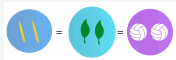


The very beginning of mathematics



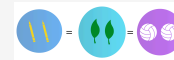
On These slides

- What is mathematics?

Sources: How do we know what we know in math history.

- Early math sources
- The Ishango bone. Interpretations.

The very beginning of mathematics



This week

- A bit about this marvelous course (and boring adminstrivia)
- What do we mean by mathematics in this course?
- Sources: How do we know what we know in math history.
 - Early math source: The Ishango bone. Interpretations.
 - Primary and secondary sources.
 - Reliability of sources
- (Maybe) The beginning of counting.

In this slides we are going to discuss

- What do we mean by mathematics in this course?
- Early math document: The Ishango bone. Interpretations.

The interpretations of the Ishango bone are the opening to reflect on one fundamental issue in this course.

- Reliability of sources

How do we know what we know and why do we believe it?

Important: log in to the platform with your SB email



1 Go to wooclap.com

2 Enter the event code in the top banner

Event code
UGHBHJ

**Write down your first name
and something you like
about math.**

Who am I?



I studied "licenciatura" in Mathematics at the University of Buenos Aires



I was born in Buenos Aires



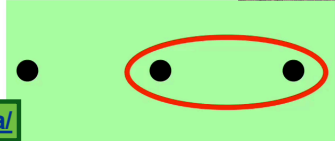
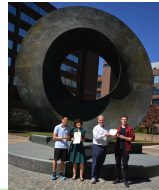
Questions?

Instagram @moirart3

I did my PhD at the Autonomia University of Barcelona



I'm a professor at Stony Brook University, New York, US



<http://www.math.stonybrook.edu/~moira/>

What is mathematics?

- Make groups of four students and go to a blackboard.
- Introduce yourself to the members of your group and exchange coordinates for future communication.
- Write down a definition of mathematics that you agree with (do not use sources other than your group to maximize your learning).
- You have 5 minutes.

What is mathematics?

Write down in Wooclap your own definition.

What is mathematics?

What is mathematics?

What do you mean by mathematics?

Fundamental type of question

All science requires mathematics. The knowledge of mathematical things is almost innate in us! This is the easiest of sciences, a fact which is obvious in that no one's brain rejects it; for laymen and people who are utterly illiterate know how to count and reckon.



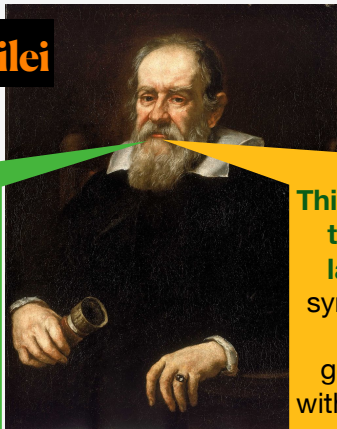
Roger Bacon. Stipple engraving, 1786.
Wellcome Library no. 6961
<https://wellcomecollection.org/works/nj73cmm?wellcomeimagesUrl=indextplus/image/V0000285.html>

Roger Bacon (1214–1294), English philosopher and scientist

Galileo Galilei

~1600

Philosophy is written in that **great book which ever lies before our eyes — I mean the universe** — but we cannot understand it if we do not first learn the language and grasp the symbols, in which it is written.



Justus Sustermans - Portrait of Galileo Galilei, 1636
Public Domain

This book is written in the mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth.

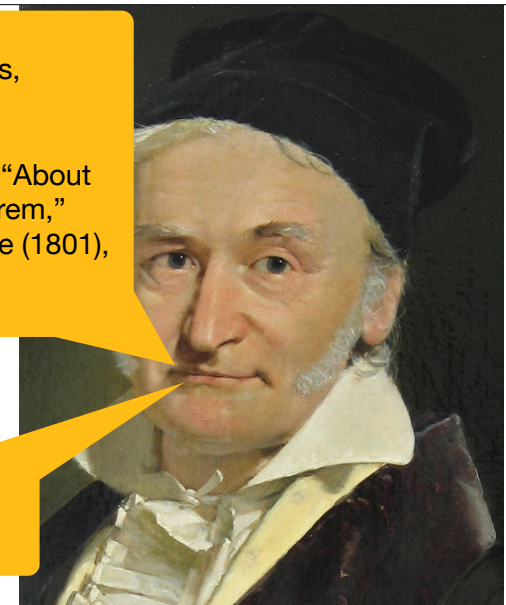
“What we need are notions, not notations.”

