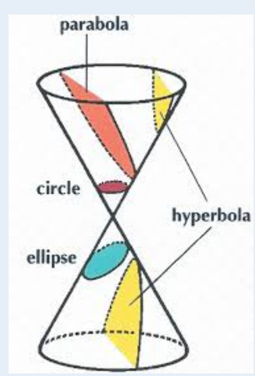
From mean proportional to conic sections

- **Hippocrates of Chios (~400 BCE)** reduced the problem of ***doubling the cube*** to finding **two mean proportionals** between two given lengths *a* and *b.* In modern terms, the unknowns *x* and *y* satisfy: a : x = x : y = y : b, or equivalently:
    1. $x^2 = a y$
    2. $y^2 = b x$
    3. $x y = a b$.
- **Menaechmus (~350 BCE)** discovered ***conic sections*** while seeking a geometric **construction** of these two mean proportionals.

Doubling the cube → Two mean proportionals → Conic sections

Conic sections

https://www.geogebra.org/m/pgu4vvtj



http://jwilson.coe.uga.edu/emt725/Conics%20Images/Conics.html

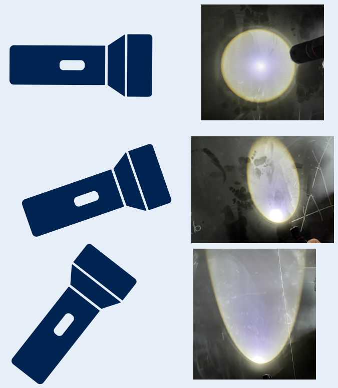
1. When α = 40° and β = 35°, the intersection is a hyperbola.
   Find all pairs (α, β) that produce a hyperbola.
2. For which values of (α, β) is the intersection an ellipse?
3. For which values of (α, β) is the intersection a circle?
4. For which values of (α, β) is the intersection a parabola?
5. Can you find any other type of curve formed by this intersection?

Compare the angle of the plane with the cone's axis (α) to the angle of the cone's side with the axis (β)

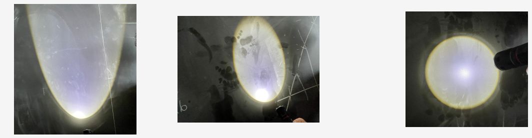
Find all the values you can — the GeoGebra app will help you visualize and test your answers.

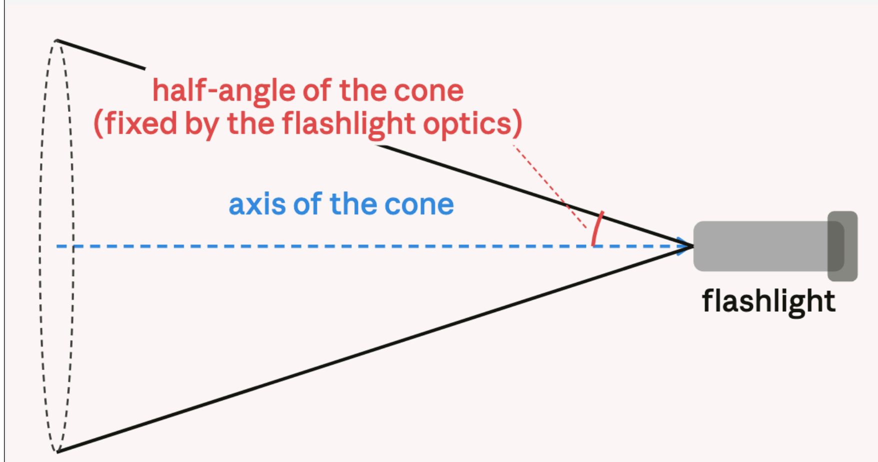
1. What curves do you see in the demonstration?
2. Explain why we see those curves.

1..
2..
3..



