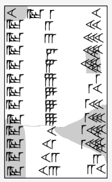
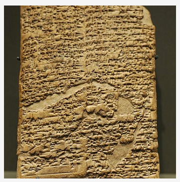
Mathematics in Mesopotamia



- Setting the Stage
- Cuneiform Writing: Birth, Decipherment, and Surviving Texts
- Mesopotamian Mathematics in a slide-nutshell
- More about base 60
- Calculation of Areas
- An amazing approximation of √2
- Tables of Reciprocals
- Plimpton 322
- Solutions of equations

Write one key feature of Egyptian mathematics in 5 words or less.

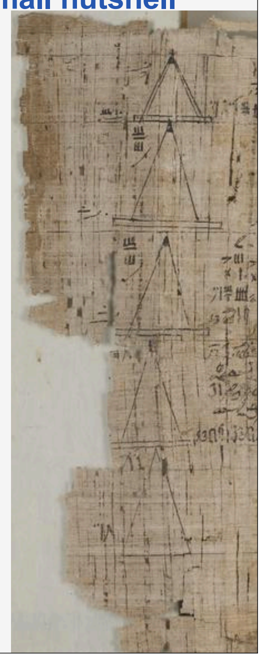
Ancient Egyptian Mathematics in a very small nutshell

Sources & Context:

- Very few surviving sources (papyrus is fragile!)
- Practical problems:
  - Example: area and volume
  - Example: fair division of loaves of bread.

Key Mathematical Features

- Linear equations solved by "false position" method
- Doubling/halving as basic arithmetic operations
- Egyptian fractions: written as sums of unit fractions (1/n)
- Surprisingly accurate π approximation.
- Some problems with theoretical interest
  - Adding $7 + 7^2 + 7^3 \ldots + 7^5$



Setting the Stage

Civilizations 3000 years' span

- Sumerian
- Babylonian
- Assyrian
- Akkadian

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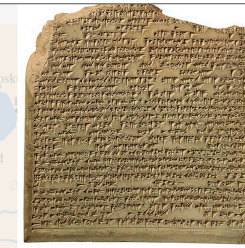

Tablet Inscribed in Babylonian with a Ritual for the Observances of Eclipses-The Morgan Library and Museum

Cuneiform Writing on Clay Tablets

- Wedge-shaped "cuneiform" script, written by trained scribes.
- Hundreds of thousands of tablets have survived.
- Most are **administrative records** (memos, receipts, wages).
- Others preserve:
  - Law codes
  - Magical rituals
  - Military campaigns and battles
  - Myths, including the Epic of Gilgamesh
  - Math!

Pick one mathematical idea from the video that you use every day. Describe how you use it.

Ancient Mesopotamia 101- National Geographic
https://youtu.be/xVf5kZA0HtQ?si=PK4TY11m37V3GYK5

Administration of surplus.

- The fertile Mesopotamian land produced **agricultural surplus,**
- That surplus was **variable** (floods, droughts, seasonal cycles).
- There was a **need to track quantities over time**: how much grain was stored, owed, distributed, or lost.

That practical pressure "pushes"abstraction

- **Numbers** to represent quantities independent of the objects themselves.
- **Calendars** to anticipate floods, planting, and harvest cycles.
- **Writing** to record obligations and inventories beyond human memory.

Abstraction emerges from the need to coordinate resources across time, people, and uncertainty.

A praise poem of King Šulgi

I am a king, offspring begotten by a king and borne by a queen.
I, Šulgi the noble, have been blessed with a favorable destiny right from the womb.
When I was small, I was at the academy, where I learned the scribal art from the tablets of Sumer and Akkad.
None of the nobles could write on clay as I could.
There where people regularly went for tutelage in the scribal art,
I qualified fully in subtraction, addition, reckoning and accounting.
The fair Nanibgal, Nisaba, provided me amply with **knowledge and comprehension.**
I am an experienced scribe who does not neglect a thing.

The poem links accounting and calculation with "knowledge and comprehension" and presents them as royal achievements. What does this suggest about how Mesopotamian culture understood the value of mathematics?

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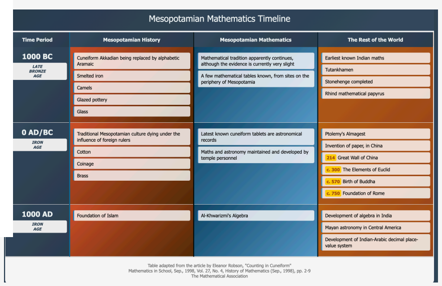
The fair Nanibgal, Nisaba" signifies one goddess, patron of Sumerian scribes.
Nanibgal meant Lady of Great Wisdom"
King Šulgi of Ur (reigned 2000 BCE)
Electronic Text Corpus of Sumerian Literature, "A praise poem of Shulgi (Shulgi B)": etcsl.orinst.ox.ac.uk/section2/tr24202.htm

https://www.math.stonybrook.edu/~moira/courses/mat336-fall2025/zmaterial/timelineMobil.html