- I wrote that sentence in “About the proof of Wilson's theorem,” *Disquisitiones Arithmeticae* (1801), Article 76

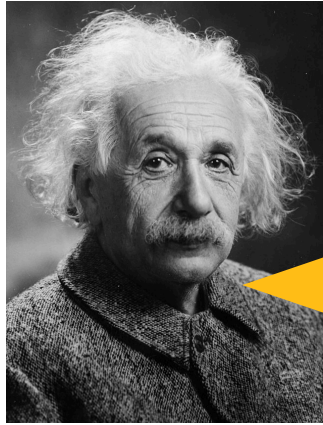
Carl Friedrich Gauss

~1800

Mathematicians stand on each other's shoulders.



Portrait of the mathematician and philosopher Carl Friedrich Gauss (1777-1855) - Public domain

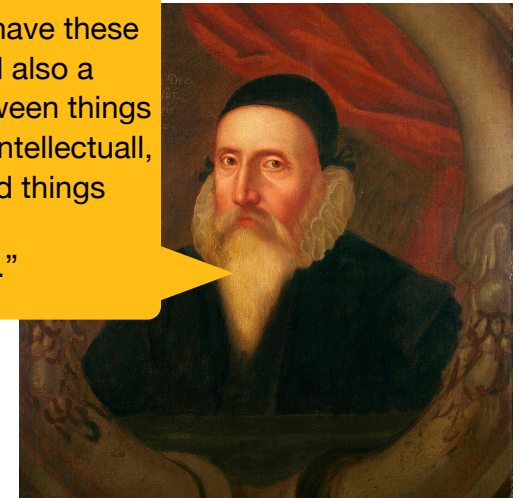


Photograph by Orren Jack Turner, Princeton, N.J. - Public domain

“Pure mathematics is, in its way, the poetry of logical ideas. One seeks the most general ideas of operation which will bring together in simple, logical and unified form the largest possible circle of formal relationships. In this effort toward logical beauty spiritual formulas are discovered necessary for the deeper penetration into the laws of nature.”

Albert Einstein, Obituary for Emmy Noether (5 May 1935).

Albert Einstein

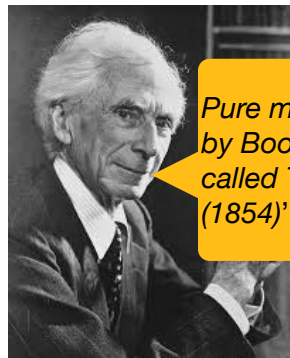


It seems that this portrait was painted when Dee was 67.

“A marveilous newtrality have these things mathematicall, and also a strange participation between things supernaturall, immortall, intellectuall, simple and indivisible, and things naturall, mortall, sensible, compounded and divisible.”

John Dee, 1527-1608. English mathematician and astrologer.

Preface to his edition of Euclid's *Elements*, 1570

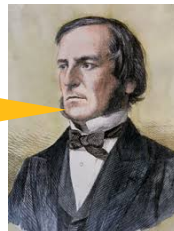


This file is made available under the Creative Commons CC0 1.0 Universal Public Domain Dedication

Bertrand Russell.

*Pure mathematics was discovered by Boole, in a work which he called *The Laws of Thought* (1854)'. (I wrote this in 1918)*

I am George Boole



George Boole, mathematician, 1815-1864 - Public Domain

What is mathematics?

In the 1950s, Sawyer described mathematics as the “classification and study of all possible patterns” .

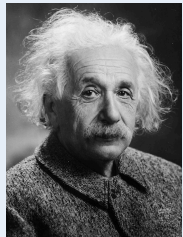
He explained that pattern was meant “to cover almost any kind of regularity that can be recognized by the mind”.

W. W. Sawyer, *Prelude to Mathematics*, 1955

What do you mean by mathematics?

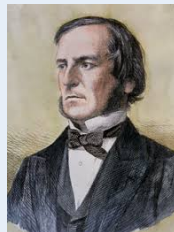
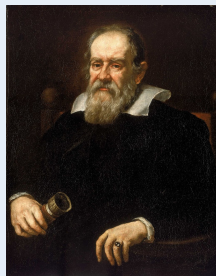
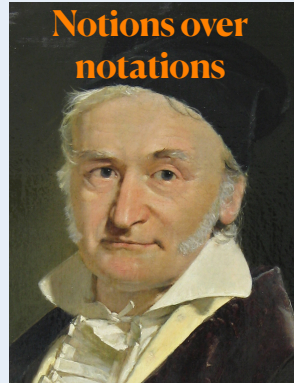
Study of ideas related to number, space, shapes, patterns, and structure, combined with logical reasoning

- Bullet point
- Lorem ipsum dolor



Photograph by Orren Jack Turner, Princeton, N.J. - Public domain

Do you find a pattern?



