



## Mesoamerica

Maya and Inca

# Incas



Inca Quipu  
Claus Ableiter nur hochgeladen aus  
enWiki - enWiki, hochgeladen von User  
Lyndsaruel



What does Machu Picchu reveal about how the Inca organized people and knowledge across hard terrain? Include one detail from the video.

<https://youtu.be/cnMa-Sm9H4k?si=0E-vatTgRHYSmRlc>

## About the Incas

### Empire Scale

- **Where/When:** Andean highlands, 1400s–1500s; empire spanned approximately 2,500 miles (~4,000 km)
- **Fall:** Spanish invasion (mid-1500s), civil war, and disease ended imperial rule.

### Organization & Communication

- **Statecraft:** Central authority; **Quechua** as lingua franca
- **Record-keeping:** **Quipus**; no **alphabetic** or **syllabic** writing system
- **Infrastructure:** ~30,000 km (~18,600 mi) road network, **chaski** relay runners, suspension bridges



Inca empire in 1500



Early Morning in Machu Picchu  
Pedro Szekeley at <https://www.flickr.com/photos/pedrosz/>

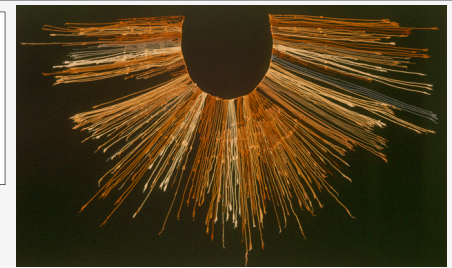
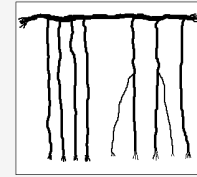




## What is a quipu?

Threads That Speak: How The Inca Used Strings to Communicate | National Geographic  
<https://youtu.be/AmPvz1kCbOw>

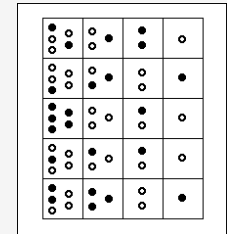
### More about incas and quipus



The largest number so far discovered on a quipu is **97,357**.

Quipu  
 By Claus Ableiter nur hochgeladen aus enWiki - enWiki, hochgeladen von User Lyndsaruell; siehe [http://en.wikipedia.org/wiki/Image:Inca\\_Quipu.jpg](http://en.wikipedia.org/wiki/Image:Inca_Quipu.jpg), CC BY-SA 3.0, <https://commons.wikimedia.org/w/index.php?curid=2386739>

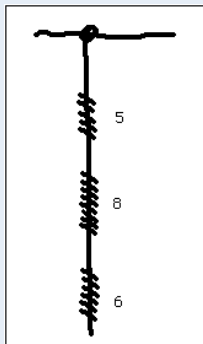
*The ordinary judges gave a monthly account of the sentences they imposed to their superiors, and they in turn reported to their immediate superiors, and so on finally to the Inca or those of his Supreme Council. The method of making these reports was by means of knots, made of various colours, where knots of such and such colours denote that such and such crimes had been punished. Smaller threads attached to thicker cords were of different colours to signify the precise nature of the punishment that had been inflicted.*



yupana and is presumed to be the counting board of the Incas.  
[https://mathhistory.st-andrews.ac.uk/HistTopics/inca\\_mathematics/](https://mathhistory.st-andrews.ac.uk/HistTopics/inca_mathematics/)

Inca Garcilaso de la Vega ~ 1600

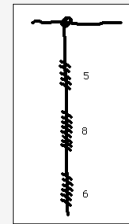
Incas used a positional number system and record their numbers in chords like the one on the photo. What number is shown on the left chord? (Units on the bottom)



Portion of a quipu  
[https://mathhistory.st-andrews.ac.uk/HistTopics/inca\\_mathematics/](https://mathhistory.st-andrews.ac.uk/HistTopics/inca_mathematics/)

Quipu  
 By P13.124 - Own work, CC BY-SA 4.0, <https://commons.wikimedia.org/w/index.php?curid=69539294>

## Quipus



Portion of a quipu  
[https://mathhistory.st-andrews.ac.uk/HistTopics/inca\\_mathematics/](https://mathhistory.st-andrews.ac.uk/HistTopics/inca_mathematics/)

Quechua was the main language of the Inca Empire.