Mesopotamian Mathematics Timeline

Time Period	Mesopotamian History	Mesopotamian Mathematics	The Rest of the World
1000 BC LATE BRONZE AGE	Cuneiform Akkadian being replaced by alphabetic Aramaic Smelted iron Camels Glazed pottery Glass	Mathematical tradition apparently continues, although the evidence is currently very slight A few mathematical tables known, from sites on the periphery of Mesopotamia	Earliest known Indian maths Tutankhamen Stonehenge completed Rhind mathematical papyrus
0 AD/BC IRON AGE	Traditional Mesopotamian culture dying under the influence of foreign rulers Cotton Coinage Brass	Latest known cuneiform tablets are astronomical records Maths and astronomy maintained and developed by temple personnel	Ptolemy's Almagest Invention of paper, in China 214 Great Wall of China c. 300 The Elements of Euclid c. 570 Birth of Buddha c. 750 Foundation of Rome
1000 AD IRON AGE	Foundation of Islam	Al-Khwarizmi's Algebra	Development of algebra in India Mayan astronomy in Central America Development of Indian-Arabic decimal place-value system

Table adapted from the article by Eleanor Robson, "Counting in Cuneiform"
Mathematics in School, Sep., 1998, Vol. 27, No. 4, History of Mathematics (Sep., 1998), pp. 2-9
The Mathematical Association

📱 **Timeline Instagram Story Activity**

https://www.math.stonybrook.edu/~moira/courses/mat336-fall2025/zmaterial/timeline-activity.html

Cuneiform Writing: Birth, Decipherment, and Surviving Texts

Mesopotamian Accounting Tokens (4000–3300 BCE)

(Image by Denise Schmandt-Besserat and the University of Pennsylvania Museum of Archaeology and Anthropology, University of Pennsylvania, Philadelphia.)

Tokens Meaning Examples

- Cone represents a measure of grain
- Tetrahedron represents a unit of work.

Envelopes:

- Tokens were stored in clay envelopes for verification.
- Engraved with tokens content

Image Denise Schmandt-Besserat and Musée du Louvre, Département des Antiquités Orientales, Paris.)

From Accounting Tokens to Writing (3300–3200 BCE)

Image Denise Schmandt-Besserat and Musée du Louvre, Département des Antiquités Orientales, Paris.)

Each circular impression → one large measure of grain; each wedge → smaller measure of grain

(Image by by Denise Schmandt-Besserat and Musée du Louvre, Département des Antiquités Orientales, Paris.))

3D tokens → 2D impressions → first writing in Mesopotamia. Impressions evolved into cuneiform script.

Writing was invented independently in: Mesopotamia, Egypt, China, Mesoamerica. Possibly also Indus Valley Civilization (Harappan) - c. 2600-1900 BCEI.

Behistun Inscription -the “cuneiform Rosetta stone”

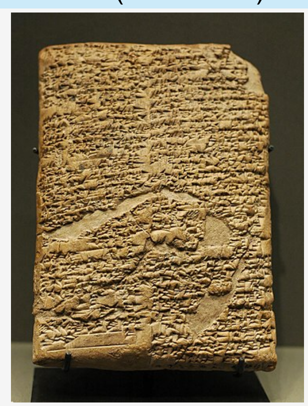
- written in three different cuneiform languages: Old Persian, Elamite, and Babylonian.
- Studied and copied by Sir Henry Rawlinson, a British army officer, beginning in 1835.

The Behistun Inscription, a large rock relief showing Darius the Great of the Achaemenid Empire punishing conspirators. Wikipedia Korosh.091 - Own work

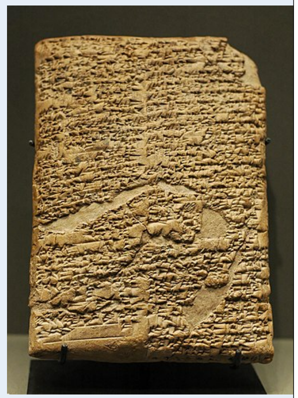
Location of the Behistun complex

Code of Hammurabi about 1700 BC

"an eye for an eye, a tooth for a tooth" (lex talionis)



How does a written law code (like Hammurabi's) change the nature of justice compared to an oral tradition?



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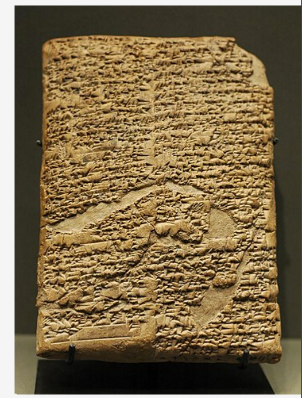


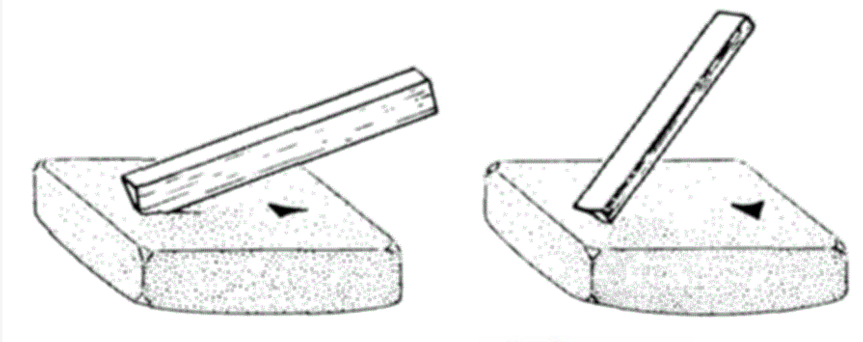
- standardizes rules,
- reduces local variability,
- shifts authority from memory to text.