I have said that the love of study is the passion most necessary to our happiness. It is an unfailing resource against misfortunes, it is an unending source of pleasures,...

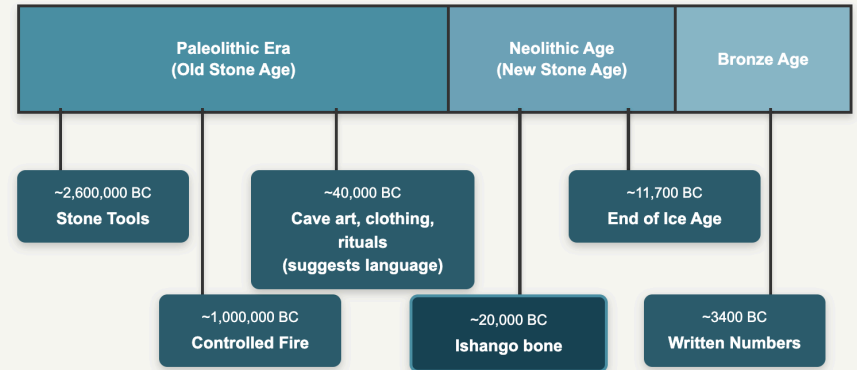
Madame du Châtelet

~1700

Gabrielle Émile Le Tonnelier de Breteuil, marquise du Châtelet (1706-1749), French mathematician and physicist - Maurice Quentin de La Tour

J'ai dit que l'amour de l'étude étoit la passion la plus nécessaire à notre bonheur : c'est une ressource sûre contre les malheurs ; c'est une ressource de plaisirs inépuisable,

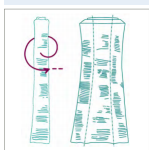
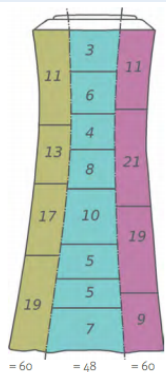
The Ishango bone and its interpretations



Ishango Bone (About 20,000 BC)

Educated guess: The purpose of the Ishango bone is:

- counting animals
- serve as a calendar
- unknown



<https://www.naturalsciences.be/en/museum/exhibitions-activities/exhibitions/250-years-of-natural-sciences/the-ishango->

Interpretations: Ishango Bone

Jean de Heinzelin (found the bone in 1960) →
 - arithmetical game,
 - counting system based on 10
 - a knowledge of multiplication by 2 and of prime numbers.

Alexander Marshack (1972) → six-month lunar calendar.

Claudia Zaslavsky (1991) → lunar calendar made by women to keep track of their monthly cycles.

Olivier Keller (2010) → I a great deal of mathematical fiction, nothing can be said definitively.



"We do not see things as they are, we see them as we are." Anais Nin, probably quoting from the Talmud.



Are other cultures so different that they are incomprehensible to us?

Let's distinguish conjectures from facts!

"All history is contemporary history."
Benedetto Croce

"As the South African archeologist David Lewis-Williams says of prehistoric art, 'Meaning is always culturally bound'"
R. Leaky - The origin of humankind

"We do not see things as they are, we see them as we are." Anais Nin, probably quoting from the Talmud.



Are other cultures so different that they are incomprehensible to us?

Challenge of prehistoric archaeology

Interpret the purpose, and meaning of marks which exhibit some regularity (length, spacing, and orientation) makes them visually indistinguishable from one another,



There are a lot of possibilities, and not methods of criteria to decide among them with the desired level of reliability and confidence.

Overmann, K. A. (2023). *The materiality of numbers: Emergence and elaboration from prehistory to present*. Cambridge University Press.

- decorative?
- conventional?
- mnemonic?
- symbolic?
- notational?
- numerical?
- astronomical?
- calendrical?
- musical?
- utilitarian?

Do these marks represent numbers?