Hint: there should be a cone in the explanation

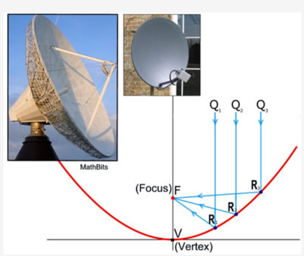




Apollonius of Perga (~200BCE)
Studied parabolas.
Current applications of properties parabolas

A parabolic dish is a surface with a cross-sectional shape of a parabola -paraboloid- used to direct light or sound waves. This concept is used by

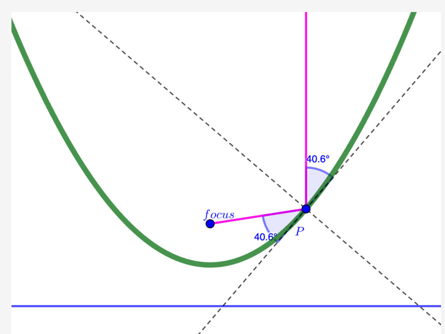
- Radio telescope antennas
- satellite dishes
- parabolic reflector,
- car's headlights and in spotlights.



https://mathbitsnotebook.com/Algebra2/Quadratics/QDParabolaApplied.htm

How a Parabolic Mirror Focuses Light

https://www.geogebra.org/m/fkqe4v5m



- https://www.geogebra.org/m/jeqqdnkh (spherical mirror for comparison)

Apollonius (~200 BCE)

Apollonius of Perga (~200BCE)

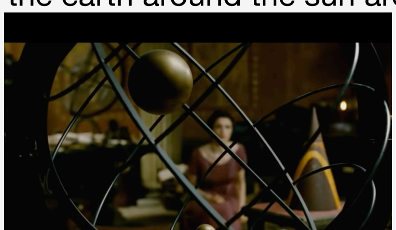
- Alexandria
- worked out more fully and generally than in the writings of others
- studied and later taught under the followers of Euclid

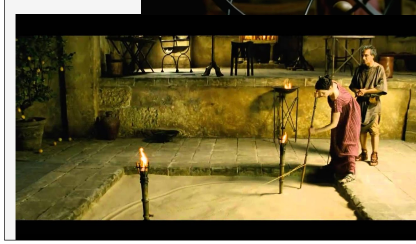
Conics - 8 books

*“**the most and prettiest of these theorems are new,** and it was their discovery which made me aware that Euclid did not work out the syntheses of the locus with respect to three and four lines, but only a chance portion of it, and that not successfully; for it was not possible for the said synthesis to be completed without the aid of the additional theorems discovered by me.”*

Hypatia (~400 CE) was a philosopher, astronomer, and mathematician who lived in Alexandria.

In the clip below from movie Agora (a fictionalized version of her life), she stars by discussing the assumption that the orbits of the earth around the sun area circles.



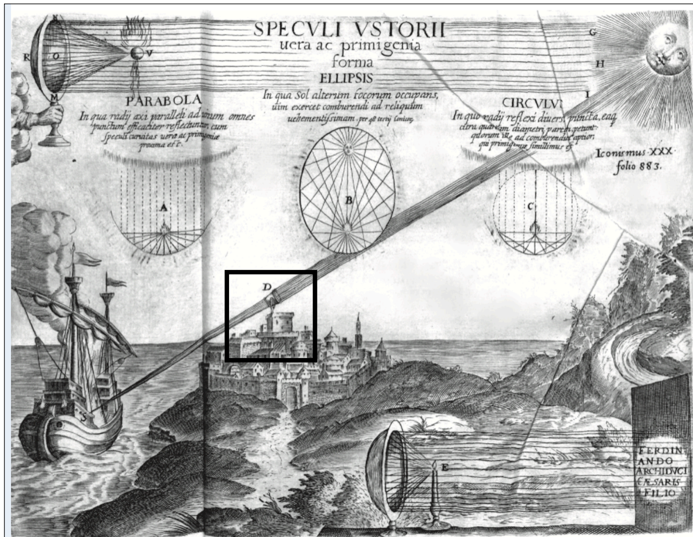


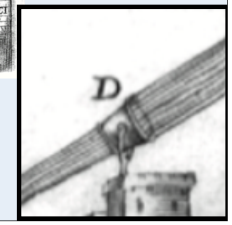
Detail from The School of Athens by Raffaello Sanzio.