Epic of Gilgamesh

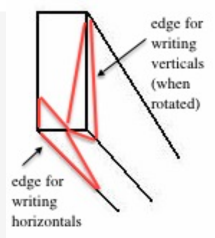
Ancient Assyrian statue currently in the Louvre, possibly representing Gilgamesh

Neo-Assyrian clay tablet. Epic of Gilgamesh, Tablet 11: Story of the Flood. Known as the "Flood Tablet" From the Library of Ashurbanipal, 7th century BC. British Museum

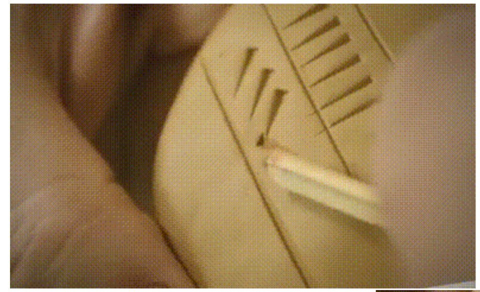
How to write in cuneiform

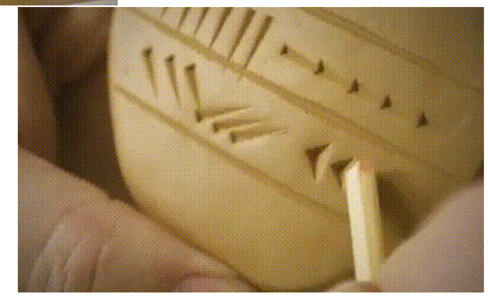


Impression of cuneiform symbols on clay tablets. (Redrawn from Neugebauer3



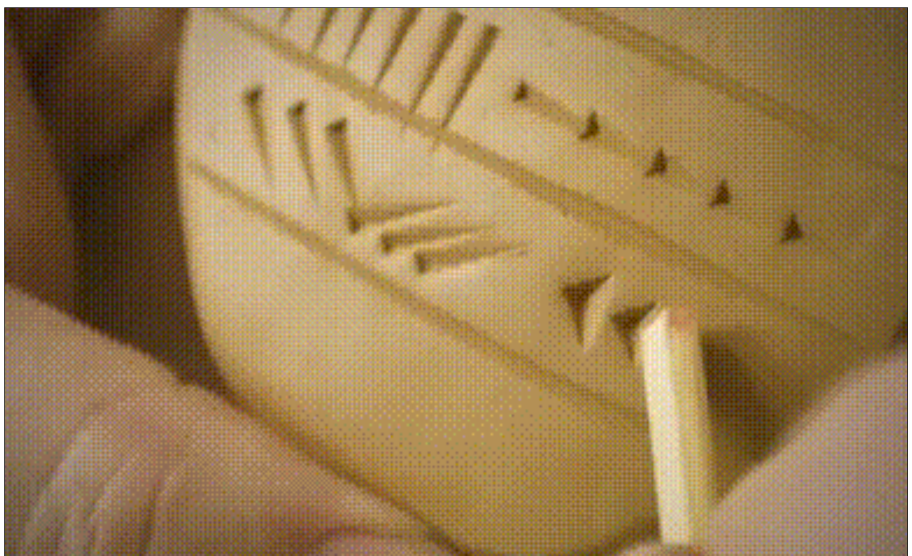
http://writingcuneiform.blogspot.com/2012/10/5-making-basic-wedges.html





Cuneiform writing

Clips from https://cuneiform.neocities.org/CWT/Figures/2_keilschrift_hwc.gif



Cuneiform writing: How it was done

https://cuneiform.neocities.org/CWT/Figures/2_keilschrift_hwc.gif

Mesopotamian Mathematics in a slide-nutshell

Overview of Mesopotamian Mathematics

Mathematical tablets are mainly:
Tables and Problems

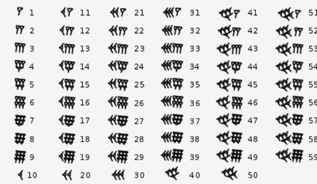
Examples
- Multiplication tables (by ***some*** numbers). Reciprocals tables
- Square roots tables
- **Pythagorean triples? Plimpton 322.**

- Rough work (solution of problems)
- Problems - Verbal techniques

Note: No tables of addition

Overview of Mesopotamian Mathematics

- Scribes used **positional** number system with base 60 for computations.
- Scribes also used other number systems.



positional number system with no zero!

Calendar

Algorithms
- Determine square roots.
- Solution of linear equations (with one or two unknowns)
- Solution of certain quadratic equations (often related to architecture and building)

Overview of Mesopotamian Mathematics

Plimpton 322

Possibly the most famous tablet; understanding it requires resisting modern labels.

Scribes' Shortcut: Multiply by This
- The defining component of an equilateral triangle was the **side** and the coefficient for the **height** was 7/8.
- The defining component of a **circle** was the **circumference**. **Coefficients:**
  - Diameter was $1/3=(0;20)_{60}$
  - Area $1/12= (0;5)_{60}$

Decoder key:
defining component = the given measure you start from
coefficient = the multiplier that converts the measure into something else.

You are now a Mesopotamian scribe.
a. Given a circle's circumference c and the coefficients for the diameter and the area, state which multiplications you would perform to compute its diameter and its area.
b. Given an equilateral triangle's side s and the coefficient for the height, state which multiplications you would perform to compute the height and area of the triangle.