Quipu (or khipu) is the Quechuan word for "knot".



Quipu  
 By P13.124 - Own work, CC BY-SA 4.0, <https://commons.wikimedia.org/w/index.php?curid=69539294>

**What it is:** A set of colored, knotted cords used to record data.

**Structure:** one main cord; attached pendant cords; optional subsidiary cords; sometimes a top cord tied opposite the pendants.

**Encoding cues:** cord color, cord order/placement, knot type, knot position, and spaces between knots.

**Where numbers are:** knots on pendants/subsidiaries (not on the main cord).

**Number system:** base-10 place value; knot clusters separated by spaces; highest place nearest the main cord.

## Quipus



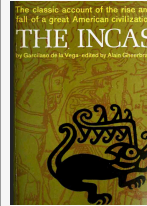
Early Morning in Machu Picchu  
Pedro Szekeley at <https://www.flickr.com/photos/pedrosz/>



Quipu  
By P13.124 - Own work, CC BY-SA 4.0, <https://commons.wikimedia.org/w/index.php?curid=69539294>

The Incas inhabited mountainous terrain in the Andes, which made transporting heavy materials difficult.

The **quipu** was **lightweight, portable, and durable** → ideal for messengers traveling long distances through difficult terrain across their vast empire.



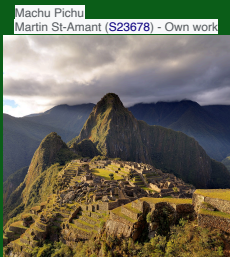
The Incas also had an excellent knowledge of **arithmetic**, and the way they counted was quite remarkable. Throughout the Empire, all **levies and taxes, both exemptions and claims, were recorded by means of knots made in colored cords.** They could add, subtract, and multiply with these knots, and in order to determine what was owed to each village, made their allotments on the basis of little pebbles and grains of corn, in such a way that there could be no mistakes in their calculations. Special bookkeepers were appointed for each section, whether it had to do with war or peace, such as taking a census of the vassals, or of levies, herds, laws, and ceremonies, and they were able to give an accounting at any moment, their collections of cords and knots being as clear and exact for them as any books. The head accountants, who were in charge of several things at a time, worked with these same means, quite as easily as the others.

Gómez Suárez de Figueroa, (Inca Garcilaso de la Vega) 1600

## Mayans



Maya  
Protoplasmakid - Own work



Machu Picchu  
Martin St-Amant (S23678) - Own work



Daniel Schwen - Own work

MONTH:

DAY:

YEAR:

CONVERT

**Long Count Date**  
**13.0.8.16.3**

13 baktun  
13 X 144,000 days = 1,872,000 days

0 katun  
0 X 7,200 days = 0 days

8 tun  
8 X 360 days = 2,880 days

16 uinal  
16 X 20 days = 320 days

3 k'in  
3 X 1 day = 3 days

Tzolk'in Date: 9 Ak'b'al  
Haab Date: 1 Yax  
Lord of the Night: G8

<https://maya.rnai.si.edu/calendar/maya-calendar-converter>

Look up "Maya culture" or "Maya script" in Google Images. What do the images suggest about the complexity or priorities of Maya civilization? Consider what you see in their art, writing, and buildings.

## Maya: Where and When, Cities and Society

- **Mesoamerica** (Yucatán, Guatemala, Belize, W. Honduras/Chiapas).
- **When:** 2000 BCE → 1600. CE Over **3,000 years!**
- **Independent city-states:** complex social hierarchy.
- **Religion:** Polytheistic, tied to nature **cycles**; priestly rituals and offerings.
- **Architecture:** stepped pyramids, palaces, ballcourts, plazas, inscriptions.



Map from Wikipedia

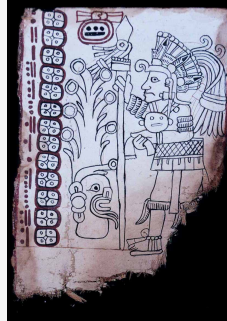


Map from Wikipedia

## Maya's Focus: Astronomy and the Passage of Time

- **Hieroglyphic writing** used for history, ritual, astronomy.
- **Base-20 mathematics** with an independent zero.
- **Calendars:** 260-day ritual + 365-day solar; Long Count chronology.
- Observatories and precise **calendrical/astronomical** tracking.

