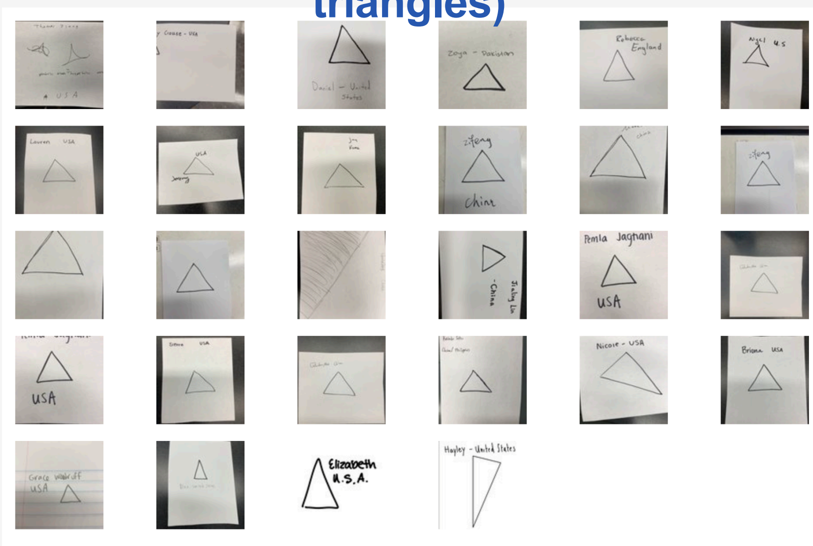
Plimpton 322

Without overthinking, quickly draw a triangle (paper or tablet) and upload it.

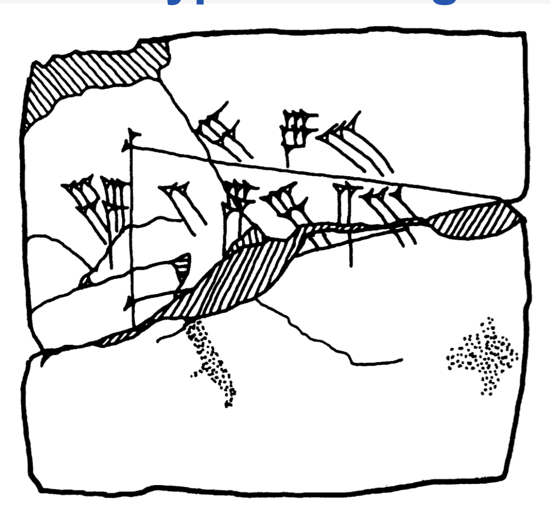
List features common to most drawings (besides the obvious fact that they are triangles)



List features common to most drawings (besides the obvious fact that they are triangles)

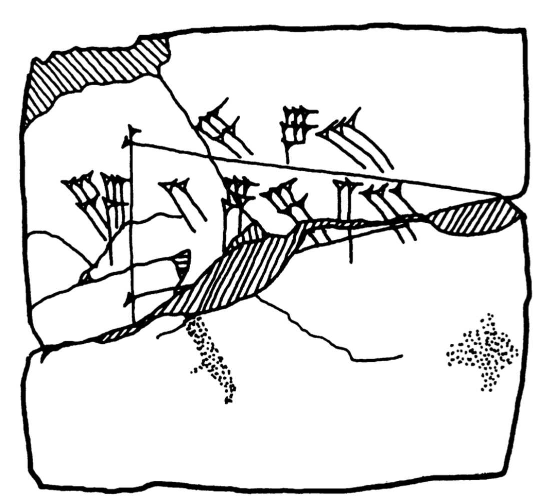
A Babylonian typical triangle

The point of triangle drawing activity is to make us aware of our biases.



M 29-15-709 (obverse). Drawing by the Eleanor Robson - Words and Pictures: New Light on Plimpton 322 - The American Mathematical Monthly , Feb., 2002, Vol. 109, No. 2 (Feb., 2002), pp. 105-120

If you had found this triangle drawing in a Mesopotamian tablet, what would you have assumed about it?"