Scribes' Shortcut: Multiply by This

- The **defining component** of an equilateral triangle was the **side** and the **coefficient** for the **height** was 7/8.
- The **defining component** of a **circle** was the **circumference**. **Coefficients:**
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b. Given an equilateral triangle's side s and the coefficient for the height, state which multiplications you would perform to compute the height and area of the triangle.

Part (a) - Circle with circumference c:

- To find diameter: Multiply c by 1/3
- To find area: Multiply c by c, then multiply the result by 1/12

Part (b) - Equilateral triangle with side s:

- To find height: Multiply s by 7/8 (to justify, use Pythagorean theorem and approximate. In modern terms, $\text{height}^2 = s^2 - (s/2)^2$.

Tables in Tablets

- Amazing aspect of the Babylonian's calculating skills: **construction of tables to aid calculation.**
- Babylonians used the formula

$$a.b = (1/2)((a+b)^2 - a^2 - b^2)$$

to make multiplication easier.

- They did not have an algorithm for long division. Instead they based their method on the fact that

$$A \% B = A \times (1/B)$$

and used tables of reciprocals **(1/B)**.

Canals: A Major Reason to Do Mathematics

Canals = essential for irrigation and transport of goods/armies

Mathematics used to:

- Compute **dimensions of canals**.
- Plan **workers × days** needed.
- Calculate **wages and expenses**.

Old Babylonian tablets record many such **canal-digging problems**.

https://mathshistory.st-andrews.ac.uk/HistTopics/Babylonian_mathematics/
K Muroi, Small canal problems of Babylonian mathematics, Historia Sci. (2) 1 (3) (1992), 173-180.

More about base 60

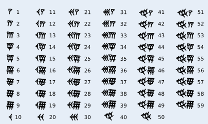
Historians have proposed several theories for the Babylonian use of base 60. Choose one and defend it.

a. Many even splits: 60 breaks evenly into many parts (2, 3, 4, 5, 6, 10, 12, 15, 20, 30), so common fractions come out neat and calculations are easier.

b. Sky counts: About 12 months × 5 “wandering stars” (Mercury, Venus, Mars, Jupiter, Saturn) = 60. (Sun and Moon were not counted as planets.)

c. Shared number for trade: Different places used different ways of counting and measuring; 60 worked as a handy common base for converting and keeping accounts.

d. Finger math: One hand can count to 12 using finger bones; the other hand tracks sets of 12 x 5 → 60.



Some of the many theories explaining why base 60!

- Theon of Alexandria (~300CE): 60 is, among all the numbers the most **convenient**, because, being the smallest among all those which have the **most divisors**, it is the easiest to handle.
- More recent theory (~1950CE): **to allow for dividing weights and measures into thirds.**
- Even more recent: Sumerian civilization must have come about through the **joining of two groups**, one of whom had **base 12** for their counting and the other having base **5 or 10.**
- Common measures theory is now widely accepted.

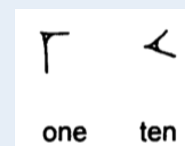
There are many theories trying to explain base 60

One can count up to 60 by pointing at one of the twelve parts of the fingers of the left hand with one of the five fingers of the right hand.

Express numbers A. and B. in Hindu-Arabic numerals

A.

B.



All Mesopotamian numerals

Figures from Robson, Eleanor. "Counting in Cuneiform." Mathematics in school 27.4 (1998): 2-9

5 meant

$5/60+40/60^2$ or

$5+40/60$ or

$5\times60+40$ or

$5\times60^2+40\times60$ or..

meant

$45/60(= 3/4)$ or

45 or

$45\times60=2700$ or..

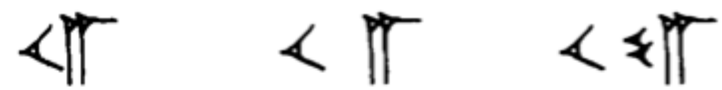
one ten

How do they know which one?

Context!

Figures from Robson, Eleanor. "Counting in Cuneiform." Mathematics in school 27.4 (1998): 2-9

Notations for intermediate zero across time



12	10 02 or 602 before 1600 BC	10 02 or 602 after 1600 BC

Figures from Robson, Eleanor. "Counting in Cuneiform." Mathematics in school 27.4 (1998): 2-9

Goal: Write a number **a=0.205** between 0 and 1 in base 60. We want numbers c_1, c_2, c_3... such that

$$\mathbf{0.205} = c_1/60 + c_2/60^2 + c_3/60^3 + \ldots$$

1) Each c_i is an integer
2) $0 \le c_i < 60$

Step 1: $c_1 = \lfloor 60\cdot 0.205 \rfloor = \lfloor 12.3 \rfloor = 12$

$a_1 = 60a - c_1 = 12.3 - 12 = 0.3$

Step 2: $c_2 = \lfloor 60a_1 \rfloor = \lfloor 60\cdot 0.3 \rfloor = \lfloor 18 \rfloor = 18$

$a_2 = 60a_1 - c_2 = 60\cdot 0.3 - 18 = 0$

Stop: If we reach i such that $a_i = 0$ the process ends.

Note: $\lfloor x \rfloor$ denotes the largest integer smaller or equal than x.