After counting **8** days forward from the dates on the left marking, Venus reemerges as the morning star



Page 7 of the Maya Codex of Mexico



Photograph of a Maya Vase. Unknown author - Art from late Classic c. 600 - 900 AD, per book "The Blood of Kings, Dynasty and Ritual in Maya Art" by Linda Schele, Mary Ellen Miller, Justin Kerr, Kimbal Art Museum, Fort Worth, 1986, plate 115 Mechanical reproduction of art more than 1,000 years old. [https://commons.wikimedia.org/wiki/File:Maya\\_vase.jpg](https://commons.wikimedia.org/wiki/File:Maya_vase.jpg)



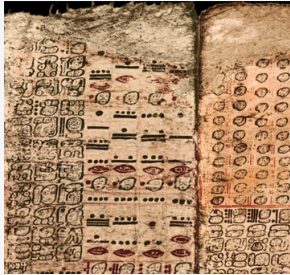
# Introduction and Mayan script



One word that captures what surprised you—or what you found remarkable—about the Maya.

# The Mayas

- had a **written language**, which help us learn about their mathematics.
- records were preserved in **bark paper** or carved in **stone**.
- priestly class who studied mathematics and astronomy.
- Remarkable development between 250 to 900 of our era.



Mayan page in tree bark  
From "Cracking the Mayan code"  
<https://www.pbs.org/wgbh/nova/mayacode/>



Mayan writing carved on stone  
From "Cracking the Mayan code"  
<https://www.pbs.org/wgbh/nova/mayacode/>



The Monuments of Yaxchilan: Lintel 14 - photograph by Jorge Pérez de Lara



## Breaking the Maya Code #2: The Maya Script

<https://youtu.be/1t5FALvtMQg>

que antes de si real y en esto no pareceran auyg plens de  
qu'risien ellos de su curiosidad. Exemplo.   
despues al cabo le pegan la parte junta. Ah. que quiere decir  
aguna por la baco tiene a. h. ante de si lo ponen ellos al  
principio con a. y al cabo desta manera  Tambie  
lo escriuen a partes de la vida y otra ma  Merayo  
no pudiera aqui ni trahera dello sino por dar cuenta entera  
de las cosas desta gente. Momhah. quiere decir no quiere ellos  
lo escriuen a partes desta manera    
Seguense sin abc.    
   
De las letras que aqui faltan carece esta lengua  
y tiene otras añadidas de la nuestra para otras  
cosas q las ha menester y ya no usan para nada destas  
sus caracteres especialmente la gente moça q au aprendido  
los nros

Image of the page from Diego de Landa's 16thC. manuscript, Relación de las Cosas de Yucatán, in which he describes the famous "de Landa alphabet". This "alphabet" shows the letters of the Spanish language and the Maya hieroglyphics symbols which were supposed to correspond with them (according to de Landa's interpretation).



## Constantine Rafinesque

Breaking the Maya Code #3: The Maya Books. <https://youtu.be/GgCf3Q7qooQ>. (3.09 minutes)

What type of Mayan hieroglyphs were deciphered first?

# Mayan Calendars

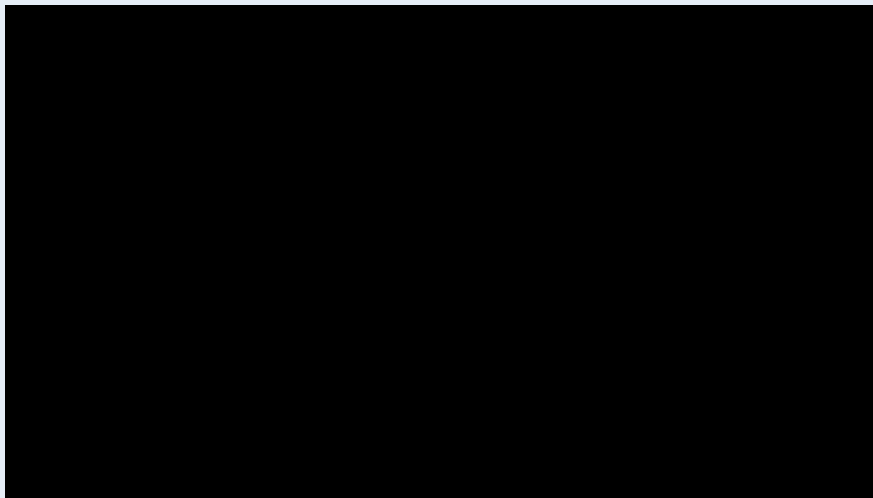
## Two (or three, depending on how we count) calendars