Hypatia arrives to the conclusion that the orbits follow another curve and not a circle. What is that curve? What is the property of that curve that she describes?

https://youtu.be/bUUB1u8xJA4

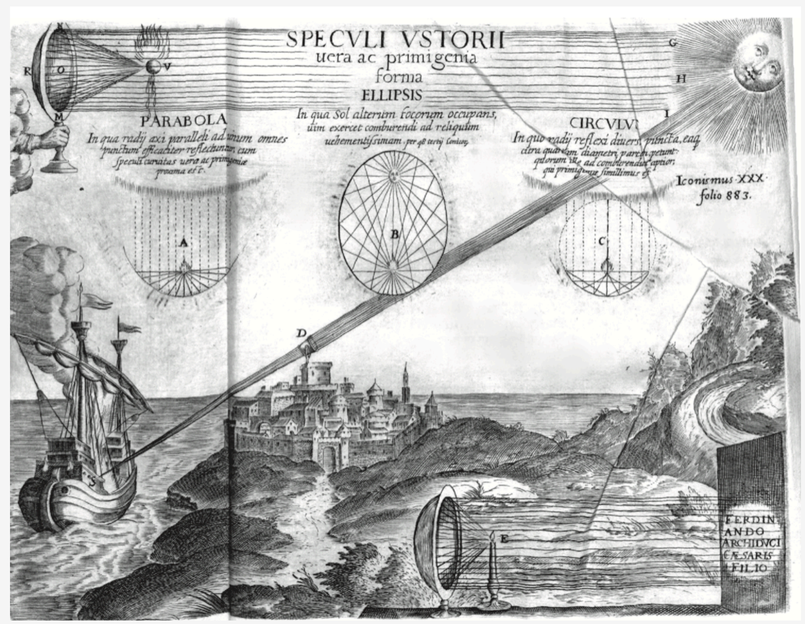
A story about Archimedes' and mirrors (with conic sections)





What's happening at point D? Why might this be important?

Archimedes' mirror



Setting fire to a ship using the sun and a mirror. Engraving from a book by our buddy Athanasius Kircher, 1671. (Wellcome Collection)

One of the many stories about Archimedes tells that he manage destroyed the Roman fleet at the siege of Syracuse in 213 BC by the application of directed solar radiant heat.

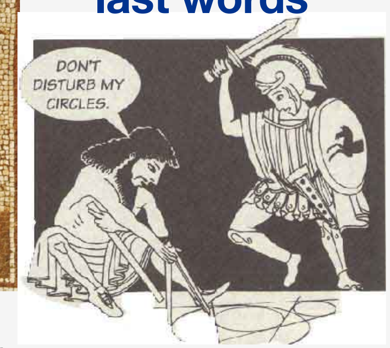
Many studies debunked this…

Stories about Archimedes

Eureka!

Archimedes last words

Death of Archimedes (Mosaic, Municipal Gallery Francfort)



Cassus: "*The head and not the lines*"

Georgios Pachymeris: "*heat me (my head) do not destroy the lines* “(drawing)

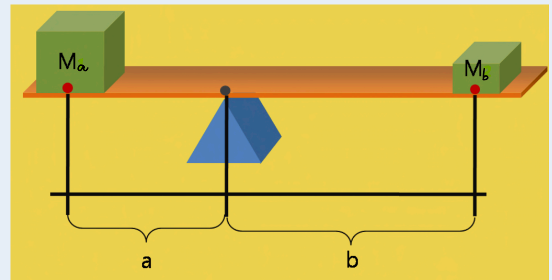
Tzetzes: “*Stay away man from my diagram*"

Unknown: "*Don't disturb my circles*"

Archimedes and the Law of the Lever

Archimedes Law of the Lever

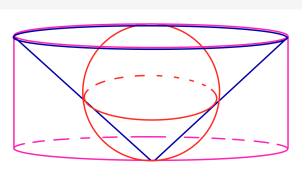
Magnitudes are in equilibrium at distances reciprocally proportional to their weights.