Goal: Write a number **a** between 0 and 1 in base 60. We want numbers c_1, c_2, c_3... such that

$$\mathbf{a} = c_1/60 + c_2/60^2 + c_3/60^3 + \ldots$$

10 Each c_i is an integer
2) $0 \le c_i < 60$

Step 1: $c_1 = \lfloor 60\cdot \mathbf{a} \rfloor$

$a_1 = 60a - c_1$

Step 2: $c_2 = \lfloor 60a_1 \rfloor$

$a_2 = 60a_1 - c_2$

Step n: $c_n = \lfloor 60a_{n-1} \rfloor$,

$a_n = 60a_{n-1} - c_n$)

Stop: If we reach i such that $a_i = 0$ the process ends.

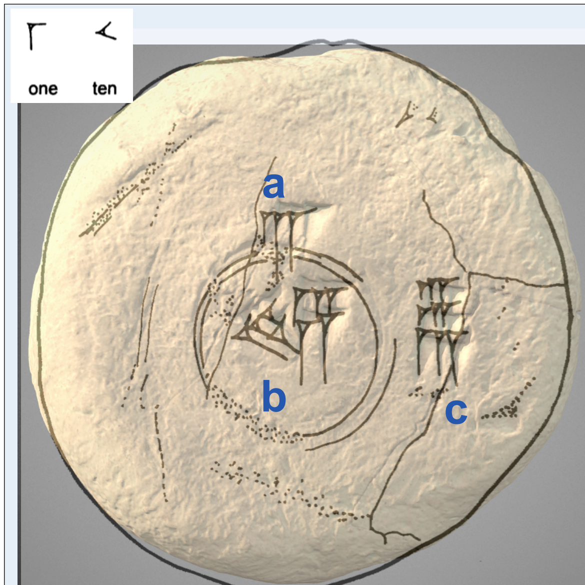
Note: $\lfloor x \rfloor$ denotes the largest integer smaller or equal than x.

Convert these fractions into base 60 (sexagesimal). Then, show what the result would look like in cuneiform.
Answer in Wooclap using base-60 notation.
Upload a screenshot of your cuneiform work.

A. 1/20
B. 21/80
C. (Optional) 1/7

Calculation of Areas

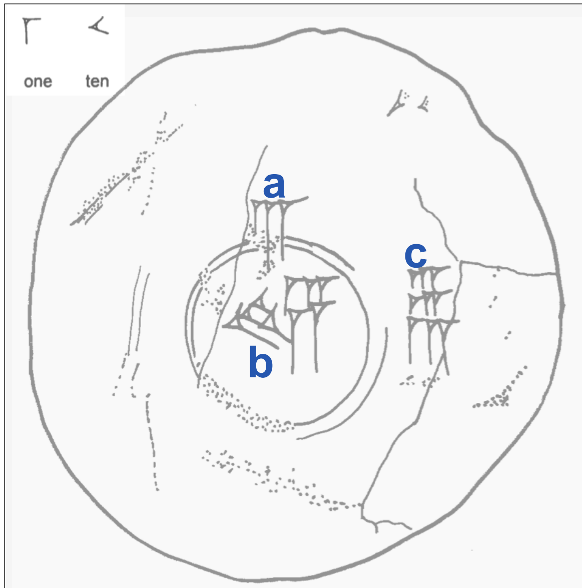
https://sketchfab.com/3d-models/the-area-of-a-circle-tablet-16f3e288b7be485297ddd0945fd10ece



Old Babylonian tables from Yale Collection
Drawing by Eleanor Robson The American Mathematical Monthly , Feb., 2002, Vol. 109, No. 2 (Feb., 2002), pp. 105-120)

1. Write a, b, c in the Hindu-Arabic number system.
2. What do these numbers represent?
3.(Optional) What is the purpose of this tablet?

Hint: To find the purpose, consider the arrangement of the numbers with respect to the circle.



Recall Scribes' Shortcut:

- The **defining component** of an equilateral triangle was the **side** and the **coefficient** for the **height** was 7/8.
- The **defining component** of a **circle** was the **circumference**. **Coefficients:**
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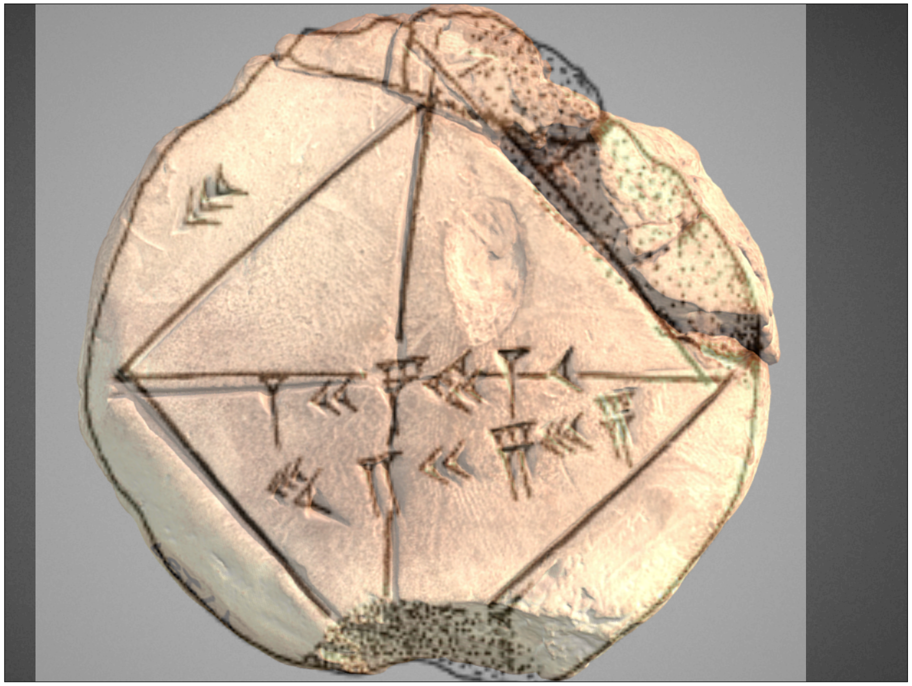
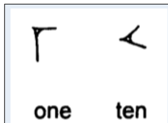
A trapezoid

