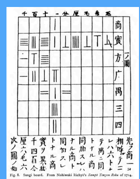
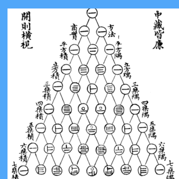
Ancient and *Medieval* Chinese Mathematics-





- Chronology
  - Nine Chapters
  - Book of Numbers and Computations
- Counting rods
- Liu Hui
- Zu Chongzhi
- Areas and Volumes -
  - π approximation
  - Volume of the sphere
- Square and cubic roots
- ***Chinese?*** reminder theorem
- Pythagorean theorem
- Systems of linear equations
- Pascal triangle
- Magic Squares

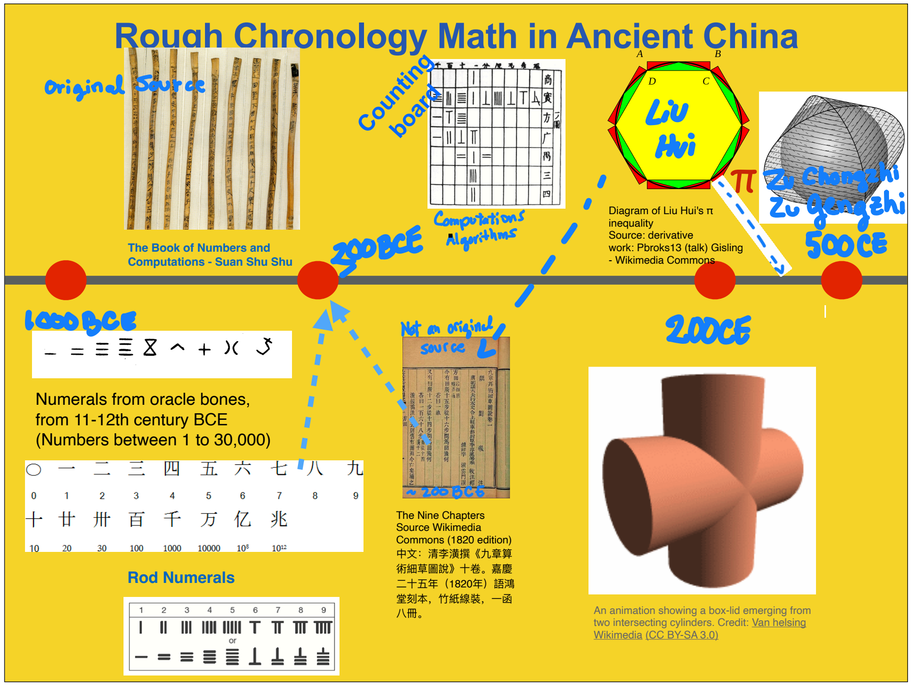
Who did math professionally in Antiquity? Priests? Scribes? Wealthy people? Bureaucrats? Somebody else? Who? (Answer the question for one or more of the following societies)

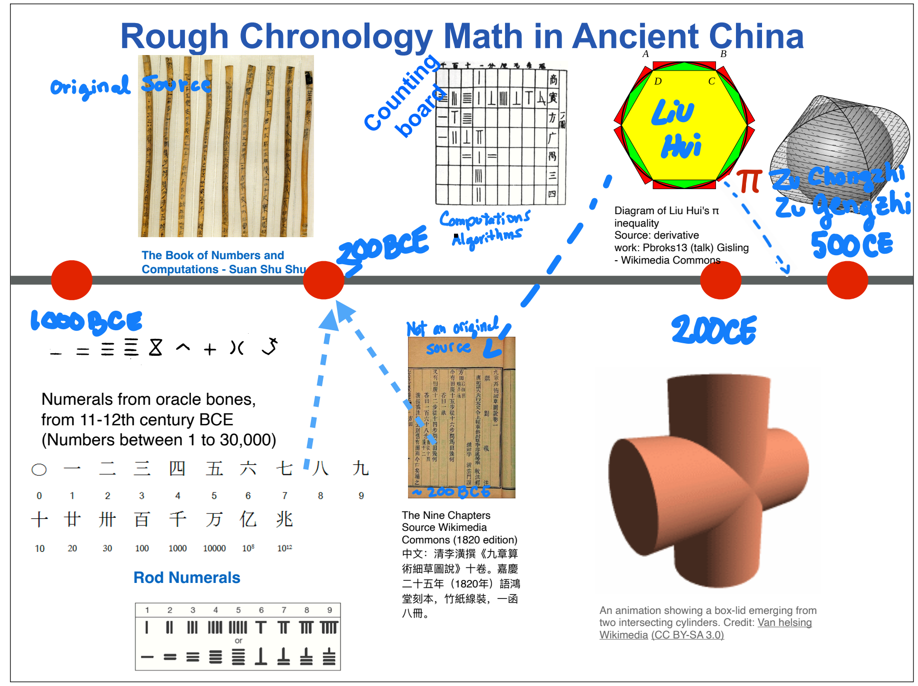
- In Egypt?
- In Babylonia?
- In the Hellenic world?
- In China?

