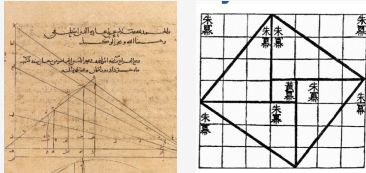
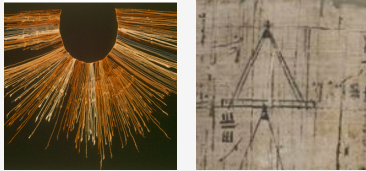


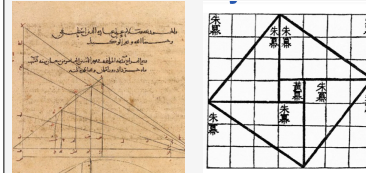
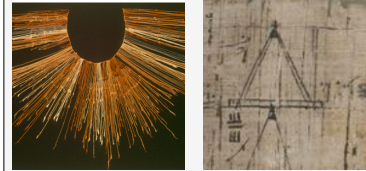
# Counting and Counting Numbers

What is Counting and How Do We Do It?



- What is counting (definitions, one-to-one correspondence)
- Recording methods (tally systems)
- Human brain and counting, connection between fingers and numbers (parietal lobe)
- Approximate number sense and object tracking

# Cultural Counting



Give a concrete example of a mathematical primary source (with words, not just links). Then, explain where you would find the document and explain how to access the text, including a translation if the original is not in English.

Blaise Pascal's "Treatise on the Arithmetical Triangle". Translations are available online and in print. For example, University of Cambridge repository.

Give a concrete example of a mathematical primary source (with words, not just links). Then, explain where you would find the document and explain how to access the text, including a translation if the original is not in English.

A concrete example of a mathematical primary source from China is **The Nine Chapters on the Mathematical Art (Jiūzhāng Suànshù)**, a foundational Chinese mathematical text compiled during the Han dynasty (around the 1st century BCE–1st century CE). The book contains problems and methods related to arithmetic, linear equations, geometry, and practical calculations.

Original manuscripts of The Nine Chapters are preserved through historical copies and commentaries, with important versions housed in major libraries and museums in China, such as the National Library of China. The original text is written in Classical Chinese. Modern readers can access the work through annotated editions and English translations, such as the translation by Kangshen Shen, John N. Crossley, and Anthony W.-C. Lun. These translations are available in academic libraries and provide both the original text and a clear English rendering.

Give a concrete example of a mathematical primary source (with words, not just links). Then, explain where you would find the document and explain how to access the text, including a translation if the original is not in English.

A common convention for publishing mathematical findings in the contemporary age are published formal papers that may be accessed via online repositories such as JSTOR.

Give a concrete example of a mathematical primary source (with words, not just links). Then, explain where you would find the document and explain how to access the text, including a translation if the original is not in English.

Papers written by the people who discovered important results are examples of mathematical primary sources (since they are first-hand sources from the person who made the discovery). More modern papers can be found in research journals or online databases like JSTOR. Older papers could also possibly be scanned/transcribed into online databases, but they could also appear in databases like the Internet Archive as well.

Example: <https://www.jstor.org/stable/2309286>

 JOURNAL ARTICLE

Pascal and the Invention of Probability Theory

[Oystein Ore](#)

*The American Mathematical Monthly*, Vol. 67, No. 5 (May, 1960), pp. 409-419 (11 page)

<https://doi.org/10.2307/2309286> · <https://www.jstor.org/stable/2309286> 

**Name one specific mathematical primary source (include title, author and date). Then state at least one of the following: 1. Where is it available? (library/website/archive) (2) How would you access it? (URL/call number/database name)**

## About the presentation and paper topic.

- Start reading.
- Ask for help if needed.
- Adjustments can be made.
- Slide deck is due soon-is.
- Remember
  - Goal of the presentation is to teach something to your classmates.
  - You can discuss a subset of the assigned topic.

# Examples of counting in different societies

## Example: Veddas of Sri Lanka counting

- Lived by eating fruit and turners of jungle plants.
- Had few words to deal with numbers. Some of these words (translated) were: a single, a couple, another one and many.
- To count, say, a pile of coconuts, a Vedda collected a heap of sticks. Then,
  - For each coconut in the pile, she took a stick.
  - Each time she took a stick, she said “another one”
  - When finished, she pointed out the pile of sticks she took and said “That many”.