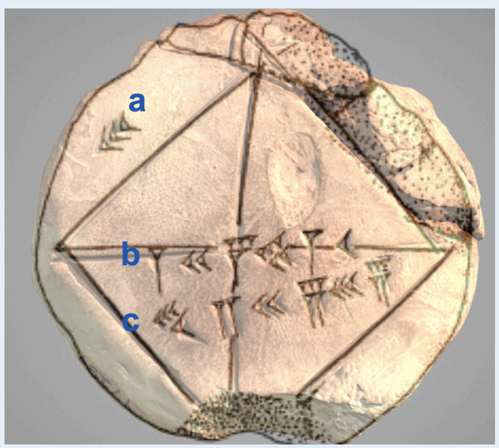
An amazing computation

A digital replica to hold in your digital hands

https://sketchfab.com/3d-models/the-diagonal-of-a-square-







Picture from Yale Collection
Drawing by Eleanor Robson

1. What are the values of a, b and c?
2. Educated guess: What do you think is the purpose of this tablet?

√2 Babylonian clay tablet (circa 1800–1600 BCE)

- Likely “written” **by a student.**
- Evidence suggests its **author copied** some of the results from an existing table of values and did not compute them himself.

- The value demonstrates one of the **greatest known computational accuracy** obtained in the ancient world.

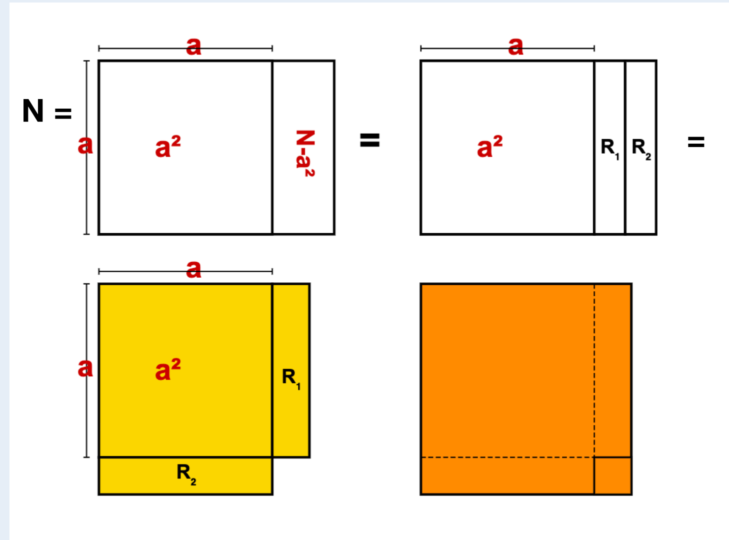
Picture from Yale Collection
Drawing by Eleanor Robson

A digital replica to hold in your digital hands

https://sketchfab.com/3d-models/the-diagonal-of-a-square-tablet-605c9d9573d14b52b8880e14c826e133?utm_medium=embed&utm_campaign=share-popup&utm_content=605c9d9573d14b52b8880e14c826e133

Some examples of notable computational accuracy in the Ancient World

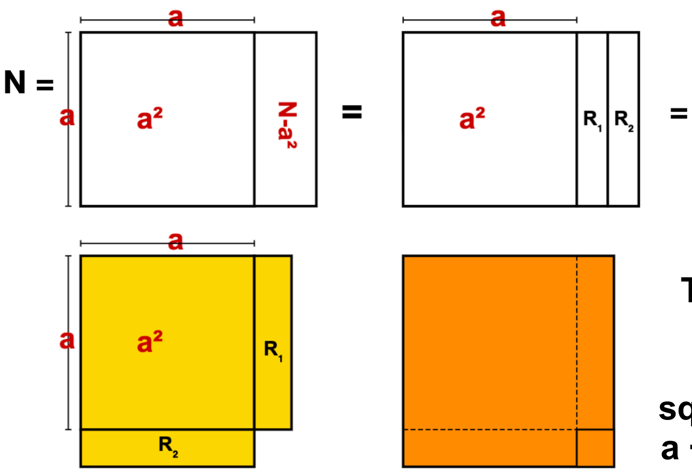
	Who and Where	When	Value	Accuracy
√2	Babylonian	1800–1600 BCE	$(1;24,51,10)_{60}$	error ≈ 6.0×10^{-7} → **6 decimals**
√2	Śulba Sūtras - India	800–500 BCE	1 + 1/3 + 1/(3·4) – 1/(3·4·34)	error ≈ 2.1×10^{-6} → **5 decimals**
π	Archimedes - Greece	3rd c. BCE	223/71 < π < 22/7 (Greek ciphered)	**≈2–3 decimals** bounds
π	Ptolemy - Alexandria	2nd c. CE	$(3;8,30)_{60}$	error ≈ 7.4×10^{-5} → **3 decimals**
π	Zu Chongzhi - China	5th c. CE	355/113	error ≈ 2.7×10^{-7} → **6 decimals**