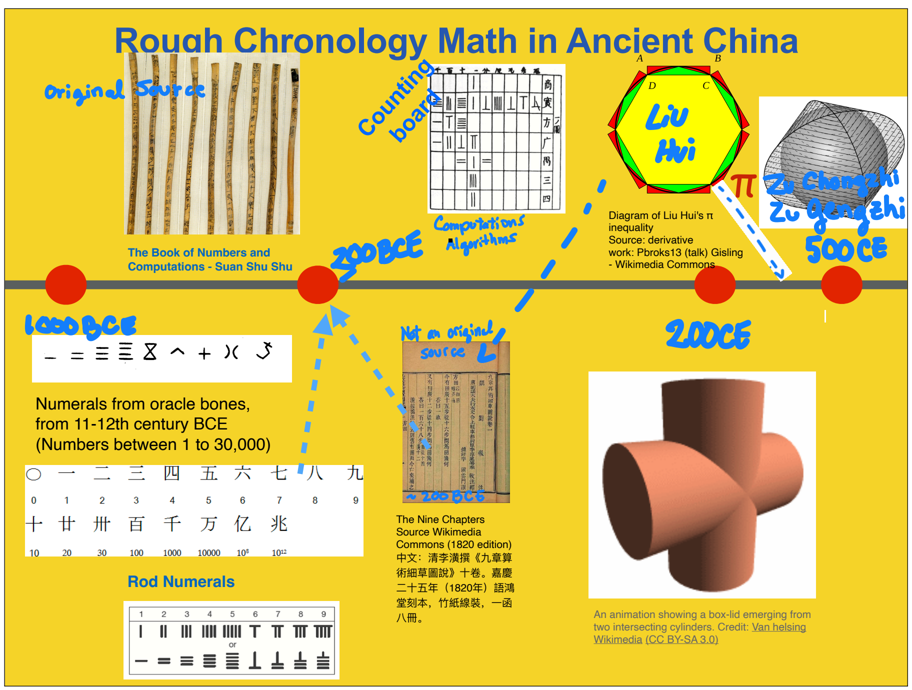
Who did math professionally in Antiquity? Priests? Scribes? Wealthy people? Bureaucrats? Somebody else? Who? (Answer the question for one or more of the following societies)