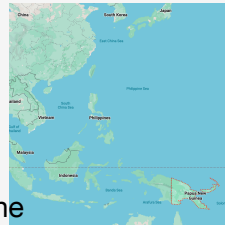
Veddas count does not use number-words



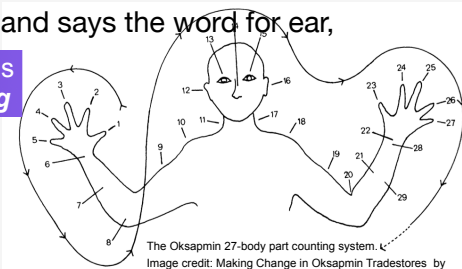
<https://sites.utexas.edu/dsb/files/2021/02/Oneness-Twoness-Threeness-How-Ancient-Accountants-Invented-Numbers.pdf>

## Example: Oksapmin from Papua New Guinea

- Begins with the thumb on one hand
- Enumerates 27 places around the body,
- Ends on the little finger of the opposite hand.
- To indicate a particular number, one points to the appropriate body part and says the body part name.
- Example: to indicate the number 12, one points to the ear which is the 12th body part and says the word for ear, “nata”.

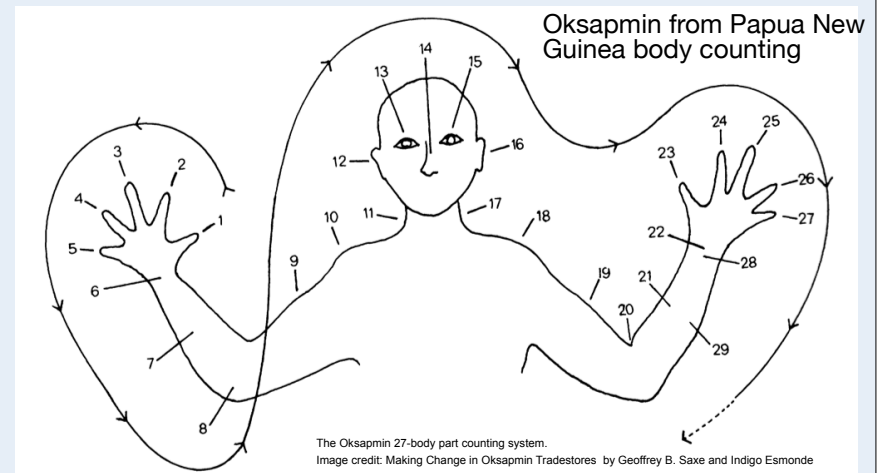


This way of counting is called **body counting**



Forms of body counting were quite spread around the globe as late as the 1800s

**"What is your age in Oksapmin body counting? Use the corresponding body part to describe it in English. For example, if you are five years old, you would write 'pinky finger,' and if you are twenty-seven, you would write 'other pinky finger'."**



## Example: Nivkh people from lower Amur River - Russia

|       | Means <i>two</i> of the following                 |
|-------|---|
| Mex   | trees, sticks, pencils and some other long things |
| Merax | leaves or pieces of textiles or other flat items. |
| Mik   | berries, balls and other round things.            |



Based on the oral accounts of Mamiya Rinzo, Sadatsugu Murakami depicted the portrait of the Smerenkul Jurchen (Nivkh people) in Hokuji Bunka Yowa (Supplementary Tales of the Northern Barbarians' Boundaries).

Example: “two leaves” is expressed as “merax x” where x is the word for leaves.

Is there any noun in English similar to those of the Nivkh people? If so, write it down.