M 29-15-709 (obverse). Drawing by the Eleanor Robson - Words and Pictures: New Light on Plimpton 322 - The American Mathematical Monthly , Feb., 2002, Vol. 109, No. 2 (Feb., 2002), pp. 105-120

https://www.nytimes.com/2010/11/23/science/23babylon.html?smid=url-share

Plimpton 322

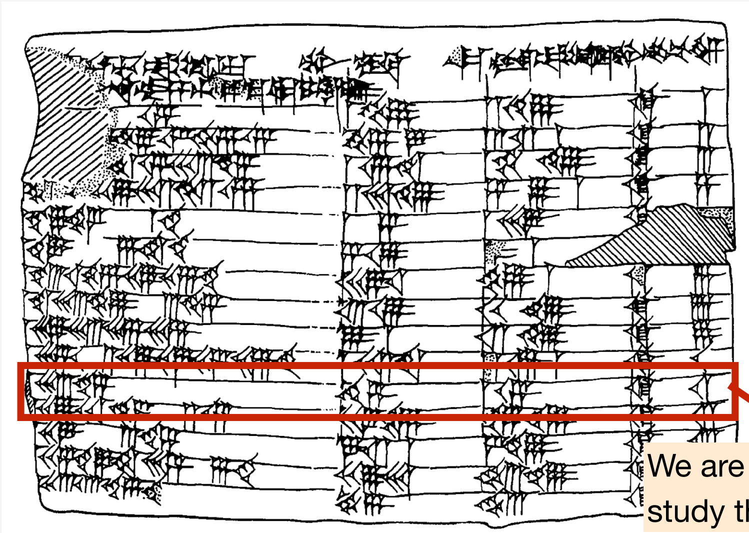
An Exhibition That Gets to the (Square) Root of Sumerian Math - NYTimes - Nov 22, 2010

- First Western owner, George A. Plimpton bequeathed to Columbia University in the mid-1930s.
- Surviving correspondence shows that he bought the tablet for $10 from a dealer called Edgar J. Banks.
- Banks told him it came from an archaeological site called Senkereh in southern Iraq, whose ancient name was Larsa
- Approximate date of the tablet: 1800 BCE.

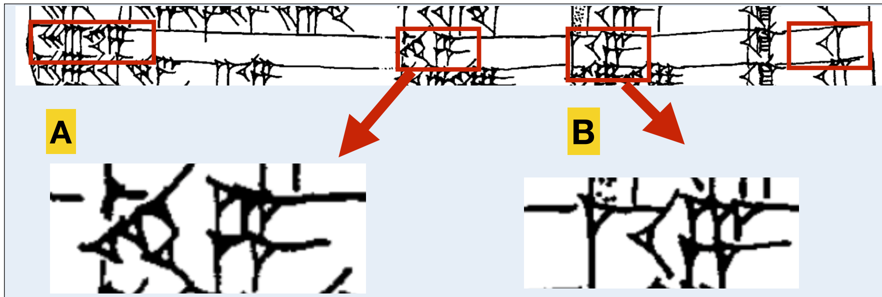
To understand Plimpton 322, in what order should we investigate the following:
A. How were the numbers calculated (algorithm or procedure)?
B. What mathematical relationships do the numbers exhibit?
C. What is the purpose of the table?
D. Who made it?
Explain your choice.

An Exhibition That Gets to the (Square) Root of Sumerian Math - NYTimes - Nov 22, 2010

Plimpton 322 - Drawing by Eleanor Robson



Plimpton 322 (Robson, Eleanor. "Neither Sherlock Holmes nor Babylon: A Reassessment of Plimpton 322." Historia Mathematica 28.3 (2001): 167-206.)



Express A or B (or both) in Hindu Arabic numerals

Plimpton 322 Transcription (and Minor Corrections)