### Calendar Round (cyclic)

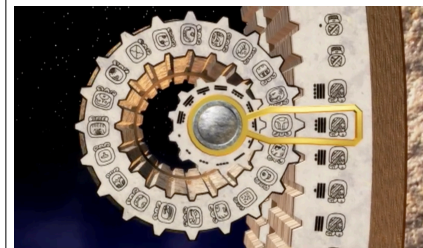
- Tzolk'in: 260-day sacred cycle (13×20)
- Haab: 365-day civil/solar calendar

**Long Count:** day count since creation;  
fixes dates.

Describe the Maya calendars. Optional: name a  
math structure that relates to them.



Breaking the Maya Code #4: The Maya Calendar <https://youtu.be/qhWltvjK9Yg> (2.18minutes)  
Breaking the Maya Code by Night Fire films (extract) <https://vimeo.com/ondemand/115559/223534703>

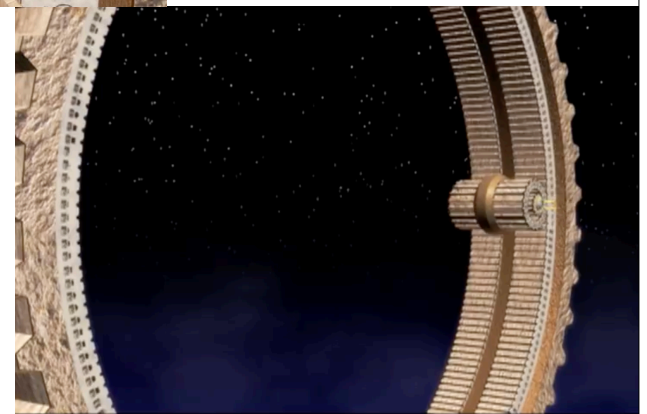


### Round calendar

- The two smaller wheels represent the Tzolk'in or sacred calendar.
- The largest wheel represents the farmers or Haab calendar

## Modular arithmetic!

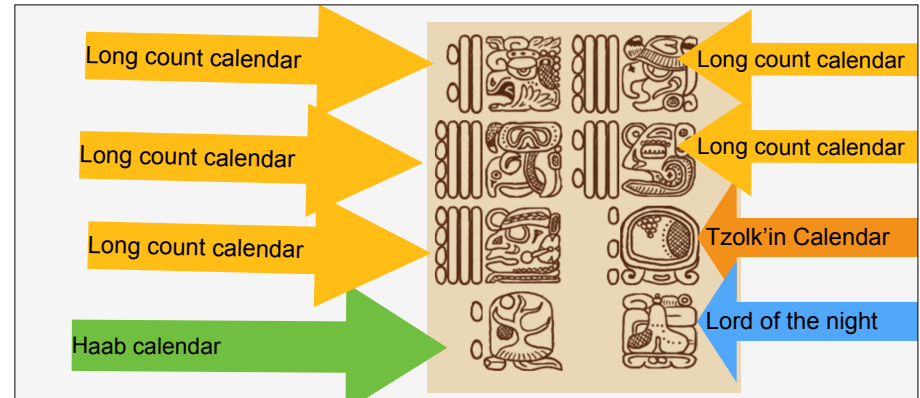
The wheels are a  
modern way of  
representing the  
calendars, not a  
Mayan  
representation.





Part 2 of. Breaking the Maya Code #4: The Maya Calendar <https://youtu.be/qhWltvjK9Yg>

1. What do the large numbers in the Dresden Codex represent?
2. (Optional) Explain the Maya Long Count—what it measures and how it works.



Based on astronomical observations, the Maya developed an elaborate system of calendars.