Table 1-2-1

| objects_being_counted        | one    | two    | three  | four   | five    | ten     |
|------------------------------|--------|--------|--------|--------|---------|---------|
| one-dimensional objects      | nex    | mex    | cex    | nux    | thox    | mxox    |
| two-dimensional objects      | nrxax  | merax  | crax   | nrx    | thorax  | mxorax  |
| three-dimensional objects    | nik    | mik    | cex    | nax    | thox    | mxox    |
| sledges                      | niř    | miř    | ceř    | nař    | thoř    | mxon    |
| boats                        | nim    | mim    | cem    | nəm    | thom    | mxon    |
| generic                      | naqř   | meqř   | caqř   | naķř   | thoqř   | mxoqř   |
| fishnet cells                | niu    | miu    | ceu    | nuu    | thou    | mxou    |
| fishnet strips               | neřqe  | meřqe  | ceřqe  | nařqe  | thořqe  | ?       |
| fishnets and fish-spears     | nvor   | mevor  | cfor   | nvr    | thovor  | mxovor  |
| special fishnets             | neo    | meo    | ceo    | nəu    | thou    | mxou    |
| poles for making fish-spears | nla    | mel    | cla    | nly    | thola   | mxola   |
| poles for drying fish        | nesk   | mesk   | cesk   | nask   | thosk   | ?       |
| boards for building boats    | nec    | mec    | cec    | nac    | thoc    | mxoec   |
| families                     | niřn   | miřn   | ceřn   | nařn   | thořn   | mxoniřn |
| generations                  | nesvax | mesvax | cesvax | nasvax | thosvax | ?       |
| places                       | navř   | mevř   | cavř   | navř   | thovř   | mxovř   |
| day's rests on one's way     | nix    | mix    | cex    | nax    | thox    | ?       |
| humans                       | nenn   | menn   | caqř   | nařy   | thory   |         |

## Concrete counting (numerosity)

**Concrete counting:** Counting systems tied to what you're counting. So the number is not fully independent of the thing counted.

The counting word tells you what kind of thing is being counted (shape, animacy, role, material, etc.), and the noun tells you which thing.

### Examples

- Various Amazonian languages have separate number words for different object types.
- Some Australian Aboriginal languages count people vs. animals vs. objects differently.

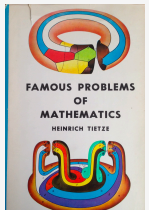
## A way of counting

A group of friends, let's call them A, B and C want to count a large number of people.

1. The people starts walking before them. Each time a person walks by, A raises a finger (and keeps it raised)
2. When all fingers of A are raised, B raises one finger and A lowers all their fingers. The procedure continues repeating appropriately 1. and 2.
3. A some point, all fingers of B are raised. Then C raises one finger and B lowers all their fingers.
4. And so on.

This type of counting was probably common but is hard to document.

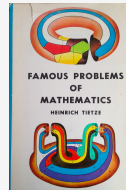
According to Tietze, this form of counting was used by a tribe from South Africa



## Counting in a planet of beings with only one hand, with three fingers in that lonely only hand

A group of friends, let's call them A, B and C want to count a large number of people.

1. The people starts walking before them. Each time a person walks by, A raises a finger (and keeps it raised)
2. When all fingers of A are raised, B raises one finger and A lowers all their fingers. The procedure continues repeating appropriately 1. and 2.
3. A some point, all fingers of B are raised. Then C raises one finger and B lowers all their fingers.
4. And so on.



<https://twitter.com/britishlibrary/status/1301807524904083456>

The British Library @britishlibrary

Finger-counting was an important way of performing calculations during the Middle Ages. These intricate diagrams demonstrate the technique. Read this article to learn more about medieval approaches to science and maths: [ow.ly/ByIk50AMw40](https://ow.ly/ByIk50AMw40) #CollectionsUnited #PolonskyPre1200

5:01 AM · Sep 4, 2020 · Hootsuite Inc.

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Illustrations of hands labelled with figures on the fingers to calculate the date of Easter and perform mathematical calculations (British Library, Egerton MS 3314, f. 73r, detail)