$(d/l)^2$ or $(s/l)^2$	Short side s	Diagonal d	Row
$(59;0;15)_{60}$	$(1;59)_{60}$	$(2;49)_{60}$	1
$(56;56;58;14;50;6;15)_{60}$	$(56;7)_{60}$	$(3;12;1)_{60}$	2
$(55;7;41;15;33;45)_{60}$	$(1;16;41)_{60}$	$(1;50;49)_{60}$	3
$(53;10;29;32;52;16)_{60}$	$(3;31;49)_{60}$	$(5;9;1)_{60}$	4
$(48;54;1;40)_{60}$	$(1;5)_{60}$	$(1;37)_{60}$	5
$(47;6;41;40)_{60}$	$(5;19)_{60}$	$(8;1)_{60}$	6
$(43;11;56;28;26;40)_{60}$	$(38;11)_{60}$	$(59;1)_{60}$	7
$(41;33;45;14;3;45)_{60}$	$(13;19)_{60}$	$(20;49)_{60}$	8
$(38;33;36;36)_{60}$	$(9;1)_{60}$	$(12;49)_{60}$	9
$(35;10;2;28;27;24;26;40)_{60}$	$(1;22;41)_{60}$	$(2;16;1)_{60}$	10
$(33;45)_{60}$	$(45)_{60}$	$(1;15)_{60}$	11
$(29;21;54;2;15)_{60}$	$(27;59)_{60}$	$(48;49)_{60}$	12
$(27;0;3;45)_{60}$	$(7;12;1)_{60}$	$(4;49)_{60}$	13
$(25;48;51;35;6;40)_{60}$	$(29;31)_{60}$	$(53;49)_{60}$	14
$(23;13;46;40)_{60}$	$(56;56)_{60}$	$(53)_{60}$	15

What mathematical relationships do the numbers exhibit?

Short side s	Diagonal d	Row	d^2-s^2
119	169	1	14,400
3367	4825	2	11,943,936
4601	6649	3	23,040,000
12709	18541	4	182,250,000
65	97	5	5,184
319	481	6	129,600
2291	3541	7	7,290,000
799	1249	8	921,600
481	769	9	360,000
4961	8161	10	41,990,400
45	75	11	3,600
1679	2929	12	5,760,000
161	289	13	57,600
1771	3229	14	7,290,000
56	106	15	8,100

The columns labeled by *Short side s, Diagonal d and Row* are three of the four columns of Plimpton 322.

The orange column ($d^2 - s^2$) wasn't on the original Plimpton 322 tablet — it was added to help you spot a pattern.
What kind of numbers do you see in this column?
(Hint: Square root)

Plimpton 322 (with first column removed and two extra columns at the end) in Hindu Arabic number system

Short side s	Diagonal d	Row	d^2-s^2	$(d^2-s^2)^{1/2}$
119	169	1	14,400	120
3367	4825	2	11,943,936	3,456
4601	6649	3	23,040,000	4,800
12709	18541	4	182,250,000	13,500
65	97	5	5,184	72
319	481	6	129,600	360
2291	3541	7	7,290,000	2,700
799	1249	8	921,600	960
481	769	9	360,000	600
4961	8161	10	41,990,400	6,480
45	75	11	3,600	60
1679	2929	12	5,760,000	2,400
161	289	13	57,600	240
1771	3229	14	7,290,000	2,700
56	106	15	8,100	90

The numbers in the added column (d^2-s^2) are all perfect squares!

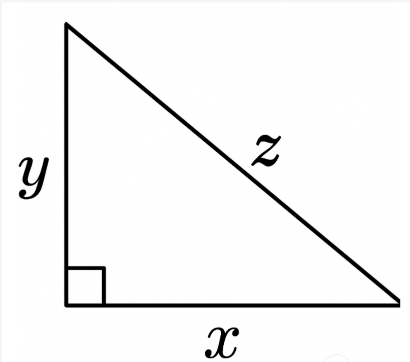
Recall: a **perfect square** is an integer whose square root is also an integer.

Plimpton 322 (with two extra columns) in Hindu Arabic number system

(d/l)2 or (s/l)2	Short side s	Diagonal d	Row	d^2-s^2	$(d^2-s^2)^{1/2}$
(1).9834028	119	169	1	14400	120
(1).9491586	3367	4825	2	11943936	3456
(1).9188021	4601	6649	3	23040000	4800
(1).8862479	12709	18541	4	182250000	13500
(1).8150077	65	97	5	5184	72
(1).7851929	319	481	6	129600	360
(1).7199837	2291	3541	7	7290000	2700
(1).6927094	799	1249	8	921600	960
(1).6426694	481	769	9	360000	600
(1).5861226	4961	8161	10	41990400	6480
(1).5625	45	75	11	3600	60
(1).4894168	1679	2929	12	5760000	2400
(1).4500174	161	289	13	57600	240
(1).4302388	1771	3229	14	7290000	2700
(1).3871605	56	106	15	8100	90

What is the purpose of the table?