- Round calendar
  - Tzolk'in calendar
  - Haab calendar
- Long count

## Mayan Calendars

<https://maya.nmai.si.edu/calendar/maya-calendar-converter>

## Today in Mayan Calendars

<https://maya.nmai.si.edu/calendar/maya-calendar-converter>



MONTH:  DAY:  YEAR:

**Long Count Date**  
**13.0.12.17.5**

13 baktun  
 13 X 144,000 days = 1,872,000 days

0 katun  
 0 X 7,200 days = 0 days

12 tun  
 12 X 360 days = 4,320 days

17 uinal  
 17 X 20 days = 340 days

5 k'in  
 5 X 1 day = 5 days

Tzolk'in Date: 2 Chikchan  
 Haab Date: 3 Yax  
 Lord of the Night: G

Round calendar  
(Haab + Tzolk'in)

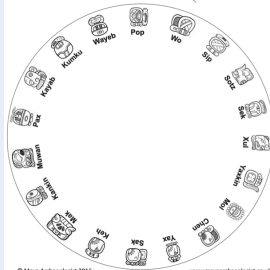
## Haab Calendar

- A solar calendar used to guide farming and civic life
- 18 “months” of 20 days each + 5 Uayeb days (unlucky/nameless)
- Each “month” had its own name (e.g., Pop, Wo, Sip)
- Days were numbered from 0 to 19 (e.g., 0 Pop, 1 Pop, ... 19 Pop, 0 Wo...)

The months of the Haab Calendar	
1. Pop	12. Keh
2. Wo	13. Mak
3. Sip	14. K'ank'in
4. Sotz'	15. Muwan
5. Sek	16. Pax
6. Xul	17. K'ayab
7. Yaxk'in	18. Kum'ku
8. Mol	19. Wayeb
9. Ch'en	(the short 5-day unlucky month)
10. Yax	
11. Sak	

Why does the Haab have 365 days —just like ours—even though it developed independently?  
Hint: What must farmers and astronomers track every year?

**Uayeb** translates to “That which has no name” — a time of caution and reduced activity.



## Tzolk'in or Sacred Calendar

- 20 day signs
- 13 sequential numbers

Each date is specified by a pair (**a, Db**), where

- $1 \leq a \leq 13$
- $1 \leq D_b \leq 20$  (We'll use numbers instead of day signs. Imix = 1, Ik=2, etc)
- The Tzolk'in cycle begins with (1, 1), continues with (2,2), (3,3), (13,13), (1,14), (2,15) and so on.

1. How many unique dates are in the Tzolk'in calendar?  
(Hint: Each day is specified by a pair, see above)
2. (Optional) If today is 10 K'an (that is, (10,4)), what will the day be 5 days later?

Day Signs	
1. Imix	11. Chuwen
2. Ik'	12. Eb'
3. Ak'bal	13. Ben
4. K'an	14. Ix
5. Chikchan	15. Men
6. Kimi	16. Kib'
7. Manik'	17. Kab'an
8. Lamat	18. Etz'nab'
9. Muluk	19. Kawak
10. Ok	20. Ajaw



Image from Breaking the Maya Code

A Maya representation of the Tzolk'in from the Madrid Codex. Time is represented by 260 dots marking a path or a journey of twenty days and thirteen numbers.



Source: Smithsonian, National Museum of the American Indian  
<https://maya.nmai.si.edu/sites/default/files/resources/The%20Maya%20Calendar%20System.pdf>

If the Haab and Tzolk'in start together (e.g., 0 Pop, 1 Imix), how many days until they realign on the same date again?

- A Haab year contains 365 days.
- A Tzolk'in cycle contains 260 days.



Image from Breaking the Maya Code



A contemporary representation of the Calendar Round, interlocking the Tzolk'in (left) with the Haab (right).  
Source: Smithsonian, National Museum of the American Indian  
<https://maya.nmai.si.edu/sites/default/files/resources/The%20Maya%20Calendar%20System.pdf>

The Haab and Tzolk'in together form the Calendar Round.

## The Calendar Round: Haab + Tzolk'in



Image from Breaking the Maya Code

- **Haab + Tzolk'in realign every 18,980 days** (= 52 Haab years = 73 Tzolk'in cycles = LCM(260,365)).
- **Calendar Round date:** a pair (Tzolk'in day, Haab day) that uniquely identifies any event within 52 years.
- **Beyond 52 years:** the **Long Count** records longer spans.
- The **52-year return** marked major cultural celebrations and ceremonies.
- Reaching one's birth date again was considered a full life cycle. A person who reaches 52 years of age, attains the special wisdom of an elder.