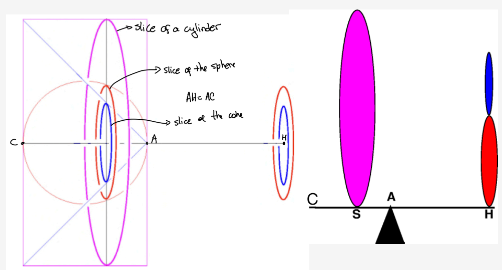


Source: Wikimedia. https://commons.wikimedia.org/wiki/File:Lever_Principle_3D.png

Write down a formula for Archimedes Law of the Lever in terms of the weights M_a and M_b and the distances a and b

Vol(Sphere) + Vol(Cone) = (1/2)Vol(Cylinder)



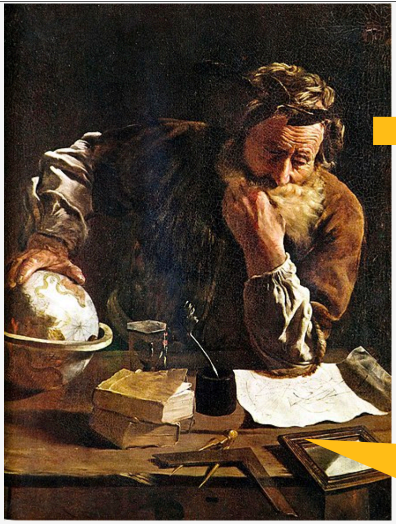


- Applied Cavalieri's principle, imagining the sections of a region balanced about a fulcrum.
- If each pair of corresponding sections balance at distances a and b, then the bodies themselves will balance at these distances
- The balancing force of the cylinder is the same if all of its mass is concentrated at its center of mass, which is the midpoint of AC.

Example of Archimedes' Method

Archimedes Law of the Lever: Magnitudes are in equilibrium at distances reciprocally proportional to their weights.

Eratosthenes, the librarian of Alexandria (~200 BCE)



Archimedes Thoughtful (also known as Portrait of a Scholar) by Domenico Fetti, 1620

Eratosthenes Teaching in Alexandria
By Bernardo Strozzi - Musée des beaux-arts de Montréal, Public Domain, https://commons.wikimedia.org/w/index.php?curid=42582581

"The proofs then of these theorems I have written in this book and now send to you (that is, Eratosthenes)" (letter from

A well at Kom Ombo, 50 km north of Syene

Image credit: https://medium.com/roaming-physicist/looking-down-a-well-in-egypt-to-measure-the-size-of-the-earth-5e12f2df0b2

Syene (Aswan)
On the summer solstice at noon
→ Sun is (almost) directly overhead
→ No shadow (or negligible)

Alexandria
On the summer solstice at noon
→ shadow is shortest.

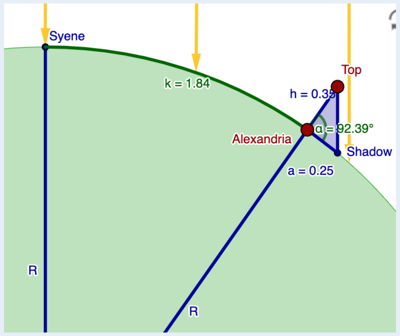
Key idea: Same Sun, same day:
no shadow in Syene vs. small shadow in Alexandria → gives angle difference.

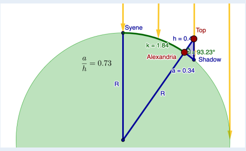
On the summer solstice, the Sun reaches its maximum height at noon in the Northern Hemisphere.

Eratosthenes on the circumference of the earth

The small triangle with vertices Alexandria, Shadow and Top is not a right-angled triangle but you may assume it is (If the picture was to scale, the error from such an approximation would be less than any other error in the measurements).