Pythagorean triples



A **Pythagorean triple** is a set of three positive integers, ***x, y* and *z*** with no common factors that satisfy the equation $x^2 + y^2 = z^2$.

"No common factors" means that the largest positive integer that divides ***x, y,*** and ***z*** is 1.

A **Pythagorean triple** is a set of three positive integers, ***x, y* and *z*** that satisfy the equation $x^2 + y^2 = z^2$.

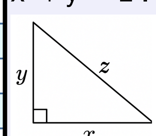
Find one Pythagorean triple that is not a multiple of (3, 4, 5) or (5, 12, 13).

Interpretation 1. of Plimpton 322:

(d/l)2 or (s/l)2	Short side s	Diagonal d	Row	d^2-s^2	$(d^2-s^2)^{1/2}$
(1).9834028	119	169	1	14400	120
(1).9491586	3367	4825	2	11943936	3456
(1).9188021	4601	6649	3	23040000	4800
(1).8862479	12709	18541	4	182250000	13500
(1).8150077	65	97	5	5184	72
(1).7851929	319	481	6	129600	360
(1).7199837	2291	3541	7	7290000	2700
		1249			
		769			
		8161			
		75			
		2929			
		289			
		3229			
		106			

Entries are generated by pairs (p, q), with no common divisor, not both odd and such that p>q.
L = 2pq,
$D=p^2+q^2$
The remaining leg is p^2-q^2

Pythagorean triples: positive integers x, y, and z, such that $x^2 + y^2 = z^2$.



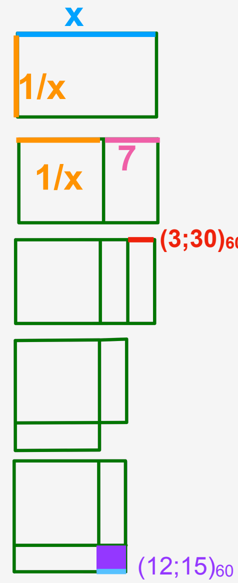
Interpretation 2 of Plimpton 322:

(d/l)2 or (s/l)2	Short side s	Diagonal d	Row	d^2-s^2	$(d^2-s^2)^{1/2}$
(1).9834028	119	169	1	14400	120
(1).9491586	3367	4825	2	11943936	3456
(1).9188021	4601	6649	3	23040000	4800
(1).8862479	12709	18541	4	182250000	13500
(1).8150077	65	97	5	5184	72
(1).7851929	319	481	6	129600	360
(1).7199837	2291	3541	7	7290000	2700
(1).6927094	799	1249	8	921600	960
(1).6426694	481	769	9	360000	600
(1).5861226	4961	8161	10	41990400	6480

Trigonometric table: if Columns L and D contain the Legs and Diagonals of right-triangles, then the values in the first column are $\tan^2$ or $1/\cos^2$. The acute angles of the triangles decrease by approximately 1°

Recall the problem: A number and its reciprocal differ by 7. Your task: form a quadratic equation whose solution gives that number.

Recall: Find a number and its reciprocal whose difference is 7 (Solve the Babylonian way)



1. You: break in half the 7 by which the reciprocal exceeds its reciprocal, and $(3;30)_{60}$ (will come up).
2. Multiply $(3;30)_{60}$ by $(3;30)_{60}$ and $(12;15)_{60}$ (will come up).
3. Append [1 00, the area,] to the $(12;15)_{60}$ which came up for you and **$(1\ 12;15)_{60}$** (will come up).
4. What is [the square-side of 1] 12;15? $(8;30)_{60}$.
5. Put down [$(8;30)_{60}$ and] $(8;30)_{60}$, its equivalent, and subtract $(3;30)_{60}$, the takiltum-square, from one (of them); append $(3;30)_{60}$ to one (of them).
6. One is **12**, the other is **5**.
7. The reciprocal is **12**, its reciprocal **5**.

$(1;\ 33\ 45)_{60}$	$(0;45)_{60}$	$(1;15)_{60}$	11
	45	75	

Row 11

$a = (1;33,45)_{60}$ → "area"
$s = (0;45)_{60}$ → "short side"
$d = (1;15)_{60}$ → "diagonal"

The scribe sets up the igi–igibi problem

- From **s**, I compute $2\cdot s = (1;30)_{60}$.
- I speak the **igi-igibi problem**: *"Find a number and its reciprocal; their difference is $(1;30)_{60}$."*

This is a simplified version of a problem that can be set up with Row 11, consistent with by modern scholarship.