What is the area of the yellow figure? What is the area of the orange figure? (Express both areas in terms of N and a)

Problem: Given a square of area N, find its side

- **Start with a good guess**: Pick **a** where a^2 is close to N (but $a^2 < N$)



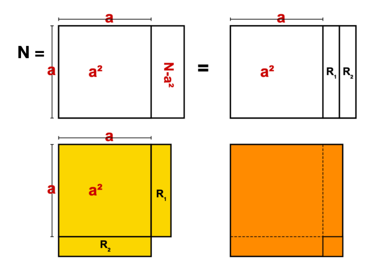
The side of the new (orange) square is a' = a + (N-a²)/(2a)

Problem: Given a square of area N, find its side

Babylonian approximation of the square root of a number N.

- **Start with a good guess**: Pick **a** where **a²** is close to **N** (and **a² < N**)
- **Form a rectangle**: Create a rectangle with area **N** and one side of length **a.** (The other side must have length **N/a**)
- **Goal**: Cut up the rectangle and rearrange the pieces to form "almost" **a** square closer to area **N**.This gives a better approximation to **√N**

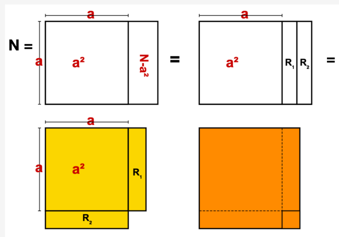
The side of the new (orange) square is a' = a + (N-a²)/(2a)



Does this algorithm rings a bell?

Babylonian approximation of the square root of a number N when $a^2 < N$

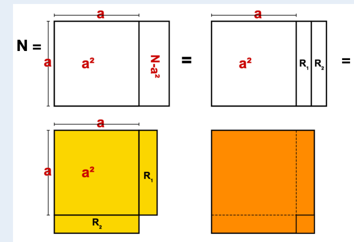
1. Find a such that a^2 is close to N and $a^2 < N$.
2. The yellow shape has area N $= a^2 + (N - a^2)$.
3. The “leftover area $(N - a^2)$ is split into two equal rectangles, each of area $(N - a^2)/2$. So their width is $(N - a^2)/(2a)$
4. Since a^2 is close to N, the area of the yellow shape (N) is close to the area of the orange square.“They differ only by the small corner square of side
5. Therefore, $\sqrt{N}$ is close to the side length of the orange square.
6. The orange square’s side length is $a + (N - a^2)/(2a)$. Thus, $a + (N - a^2)/(2a)$ is the new approximation to $\sqrt{N}$.

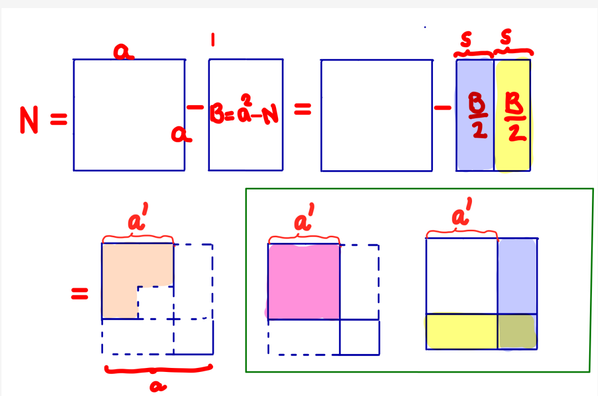


Using the algorithm, compute an approximation of $\sqrt{2}$ starting with a=1

- **Start with a good guess**: Pick **a** where **a²** is close to **N** (but **a² < N**)
- **Form a rectangle**: Create a rectangle with area **N** and one side of length **a.** (The other side must have length **N/a**)
- **Goal**: Cut up the rectangle and rearrange the pieces to form “almost” **a** square closer to area **N.** This gives a better approximation to **$\sqrt{N}$**

The side of the new (orange) square is $a' = a + (N-a^2)/(2a)$





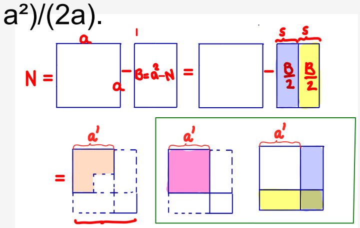
Babylonian meth to find square roots, using a approximation with $a^2>N$.

Given **N** and **a** such that:
- N>0, a>0
- $a^2>N$
- a^2 is close to N (thus a is close to $\sqrt{N}$)
- Hint: Write **s** in terms of **N** and **a**

Following the figures, find a’ such that (in terms of a and N) such that $(a')^2$ is even closer N than a^2 (so a’ is even closer than a to $\sqrt{N}$)

Babylonian approximation of the square root of a number N when $a^2 > N$

1. Pick a so that a^2 is close to N and $a^2 > N$.
2. The yellow shape has area N $= a^2 - (a^2 - N)$.
3. Split the excess area $(a^2 - N)$ into two equal rectangles, each of area $(a^2 - N)/2$, so their width is $(a^2 - N)/(2a)$.
4. Remove one strip from the right and one from the bottom of the $a \times a$ square to get the orange shape. Since the two stripes share a small square, we remove it too.
5. The small missing corner has side $(a^2 - N)/(2a)$, so the orange area is close to N.
6. Therefore $\sqrt{N}$ is close to the side of the orange square.
7. The orange square’s side is $a - (a^2 - N)/(2a)$. Thus the new approximation is $a' = a - (a^2 - N)/(2a) = a + (N - a^2)/(2a)$.