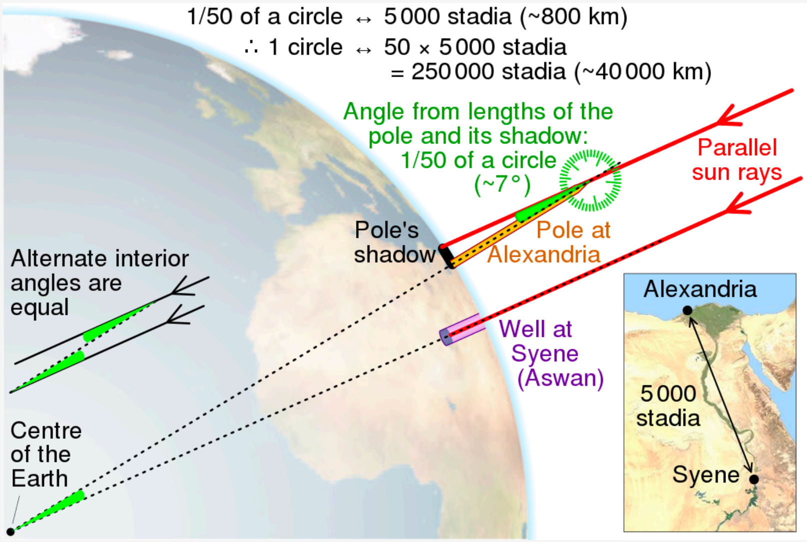
1. Note that the angles β and γ are equal. Explain why.
2. Find the diameter of the circumference of the earth in terms of h and a, the two legs of the small triangle. (Hint: Start by finding R, the radius of the "green" circle above, in terms of a and h).
3. Why can we assume that the rays of the sun are parallel?





Discussion in groups. Answer in Wooclap.
https://www.geogebra.org/m/ty5ych75

Illustration of the method Eratosthenes used to calculate the circumference of the Earth

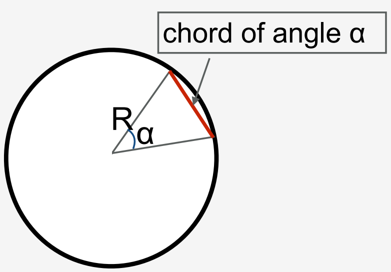


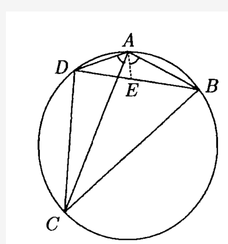
Claudius Ptolemy (100 – 170 CE)

Claudius Ptolemy ~100AD

Mathematical Collection (Almagest) - Mathematical Astronomy

- Astronomer and physicist (hence, mathematician of his era)
- Trigonometry - Table of chords
  - Solved plane triangles
  - Solved spherical triangles





Claudius Ptolemy ~100AD

I know that I am mortal and the creature of a day; but when I search out the massed wheeling circles of the stars, my feet no longer touch the Earth, but, side by side with Zeus himself, I take my fill of ambrosia, the food of the gods.

(Ptolemy's Almagest)

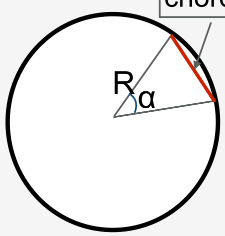
Claudius Ptolemy Table of Chords

The idea of ***angles*** seems to have appeared in Greece between 6th and 5th century BCE.

Trigonometry is the branch of mathematics that studies relationships between side lengths and angles of triangles.

Ptolemy made a table of correspondence between angles and their chords. (This is the earliest form of trigonometry we know of)

chord of angle α



Ptolemy's treatise on astronomy, written around 150 CE, explains how to construct the table.

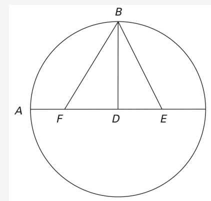
Claudius Ptolemy ~100AD

- Worked with a circle of radius 60 (because he did calculations in base 60) **Question: Why base 60?**
- Computed chord(36°). How?

Main idea: computed chord tables of certain angles using

- propositions from Euclid's elements
- square roots (or approximations)

In the Almagest, Ptolemy announces that he is going to employ in general the base 60 system, in order to avoid "**the embarrassment of the fractions"**



Example: Suppose that E bisects DC

Find F so EF=EB

Then (prove it!) DF is the side of a decagon inscribed on a circle with radius DC.

Ptolemy computed DF=chord(36°):

$$DF^2=(EF-ED)^2=(EB-ED)^2=((BD^2+ED^2)^{1/2} - ED)^2=$$

$$((60^2+30^2)^{1/2} - 30)^2=(37;4;55)_{60}$$

Claudius Ptolemy ~100AD

- regular dodecagon -> chord(30°)
- regular decagon-> chord(36°)
- chord(36°) and chord(30°) ->chord(6°)
- Bisecting chord(3°)->chord(1° 30')
- Upper and lower values of the chord(1°)
- Finally, a table of 360 entires, with chords of at 30f' increments up to 180°.