The student's solution

- Halve the difference: $(1;30)_{60} \div 2 = s = (0;45)_{60}$.
- Make a square and append one: $1 + s^2 = a = (1;33,45)_{60}$.
- Take the square-side of a: $\sqrt{a} = d = (1;15)_{60}$.
- Take the two numbers:
  - $x = d + s = (2;00)_{60}$, and
  - $1/x = d - s = (0;30)_{60}$.

Note: x × 1/x is a power of 60.

Words and Pictures: New Light on Plimpton 322 https://doi.org/10.1080/00029890.2002.11919845

Interpretation 3 of Plimpton 322

(d/l)2 or (s/l)2	Short side s	Diagonal d	Row	d^2-s^2	$(d^2-s^2)^{1/2}$
(1).9834028	119	169	1	14400	120
(1).9491586	3367	4825	2	11943936	3456
(1).9188021	4601	6649	3	23040000	4800
(1).8862479	12709	18541	4	182250000	13500
(1).8150077	65	97	5	5184	72
(1).7851929	319	481	6	129600	360
(1).7199837	2291	3541	7	7290000	2700
(1).6927094	799	1249	8	921600	960
(1).6426694	481	769	9	360000	600
(1).5861226	4961	8161	10	41990400	6480
(					)
(1)					00
(1)					0
(1)					00
(1)					0

Teaching tool: The tablet gives examples (or starting values) for generating exercises (igi-igibi problems), possibly used by teachers to create math problems for students.

How were the numbers calculated (algorithm or procedure)? Who made it?

The Reciprocal Pair Method: Row 11 of Plimpton 322

Short side s	Diagonal d	Row	r
45	75	11	2
$(45)_{60}$	$(1;15)_{60}$		$(2;0)_{60}$

Step 1: Choose a regular number → r = 2

Step 2: Find r's reciprocal → 1/r = 1/2

Step 3: Half-difference and half-sum →

s/l = (r – 1/r)/2 = 3/4 = 45/60

d/l = (r + 1/r)/2 = 5/4 = 75/60

Step 4: Rescale (multiply by 60) →

s = 45

d = 75

Then l = 60.

The Reciprocal Pair Method: Row 1 of Plimpton 322

Short side s	Diagonal d	Row	r
119	169	1	12/5
$(1;59)_{60}$	$(2;49)_{60}$		$(2;24)_{60}$

Step 1: Choose a regular number → r = 12/5 = $(2;24)_{60}$

Step 2: Find r's reciprocal → 1/r = 5/12 = $(0;25)_{60}$

Step 3: Half-difference and half-sum →
s/l= (r – 1/r)/2 = 119/120
d/l = (r + 1/r)/2 = 169/120

Step 4: Rescale (multiply by 120) →

s = 119

d = 169

Then l =120.

The Reciprocal Pair Method: Row 5 of Plimpton 322

Short side s	Diagonal d	Row	r
65	97	5	9/4
$(1;59)_{60}$	$(2;49)_{60}$		$(2;15)_{60}$

Find s and d on this case, following the previous example (below)

Example from row 1

Step 1: Choose a regular number → r = 12/5 = $(2;24)_{60}$

Step 2: Find r's reciprocal → 1/r = 5/12 = $(0;25)_{60}$

Step 3: Half-difference and half-sum →
s/l= (r – 1/r)/2 = 119/120
d/l = (r + 1/r)/2 = 169/120

Step 4: Rescale (multiply by 120) →

s = 119

d = 169

Then l =120.

Short side s	Diagonal d	Row	r
119	169	1	12/5
$(1;59)_{60}$	$(2;49)_{60}$		$(2;24)_{60}$

The Reciprocal Pair Method: Row 5 of Plimpton 322

Short side s	Diagonal d	Row	r
65	97	5	9/4
$(1;59)_{60}$	$(2;49)_{60}$		$(2;15)_{60}$

Step 1: Choose a regular number → $9/4 = (2;15)_{60}$

Step 2: Find r's reciprocal → $1/r = 4/9 = (0;6,40)_{60}$

Step 3: Half-difference and half-sum →
$s/l = (r - 1/r)/2 = (9/4 - 4/9)/2 = (65/36)/2 = 65/72$
$d/l = (r + 1/r)/2 = (9/4 + 4/9)/2 = (97/36)/2 = 97/72$