Forms of body counting were quite spread around the globe as late as the 1800s

## Navajo Number Names: Fingers and Mathematics

| Number | Navajo        | Approximate Translation | Arithmetic |
|--------|---------------|-------------------------|------------|
| 1      | Łá            | Little finger           | 1          |
| 2      | Náka          | Ring finger             | 2          |
| 3      | Tát'j         | Middle finger           | 3          |
| 4      | Dij           | Index finger            | 4          |
| 5      | Ashdla'       | The hand is finished    | 5          |
| 6      | Łáts'ij       | One past the hand       | 5 + 1      |
| 7      | Tso           | Two past the hand       | 5 + 2      |
| 8      | Tsoo t'áá     | Three past the hand     | 5 + 3      |
| 9      | Náhaq         | Four past the hand      | 5 + 4      |
| 10     | Benáá         | Both hands finished     | 5 + 5      |
| 11     | Benáá íá      | Ten and one             | 10 + 1     |
| 15     | Benáá ashdla' | Ten and five            | 10 + 5     |
| 20     | Náhaq benáá   | Two tens                | 2 × 10     |

## Review about counting

- Almost (if not all) societies have/had some awareness of number.
- Counting is establishing a **one-to-one correspondence** between two sets.
- Counting is closely **linked to fingers**.
- The concept of number, although basic is **very hard to define rigorously**. (Recall the barber's paradox)
- After counting, a need to record the result of the counting arises. There are different ways of **keeping records** of "how many", for instance
  - Making scratches
  - Using objects (sticks, pebbles, shells)
  - Tying knots.



Image credit: <https://creativekindergartenblog.com/one-to-one-correspondence-intervention-for-kindergarten/>

# History of mathematics hidden in language

## How can we explore the origins of numbers?

### Some approaches:

- Comparing past and present number systems
- Analyzing notations in ancient number systems
- Investigating the origins and evolution of number words

## From Two to Three: A Giant Leap

### Three means "many" in many languages:

- English: "three" and "through"
- French: "trois" and "très" (very)
- Latin: "trans" (beyond) suggests surpassing a limit

### Evidence in ordinal numbers:

- "First" = before all others
- "Second" = "the other" (Latin: following)
- "Third, fourth, fifth..." = systematic counting

The struggle: Moving from "few" to "many"

## Early word numbers: Observe patterns

| Number | English | Gothic | Latin    | Ancient Greek | Welsh  | Sanskrit | Basque    |
|--------|---------|--------|----------|---------------|--------|----------|-----------|
| 1      | one     | ains   | unus     | heis          | un     | eka      | bat       |
| 2      | two     | twai   | duo      | dyo           | dau    | dva      | biga      |
| 3      | three   | threis | tres     | treis         | tri    | tri      | hirur     |
| 4      | four    | fidwor | quattuor | tettares      | pedwar | catur    | laur      |
| 5      | five    | fimf   | quinque  | pente         | pump   | panca    | bortz     |
| 6      | six     | saihs  | sex      | hex           | chwech | sad      | sei       |
| 7      | seven   | sibun  | septem   | hepta         | saith  | sapta    | zazpi     |
| 8      | eight   | ahtau  | octo     | okto          | wyth   | asta     | zortzi    |
| 9      | nine    | niun   | novem    | ennea         | naw    | nava     | bederatzi |
| 10     | ten     | taihun | decem    | deka          | deg    | dasa     | hamar     |

A counting system is quite a complex construction, with different historical structures overlaying one another

## Describe all patterns you can find in this table

| Number | English   | Gothic       | Latin         | Ancient Greek   | Welsh             | Sanskrit    | Basque     |
|--------|-----------|--------------|---------------|-----------------|-------------------|-------------|------------|
| 11     | eleven    | ainlif       | undecim       | hendeka         | un ar ddeg        | ekaadasha   | hamaika    |
| 12     | twelve    | twalif       | duodecim      | dodeka          | deudddeg          | dvaadashan  | hamabi     |
| 13     | thirteen  | þriehund     | tredecim      | treiskaideka    | tri ar ddeg       | trayodasha  | hamahiru   |
| 14     | fourteen  | fidwartehund | quattuordecim | tessareskaideka | pedwar ar ddeg    | chaturdasha | hamalau    |
| 15     | fifteen   | fimftehund   | quindecim     | pentekaideka    | pymtheg           | panchadasha | hamabost   |
| 16     | sixteen   | saihstehund  | sedecim       | hexkaideka      | un ar bymtheg     | shodasha    | hamasei    |
| 17     | seventeen | sibuntehund  | septendecim   | heptakaideka    | dau ar bymtheg    | saptadasha  | hamazazpi  |
| 18     | eighteen  | ahtautehund  | duodeviginti  | oktokaideka     | deunaw            | ashtadasha  | hemezortzi |
| 19     | nineteen  | niuntehund   | undeviginti   | enneakaideka    | pedwar ar bymtheg | navadasha   | hemeretzi  |
| 20     | twenty    | twaitigjus   | viginti       | eikosi          | ugain             | vimsatih    | hogoi      |