# Long count calendar

What numbers are shown above glyphs a–d, (optional e)?



Images from the documentary Breaking the Mayan Code <https://nightfirefilms.org/breaking-the-maya-code/>



The long count calendar counts how much time elapsed since the creation date.

Images from the documentary Breaking the Mayan Code <https://nightfirefilms.org/breaking-the-maya-code/>

# Long count calendar

A long count calendar date is written as 2.3.4.5.6.



Reading the Maya Calendar  
<https://maya.rmit.ac.edu/calendar/maya-calendar-converter>

• **Long Count** = elapsed days since the creation date.

• **Units:**

- kin = 1 day
- uinal = 20 kin (≈ 20 days ~ “month”)
- tun = 18 uinal = 360 d (≈ “year” minus 5 days)
- k’atun = 20 tun = 7,200 d (≈ 19.7 years)
- bak’tun = 20 k’atun = 144,000 d (≈ 394.3 years)

• Example shown: 9.14.19.8.0 = 9 bak’tun, 14 k’atun, 19 tun, 8 uinal, 0 kin  
 (= 1,404,760 days ≈ 3,847.3 years)



## How many years are in a bak’tun? Assume 365-day years; calculators allowed.

• **Long Count** = elapsed days since the creation date.

• **Units:**

- kin = 1 day
- uinal = 20 kin
- tun = 18 uinal
- k’atun = 20 tun
- bak’tun = 20 k’atun

• **Example:** 9.14.19.8.0 = 9 bak’tun, 14 k’atun, 19 tun, 8 uinal, 0 kin = 1,403,800 days ≈ 3,846.0 years (using 365-day years) after the creation date.

- A Long Count date and a Calendar Round date were used by the Maya to place mythical and historical events in chronological order.
- An era (full cycle) is composed of 13 bak’tuns.
- Note that this is similar to the Gregorian calendar system that counts days, months, years, centuries and millennia.

## What is the day after the date below in the Long Count calendar? Write it as *bak’tun.k’atun.tun.uinal.kin*

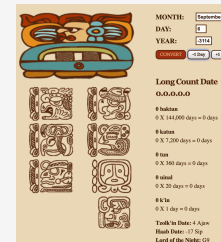


Teams of 3

- kin = 1 day
- uinal = 20 kin
- tun = 18 uinal
- k’atun = 20 tuns
- bak’tun = 20 k’atun



From site of etno.C. Designed with the official creation date of 13.0.0.0.0 (21/23 Dec 2012, correlation-dependent) → completion of 13 bak'tuns since the creation date.  
 Page 11, 2011-2012. © The University of Queensland  
 @ethnology



<https://maya.rmit.ac.edu/calendar/maya-calendar-converter>

## The 2012 “end of the world”?

• **Creation Date in Maya inscriptions:**

- **Long Count:** 13.0.0.0.0; Tzolk’in: 4 Ajaw 8 Kumk’u
- ~ 11 Aug 3114 BCE.

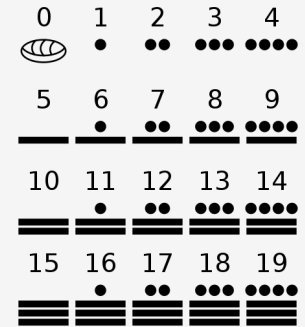
• **2012 “end of the world” hype:** Long Count reached 13.0.0.0.0 (21/23 Dec 2012, correlation-dependent) → completion of 13 bak’tuns since the creation date.

• Maya texts celebrate such **completions with renewal rites**, and they even compute far-future dates (e.g., 1.0.0.0.0.0, five zeros), showing the count (and the world!) continues.