- Assume that prehistoric marks on nonperishable materials like bones were numbers. Then, these numbers
 - most likely would have been realized and elaborated through the use of the **fingers (and toes)** (as the hands/feet are the material form most commonly used today in emerging number systems.)
 - would also likely have been in existence for some period of time – **long enough to motivate not only their being recorded**, but also the use of a **nonperishable** substance for it.
- But with two debated exceptions – the Ishango and Les Pradelles artifacts – **no prehistoric artifact has marks organized in ways that unambiguously identify them as numbers.**
- The earliest notations with **unambiguous numerical organization**, the Mesopotamian impressions, **occur 5000 or 6000 years ago.**
- **The late emergence of unambiguously numerical marks suggests that Palaeolithic marks either did not represent numbers**

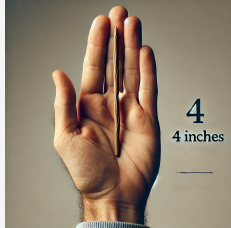
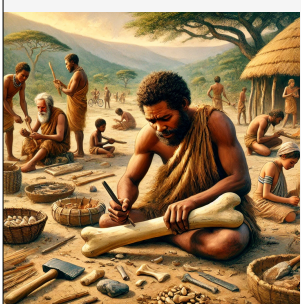
Overmann, K. A. (2023). *The materiality of numbers: Emergence and elaboration from prehistory to present*. Cambridge University Press.

The use of AI

Images made by ChatGPT in 2023 of a person carving the Ishango bone



The size of the Ishango bone is about 4 inches



Prompts to ChatGPT and Perplexity in 2025. Make a realistic image of a person carving the Ishango bone.

The size of the Ishango bone is about 4 inches



ChatGPT



Prompts to ChatGPT and Perplexity in 2025. Does the person have to be alone? Is it possible there were other human beings around?

The size of the Ishango bone is about 4 inches

Perplexity

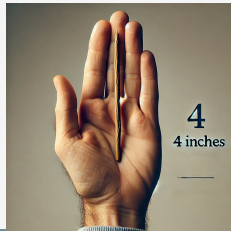


Claude says: Better detail of the carving process, stronger community feel, but potentially misleading geographic/cultural context

Short hair?

No beard?

ChatGPT



Claude says: Accurate African savanna setting, culturally appropriate context

Skin color?

Gender?

The use of the Web



From the Boston University Website
<https://www.bu.edu/africa/outreach/teachingresources/outreach-teachingresources-mathematics/>

The Lebombo bone (top) is the oldest known mathematical artifact. It is a tally stick with 29 distinct notches that were deliberately cut into a baboon's fibula. It was discovered within the Border Cave in the Lebombo Mountains of Eswatini. The Lebombo bone (bottom) resembles a calendar stick still used in Namibia. See more about these artifacts under "Other Resources" below.



Mathematical Treasure: Ishango Bone
 MAA website

<https://old.maa.org/press/periodicals/convergence/mathematical-treasure-ishango-bone>



From the Boston University Website
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Mathematical Treasure: Ishango Bone
 MAA website
<https://www.maa.org/press/periodicals/convergence/mathematical-treasure-ishango-bone>

Why do you think I showed these two photos? Can you extract a conclusion to apply to your research in this class?

How can we deduce information from bones?

Reindeer antler of Little Salt Spring, Florida , about 8000BCE



The reindeer antler of Little Salt Spring, Florida, with a colleague's hand for scale. Photograph by Caleb Everett.

- 29 incisions
 - very regular
 - about five millimeters long
 - the spacing between them is consistent
- suggests that the marks were made systematically.*
- smaller etchings in one-to-one alignment with the larger incisions and next to them,
- suggest that quantities were ticked off along that progression.*

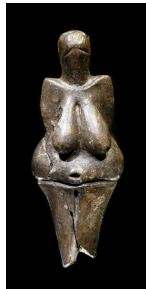
Source: *Numbers and the Making of Us: Counting and the Course of Human Cultures*, by Caleb Everett., 2017, Harvard University Press

Dolní Věstonice (in Czech Republic) archeological site

- From (roughly) 27,000 to 20,000 B.C
- Many archeological artifacts.
- Representations of men, women, and animals, along with personal ornaments, human burials and engravings.

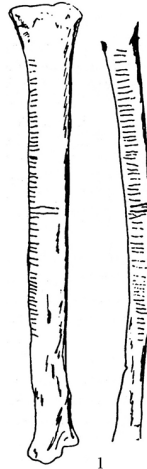


Carved female head from Dolní Věstonice, Krahuletz-Museum (replica)
Source: Wikipedia



Venus of Dolní Věstonice, the earliest discovered use of ceramics (29,000 BCE – 25,000 BCE) by Petr Novák, Wikipedia

Tally Sticks of the Stone Age
Andrej Kapcar
https://www.academia.edu/18709816/Tally_Sticks_of_the_Stone_Age



The Wolf Bone of Dolní Věstonice

Did Neanderthals count?