## Hidden History in Number Words

| Number | English   | Gothic       | Latin         | Ancient Greek   | Welsh             | Sanskrit    | Basque     |
|--------|-----------|--------------|---------------|-----------------|-------------------|-------------|------------|
| 11     | eleven    | ainlif       | undecim       | hendeka         | un ar ddeg        | ekaadasha   | hamaika    |
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| 20     | twenty    | twaitigjus   | viginti       | eikosi          | ugain             | vimsatih    | hogoi      |

- "Eleven" = "one left" – words close in English and Gothic.
- "Twelve" = "two left" – words close in English and Gothic.

We can infer that in here were northern European tribes whose counting words went up only to 'ten' then counted leftovers

## Hidden History in Number Words

### Base-20 Traces

- French: "quatre-vingts-sept" =  $4 \times 20 + 7 = 87$
- English (Bible Psalm 90:10): "three score and ten" =  $3 \times 20 + 10 = 70$

Mixed systems show historical layers

Can you remember a historical occurrence of the word "score"?  
(Where score mean 20)

Four score and seven years ago our fathers brought forth on this continent, a new nation, conceived in Liberty, and dedicated to the proposition that all men are created equal.

Now we are engaged in a great civil war, testing whether that nation, or any nation so conceived and so dedicated, can long endure. We are met on a great battle-field of that war. We have come to dedicate a portion of that field, as a final resting place for those who here gave their lives that that nation might live. It is altogether fitting and proper that we should do this.

But, in a larger sense, we can not dedicate -- we can not consecrate -- we can not hallow -- this ground. The brave men, living and dead, who struggled here, have consecrated it, far above our poor power to add or detract. The world will little note, nor long remember what we say here, but it can never forget what they did here. It is for us the living, rather, to be dedicated here to the unfinished work which they who fought here have thus far so nobly advanced. It is rather for us to be here dedicated to the great task remaining before us -- that from these honored dead we take increased devotion to that cause for which they gave the last full measure of devotion -- that we here highly resolve that these dead shall not have died in vain -- that this nation, under God, shall have a new birth of freedom -- and that government of the people, by the people, for the people, shall not perish from the earth.

Abraham Lincoln The Gettysburg Address  
Gettysburg, Pennsylvania November 19, 1863

## How Old Is Our Counting System?

### Language family evidence:

- Most European and some Asian languages descend from Indo-European, spoken around 4000 BC.
- Similar words for "hundred" across these languages
- Suggests counting to 100 existed in original language

### Before Indo-European:

- Counting by twenties (Basque, traces in French/Welsh) in Europe before Indo-European.
- Conclusion: Or spoken counting systems are 5000-8000 years old

## Evolution of the Word "Digital"



- Originally referred to fingers (from Latin digitus).
- Later came to describe things represented in distinct units (digits), which led to its association with discrete values in contrast to analog (continuous) representations.
- Extended to binary data & computers → Broader use: digital media, communication, transformation.

| Word     | Origin  | Meaning                              |
|----------|---|--------------------------------------|
| Digital  | Latin 'digitalis' (of the finger)                 | Relating to fingers, digits          |
| Digit    | Latin 'digitus' (finger, toe)                     | Finger, used for counting            |
| Calculus | Latin 'calculus' (small stone)                    | Small stone used for counting        |
| Tally    | Latin 'talea' (stick, cutting)                    | A stick with notches for counting    |
| Reckon   | Old English 'gerecenian' (to recount)             | To count or explain                  |
| Score    | Old Norse 'skor' (notch, tally)                   | Notches used for counting            |
| Five     | Proto-Indo-European 'penkwe' (five, hand-related) | Five, related to counting on fingers |
| Compute  | Latin 'computare' (to reckon, count)              | To calculate or count                |
| Notation | Latin 'notare' (to mark, note)                    | Marking numbers or symbols           |