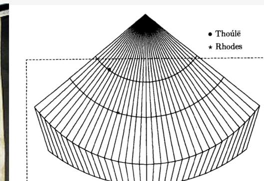
περιφε-ρειῶν	εὐθειῶν			ἑξηκοστῶν			
∠′	ο	λα	κε	ο	α	β	ν
α	α	β	ν	ο	α	β	ν
α∠′	α	λδ	ιε	ο	α	β	ν
β	β	ε	μ	ο	α	β	ν
β∠′	β	λζ	δ	ο	α	β	μη
γ	γ	η	κη	ο	α	β	μη
γ∠′	γ	λϑ	νβ	ο	α	β	μη
δ	δ	ια	ιϛ	ο	α	β	μζ
δ∠′	δ	μβ	μ	ο	α	β	μζ
ε	ε	ιδ	δ	ο	α	β	μϛ
ε∠′	ε	με	κζ	ο	α	β	με
ϛ	ϛ	ιϛ	μϑ	ο	α	β	μδ
ϛ∠′	ϛ	μη	ια	ο	α	β	μγ
ζ	ζ	ιϑ	λγ	ο	α	β	μβ
ζ∠′	ζ	ν	νδ	ο	α	β	μα
η	η	κβ	ιε	ο	α	β	μ
η∠′	η	νγ	λε	ο	α	β	λϑ
ϑ	ϑ	κδ	νά	ο	α	β	λη
ϑ∠′	ϑ	νϛ	ιγ	ο	α	β	λζ
ι	ι	κζ	λβ	ο	α	β	λε
ι∠′	ι	νη	μϑ	ο	α	β	λγ
ια	ια	λ	ε	ο	α	β	λβ
ια∠′	ιβ	α	κα	ο	α	β	λ
ιβ	ιβ	λβ	λϛ	ο	α	β	κη

Beginning of Ptolemy table of chords

https://www.wilbourhall.org/pdfs/HeibergAlmagestComplete.pdf

Claudius Ptolemy ~100AD

Ptolemy's world map, reconstituted from Ptolemy's Geography (circa 150) in the 15th century, indicating "Sinae" (China) at the extreme right, beyond the island of "Taprobane" (Ceylon or Sri Lanka, oversized) and the "Aurea Chersonesus" (Southeast Asian peninsula).



Ptolemy method of making (The History of MathematicsA Brief Course - Roger Cooke · 2011)

One of the first scholars to look at the problem of representing large portions of the Earth's surface on a flat map.

Diophantus (~200 CE) and The birth of literal algebra

Diophantus - (possibly ~250AD) -Alexandria

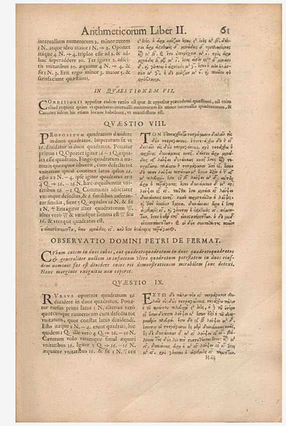
Arithmetica

- Major advance in the solution of equation: Introduction to symbolism.
  - Linear and quadratic equations
  - Higher degree equations
  - The method of false position

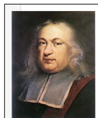
ALGEBRA

$$K^{\Upsilon}\alpha\,\varsigma\gamma\,\pitchfork\,\Delta^{\Upsilon}\gamma\,\mathring{M}\alpha$$