Hyena Bone. about 60.000 years old. found in France



Prehistoric accounting? Markings made on a hyena bone by a Neanderthal might have recorded numerical information.

According to a report published in *Nature*, the femur bone bears nine notches which are 'strikingly similar and approximately parallel, as if they were meant to signify something'

Barras, C. (2021). How did Neanderthals and other ancient humans learn to count?. *Nature*, 594(7861), 22-25.

<https://media.nature.com/original/magazine-assets/d41586-021-01429-6/d41586-021-01429-6.pdf>

Administrative stuff

Course Website and Wooclap

- We are going to use the interactive platform Wooclap for questions, polls and surveys.
- You will be able to answer the questions in this platform from a web browser or a smart phone app.

Course Website

<https://www.math.stonybrook.edu/~moira/courses/mat336-sp2026/>

Note: A quick way to find the course website is googling my name, go to my website and find there the link for the course.

Topics for paper and presentation

Everything is on the course schedule!

MAT 336: History of Mathematics

Course Schedule - Lecture 1 - Spring 2026

Note: This schedule may change as needed. Check regularly for updates.

#	Date	Topic	Quizzes - HW	Paper/Present
1	Jan 27	Math History Introduction – Info about the course		
2	Jan 29	Reliable Sources – Counting Foundations	Info and topic choice form	

There is a link to the course schedule in Brightspace

Your Research Assignment

- Presentation and paper on assigned math history topic
- **Deadline: Complete topic choice form by Thursday 29/2**, I'll match topics to preferences to maximize "happiness"
- All papers have the same due date
- Find everything on the course schedule (linked in Brightspace):
 - Presentation dates
 - Topic choice form
 - All assignment deadlines
 - Topic list with sources

Topics We Will Discuss in this Course

Topics We Will Discuss in this Course

- The beginning of mathematics
- Number systems
- Sources for studying history.
- Ancient Egypt
- Ancient Mesopotamia
- Around the world
- Hellenic Mathematics
- Ancient and Medieval China
- Ancient and Medieval India
- Ancient and Medieval Islamic world
- European Renaissance
- Calculus
- Selected topics of modern mathematics

If you have a special interest in a math history topic, let me know. We might be able to cover it.

Topics (cont.)

- The beginning of mathematics
- Number systems
- Sources for studying history.
- Ancient Egypt
- Ancient Mesopotamia
- Around the world
- Hellenic Mathematics
- Ancient and Medieval China
- Ancient and Medieval India
- Ancient and Medieval Islamic world
- European Renaissance
- Calculus
- Selected topics of modern mathematics

We will go deeper than wider, really understanding mathematical thinking in selected areas rather than surveying everything superficially

If you have a special interest in a math history topic, let me know. We might be able to cover it.

The Presentation - Requirements

Engage your audience!



Foto credit: The Simpsons

- The goal is to **teach something to the class**.
- It is strongly encouraged to **include a learning activity** for the class to help. (you can have a few extra minutes in this case, let me know in advance)
- **150 words!!!** at most in the slides (that is, about 15 per slide)
- Minimal notes in paper to help your memory are fine. See precise guidelines in the course website.

The Presentation - Support

Engage your audience!

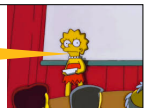


Foto credit: The Simpsons

- Speaking in public can be scary, but we will be a kind, supporting audience, rooting for you.
- If you need to break any of the rules to give a better presentation, discuss it with me beforehand.
- You are welcome to make an appointment with me to do a rehearsal of your presentation

More about the course

Communications spelled out

- Check email and Brightspace regularly
- Absences/missed work → use form (not email)
- Missed content → ask classmates
- Grading questions → contact grader first
- Email response: 1-3 business days
- Issues affecting participation → contact me early

Tips to succeed in this course

- Keep track of all due dates and **plan ahead**.
- Complete all **assignments on time**.
- Set aside **focused, distraction-free time** for this course.
- Check Brightspace and Stony Brook email regularly.
- Be **present** in the lectures.
- Let me know if you need **help**.