Step 4: Rescale (multiply by 72) →

s = 65

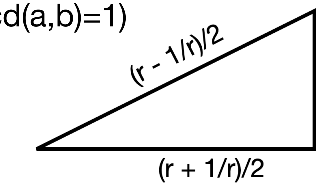
d = 97

Then, l = 72

Justifying the Reciprocal Pair Method

Assume $r > 1$ is a positive rational,
Write $r = a/b$ in lowest terms (that is, $\gcd(a,b)=1$)
Then

- $(r - 1/r)/2 = (a^2 - b^2)/(2ab)$
- $(r + 1/r)/2 = (a^2 + b^2)/(2ab)$



Remark: $\gcd(a.b, a^2 + b^2) = 1$ and $\gcd(a.b, a^2 - b^2) = 1$ (because $\gcd(a, b)=1$ implies $\gcd(a, b^2)=1$ and $\gcd(b, a^2)=1$).

Then be obtain Pythagorean triples:

- $((a^2 - b^2)/2, (a^2 + b^2)/2, ab)$ if both a and b are odd
- $(a^2 - b^2, a^2 + b^2, 2ab)$ if one of them a or b is even.

Plimpton 322: Evidence for a Teacher's Resource

- The choice of **regular numbers** follows typical scribal school practice.
- Rows can be generated from regular numbers using the **reciprocal pair method**, a standard computational procedure.
- **Structured** like other Old Babylonian school tables: the same setup repeated with varying numbers.
- The **error pattern** is consistent with copying and scaling reciprocals.

Most modern scholars agree that this is the most **historically consistent** interpretation.

Plimpton 322: Evidence against Pythagorean triples interpretation

- No similar table found anywhere in the Mesopotamian.
- The ordering of the missing columns would violate standard formatting rules
- Column I has no natural explanation.

Plimpton 322: Evidence against Trigonometry Interpretations

- No textual or material evidence for angle-based reasoning in Old Babylonian geometry.
- No evidence that scribes conceptualized “trigonometry” in any form.
- The tablet does not present ratios of sides in a way consistent with later trigonometric tables.
- The ordering of rows reflects numerical structure, not angular progression.

Even groups answer {1,A}, odd groups answer {2,B}

1. What was the primary use of tables of reciprocals in Mesopotamian mathematics?

2. Looking at Plimpton 322, what mathematical pattern connects the short side and diagonal in each row?
Hint: Recall the “orange column” we added in class.

A. Different scholars have proposed different interpretations of Plimpton 322. What does the debate (trigonometric table, Pythagorean triples, teaching tool) show us about the way historians study ancient texts?

***B*. What did the triangle drawing activity reveal about cultural assumptions in mathematics?**

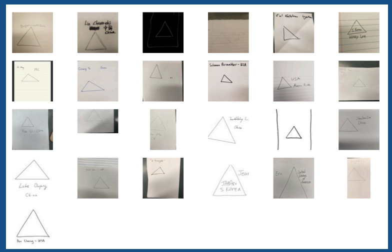
Form groups of 3–4.
Choose one speaker.
Write one agreed-upon answer for each assigned question.
Do not write until all group members agree.
You have 6 minutes.

"Further, if we believe that **Plimpton 322 was intended to be a list of parameters to aid the setting of school mathematics problems** (and the typological evidence suggests that it was), the question '**how was the tablet calculated?**' does not have to have the same answer as the question 'what problems does the tablet set?' The first can be answered most satisfactorily **by reciprocal pairs,** as first suggested half a century ago, and the second by some sort of right-triangle problems.”

E. Robson

Robson, Eleanor (August 2001), "Neither Sherlock Holmes nor Babylon: a reassessment of Plimpton 322", Historia Math., 28 (3): 167–206, doi:10.1006/hmat.2001.2317,

Ancient mathematical texts and artefacts, if we are to understand them fully, must be viewed in the light of their mathematico-historical context, and not treated as artificial, self-contained creations in the style of detective stories.
Eleanor Robson



Calendar

Fragment of a circular clay tablet with depictions of constellations (planisphere). Neo-Assyrian. - British Museum

Concrete impact:
360 degrees angle
60 minutes in an hour
60 seconds in a minute