$$x^3 - 3x^2 + 3x - 1$$

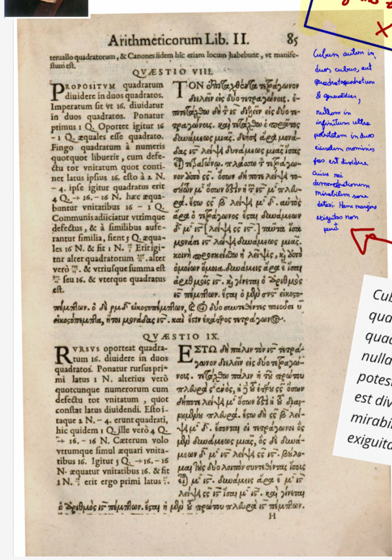


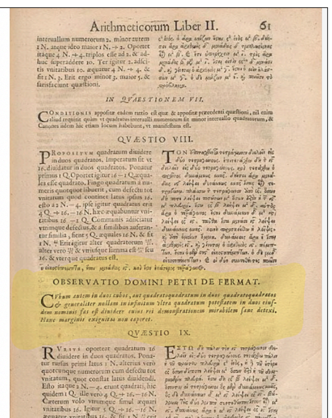
Problem II.8 in the Arithmetica (edition of 1670), annotated with Fermat's comment which became Fermat's Last Theorem.
Wikipedia



Fermat
~1600

Fermat last theorem
Is $n \geq 3$, there are no positive integers
x, y and z such that
$x^n + y^n = z^n$





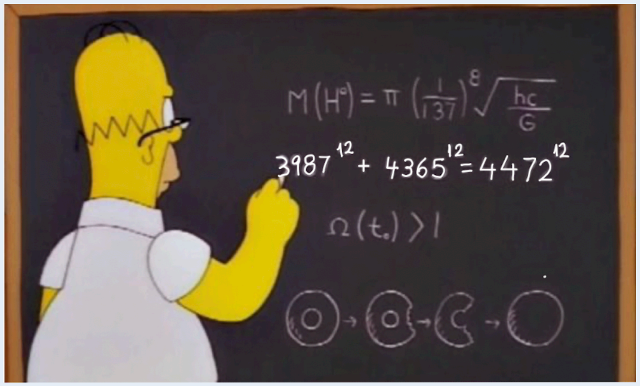
Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos & generaliter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est dividere cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.

It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. I have discovered a truly marvellous proof of this, which this margin is too narrow to contain.

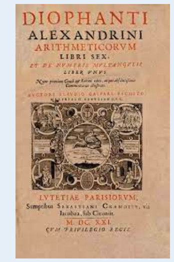
Fermat last theorem
Is $n \geq 3$, there are no positive integers
x, y and z such that
$x^n + y^n = z^n$

Andrew Wiles
presented a proof in 1993 - 1994

Use a gadget (phone, computer..) that can only use numbers of 10 digits to compute first $3987^{12}+4365^{12}$ and second, 4472^{12}.
Have we (jointly with Homer Simpson) disproved Fermat Last Theorem? Why or why not?



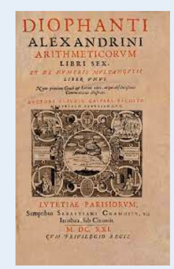
Solve the following problem by Diophantus.
To split a given number (100) in two parts having a given difference (40).



Solve the following problem by Diophantus.
To split a given number (100) in two parts having a given difference (40).

In fact, let the given arithmos be 100, and the excess M̊ 40. To find the arithmoi. Let the smaller be assigned, ϛ 1. Therefore, the larger will be ϛ 1 M̊ 40. Therefore, together they become ϛ 2 M̊ 40. But M̊ 100 are given. Therefore, M̊ 100 are equal to ϛ 2 M̊ 40. And from similars are similars. I subtract from 100, M̊ 40 [and from the 2 arithmoi and of 40 units similarly 40 units]. Remainders ϛ 2 equal M̊ 60. Therefore each ϛ becomes M̊ 30. For the actualities. The smaller will be M̊ 30 and the larger M̊ 70, and the demonstration is obvious.

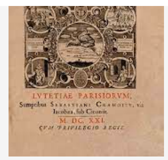
To split a given number (60) in two parts having a given ratio (3:1).



To split a given number (60) in two parts having a given ratio (3:1).

2. To divide the prescribed arithmos into two arithmoi in a ratio that's given.

In fact, let it be prescribed to divide 60 into two arithmoi in a ratio 3-times. Let the smaller be assigned ς 1. Therefore, the larger will be ς 3, and the larger is triple the smaller. It is required the two be equal to M̊ 60. But the two added are ς 4. Therefore, ς 4 are equal to M̊ 60. Therefore, ς is M̊ 15. Therefore, the smaller will be M̊ 15 and the larger M̊ 45.



To find two numbers in a given ratio and such that their difference is also given.

Given ratio 5 : 1, given difference 20.