What I expect from everybody (including myself) in the classroom

- Respect for each other and for the different societies and cultures we will find.
- Kindness
- Open mind.
- Effort
- Being present during lectures

**I hope for growth mindset and
curiosity.**

More about this course

- The main goal of class activities and Wooclap questions is to support you in learning and thinking, not to evaluate you. They also help me gauge your understanding.
- Constructive feedback is always welcome.
- You're welcome (but not required) to discuss any classroom issue that affects you.

There are no dumb questions about course content.

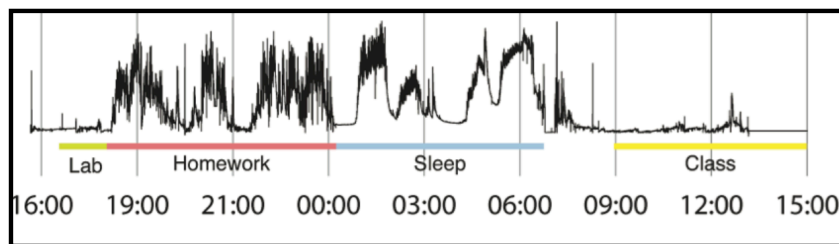
(All logistics—deadlines, schedules, assignments—are posted on the course website and Brightspace)

Fixed mindset

Growth mindset

Instead of thinking	Try this
I am not good at this	What am I missing?
I can't make it any better	I can improve if I keep trying
I made a mistake. Therefore, I am not smart.	What can I learn from this mistake?
This is too hard. I give up	This might take longer than I expected
The problem is that X is smarter than me	I'll try to learn how X does it.
This is too easy for me	Can I understand this more deeply?

Note: Lectures barely register -
Active learning lights up the brain



Brain on lecture vs. brain on active learning

Table from an old slide of Eric Mazur (<http://ericmazur.com/about.php>)

A picture of a lecture I will work on **avoiding**



Image credit: Could not find it

contrast with

"I hear and I forget
I see and I remember.
I do and I understand"
Confucius

All the activities of class
are tools to help you learn.

Active learning increases student performance in science, engineering, and mathematics

Scott Freeman¹*, Sarah L. Eddy², Miles McDonough³, Michelle K. Smith⁴, Nnadozie Okoroafor⁵, Hannah Jordt⁶, and Mary Pat Wenderoth¹

¹Department of Biology, University of Washington, Seattle, WA 98195; and ²School of Biology and Ecology, University of Maine, Orono, ME 04469

Edited* by Bruce Alberts, University of California, San Francisco, CA, and approved April 15, 2014 (received for review October 8, 2013)

To test the hypothesis that lecturing maximizes learning and course performance, we metaanalyzed 225 studies that reported learning interventions varied widely in intensity and implementation, and included approaches as diverse as occasional group

PNAS

<https://www.pnas.org/content/111/23/8410>

Image created by AI (DALL-E), illustrating something analogous to the use of AI to complete assignments



More about prehistoric mathematics

Photo Credit Sawright, Caroline, (2014). ARICSPAL Essay: Ochre and the African middle stone age record. - 70,000 years old - found in Blombos Cave in South Africa



70,000 years old - found in South Africa

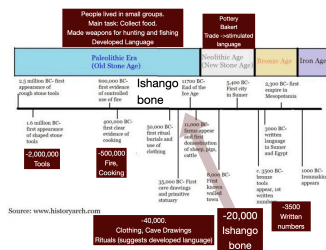
Recall: Ishango bone, 20,000BCE

Can you see any math in these photos?

Reproductions one of Lascaux artworks by Jack Versloot (about 17,000 years old)



17,000 years old, found in France



Blanchard “calendar” bone



Photo: Don Hitchcock 2015

Source: Facsimile, Musée d'Archeologie Nationale et Domaine, St-Germain-en-Laye

This bone suggests that its creator(s) were deliberately representing numerical concepts materially.

The farther we delve into the past, the more we find mathematics entangled with accounting, surveying, astronomy, and the general administration of empires. **Mathematics arises wherever people think about the physical world or about the world of ideas embodied in laws and even theology.** It grows like a plant, from a seed that germinates and later ramifies to produce roots, branches, leaves, flowers, and fruit. It is constantly growing.
Roger Cooke - The history of Mathematics - Third Edition

Take a moment to reflect on today's discussion. In a few sentences, summarize the key ideas and topics we explored—no need to include anything about course logistics or administrative details. If something stood out to you or sparked a lingering question, feel free to include that too! Make sure you save your text.