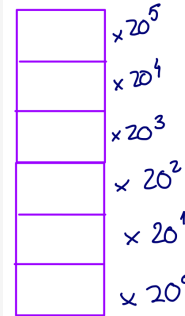
# All calendars again

## Mayan number system

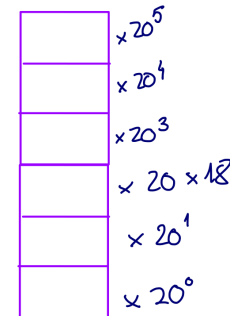
Write down 365 in the Mayan number system. (numbers are written vertically, lower units at the bottom)



Base 20



Modified base 20



Mayan numerals - Wikipedia

## Haab Calendar

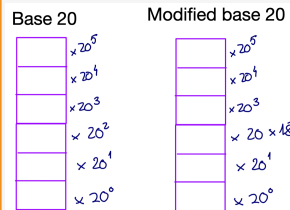
When days were counted, the third-place unit, instead of being 20.20 was 20.18.

This was probably because there are 360 days in the “regular” part of the 365 days calendar known as the *Haab* and the other 5 days were considered “unlucky” (and so best not included in the count)

Translate a Mayan number on to Hindu-Arabic in two ways (units at the bottom):

1. Assuming Mayan number system was “honest” base 20

2. Assuming the “calendar way” (that is days were counted, the third-place unit, instead of being 20x20 was 20x18).



## All calendars

Calendar Round

- **Haab:** 365-day solar.
- **Tzolk'in:** 260-day ( $13 \times 20$ ).
- **Together:** Calendar Round → dates repeat every 52 Haab years.

Long count

- Counts days since a fixed epoch (creation date).
- Written a.b.c.d.e = bak'tun.k'atun.tun.uinal.kin.  
Value in days:  $144,000a + 7,200b + 360c + 20d + e$ .
- Period endings (e.g., 13.0.0.0.0) mark completions; the count continues (texts even project to 1.0.0.0.0.0).

# And more...

## Codex-Style Vessel CE 700–750.

Two teaching scenes with Itzam—an elderly creator/atlas-like deity—  
instructing four students



**Itzam ID cues:** Aged face; netted headdress with a brush tucked in.

**Scene 1:** Itzam uses a pointer and a folded codex; a speech thread with **bar-and-dot numerals suggests arithmetical/calendrical calculation.**



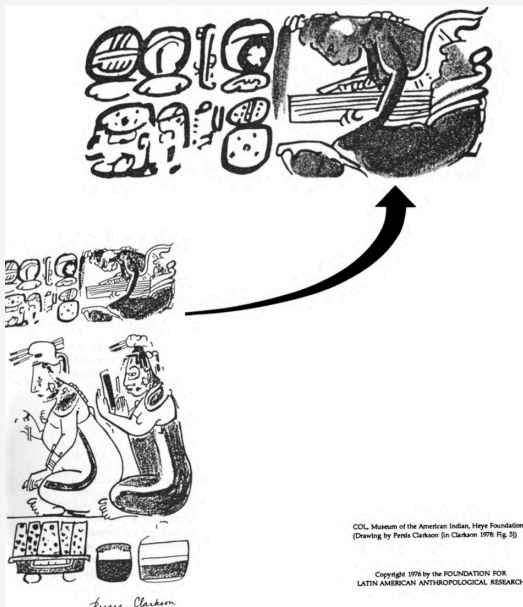
**Scene 2:** Itzam taps the ground while addressing two students; spoken glyphs on a speech thread.

**Takeaway:** Elite scribal education in mathematics/calendar lore framed as instruction from a creation deity.

<https://kimbrellart.org/collection/vap-200404>

## Answer at least two:

1. Name one concrete thing you remember about the Maya.  
(Hint: writing, zero/base-20, pyramids, maize, deities, astronomy)
2. What's the difference between the Tzolk'in and the Haab, and what did each mainly guide?  
(Hint: Tzolk'in = 260 ritual, Haab = 365)
3. What are the Tzolk'in and the Haab, and how do they combine to make the Calendar Round?  
(Hint: two cycles interlock, repeat every 52 Haab years)
4. What does 13.0.0.0.0 mean in the Long Count, and why did some people talk about 2012?  
(Hint: days since creation, 13 bak'tuns completed, correlation sets date)
5. Reflect: What most changed your impression of the Maya—writing, timekeeping, cities, or continuity—and why?



A female  
Mayan scribe?  
Still debated...

CCIL, Museum of the American Indian, Heye Foundation  
(Drawing by Francis Clarkon [in Clarkon 1976 Fig. 5])  
Copyright 1976 by the FOUNDATION FOR  
LATIN AMERICAN ANTHROPOLOGICAL RESEARCH

Francis Clarkon