- **What were some main features of Mesopotamian mathematics?** *(Think: number system, $\sqrt{2}$ approximation, reciprocal tables…)*
- **Why was base 60 practical for calculations?** *(Hint: divisors, fractions, and reciprocals.)*
- **How did Mesopotamians perform division?** *(Remember how they used tables…)*
- **What do we learn from the $\sqrt{2}$ tablet? (***Consider the value written, its accuracy, and how it was used.)*
- **How did Mesopotamians achieve accuracy in their work?** *(Think about pre-calculated values and reference lists.)*
- **How was mathematical knowledge preserved and passed on?***(Hint: scribal schools, coefficient lists, training.)*

Mesopotamia and the Pythagorean theorem

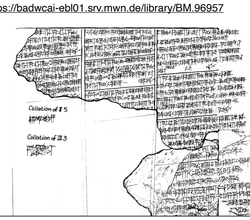
Mesopotamia and the Pythagorean theorem

- The theorem is ascribed to Pythagoras by Proclus and several later commentators (based on Proclus)
- This is one of many improbable legends ascribed to Pythagoras in late Antiquity
- Old Babylonian texts (c. 2000 BC) show the principle was already known

Pythagoras Emerging from the Underworld, 1662

By Salvator Rosa, Public Domain, https://commons.wikimedia.org/w/index.php?curid=7119956

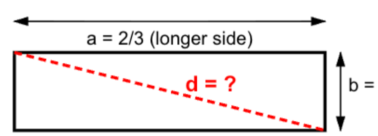
https://badwcai-ebl01.srv.mwn.de/library/BM.96957



See Three Old Babylonian Methods for Dealing with "Pythagorean" Triangles Author(s): Eleanor Robson Source: Journal of Cuneiform Studies, 1997, Vol. 49 (1997), pp. 51-72

A Babylonian tablet from around 2000 BCE asks for the diagonal of a rectangle whose sides are 2/3 and 1/6. The scribe gives the following procedure

- Square the shorter side, 1/6, to obtain 1/36.
- Take the reciprocal of the longer side, 2/3, which is 3/2, and multiply it by 1/36.
- The result is 1/24.
- Take half of this value to get 1/48.
- Add this to 2/3.
- The result is 11/16, which is the diagonal.



- Square the shorter side, b: b^2
- Take the reciprocal of the longer side, a, which is 1/a, and multiply it by b^2.
- The result is $(1/a) \times b^2 = b^2/a$
- Take half of this value: $(b^2/a)/2 = b^2/(2a)$
- Add this to a: $a + b^2/(2a)$
- **This gives the approximation for the diagonal: $d \approx a + b^2/(2a)$**

$d = (a^2 + b^2)^{1/2}$

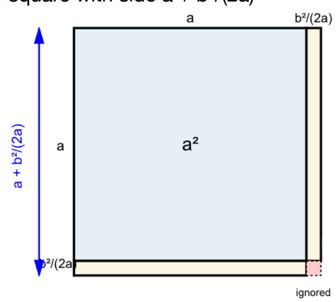
$\approx a + (1/2)\, b^2\, (1/a) = a + b^2/(2a)$

This is a particular case of the approximation when $b \ll a$

$$\mathbf{d = (a^2 + b^2)^{1/2} \approx a + b^2/(2a)}$$

Here is an explanation of the approximation $a + b^2/(2a) \approx \sqrt{(a^2 + b^2)}$.

1. Start with a square of side a (area = a^2)
2. We want to add an area b^2 to this square and obtain a shape that is almost a square (a square minus a tiny square), in such a way that we can compute the "side" of this "almost square."
3. This is done by attaching two rectangular strips along two adjacent sides of the square.
4. Each strip has area $b^2/2$ and dimensions $a \times (b^2/2a)$.
5. The a^2-square with the added strips create an almost-square with side $a + b^2/(2a)$
6. The missing corner has area $(b^2/(2a))^2$



Formula: $d \approx a + b^2/(2a)$ where $a = 2/3$, $b = 1/6$

Algebraically, this means:

$(a + b^2/(2a))^2 = a^2 + 2a \cdot (b^2/(2a)) + (b^2/(2a))^2 = a^2 + b^2 + (b^2/(2a))^2$

The area of the almost square is $(b^2/(2a))^2$, so: $(a + b^2/(2a))^2 \approx a^2 + b^2$

The side of the almost square is **$a + b^2/(2a) \approx \sqrt{(a^2 + b^2)}$**

Why the corner is negligible:

- Corner area = $(b^2/(2a))^2 = b^4/(4a^2)$
- In the example: $b = 1/6$, $a = 2/3$
- Corner = $(1/36)^2/(4 \cdot 4/9) = (1/1296)/(16/9) \approx 0.0004$
- Completely negligible!

The figure **is** the algebraic approximation made visible. The ignored shaded corner corresponds exactly to the dropped term in the Taylor expansion.

Tables of Reciprocals