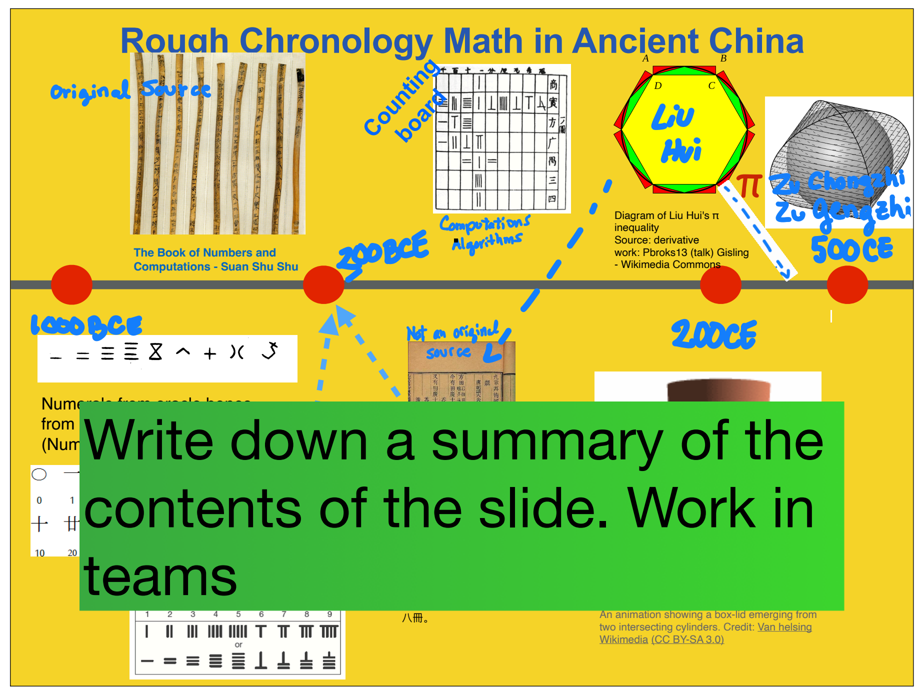
- Egypt
  - Bureaucrats and government officials
  - "Thus the mathematical sciences originated in the neighborhood of Egypt, because there the priestly class was allowed leisure." Aristotle (Metaphysics):
- Babylonia
  - scribes (administrators depending on the state.)
- Hellenic world
  - Philosophers and scholars, administrators, engineers and architects, astronomers...
- China d
- Bureaucrats but after a hard test

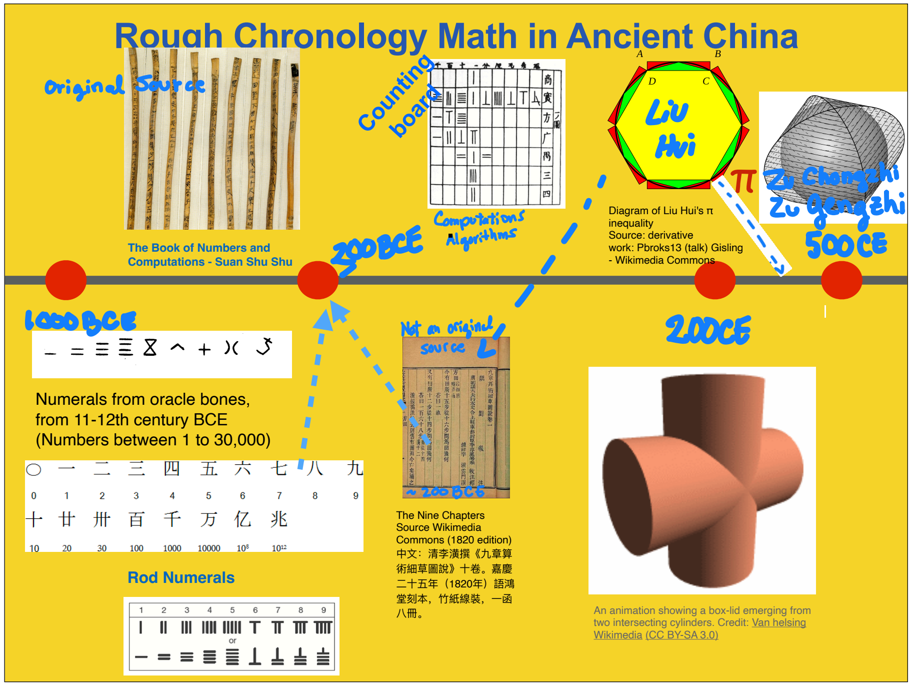
Rod numerals, counting boards and matrices


Rough Chronology Math in Ancient China
Original Source
The Book of Numbers and Computations - Suan Shu Shu
Counting board
Computations Algorithms
200 BCE
Liu Hui
Diagram of Liu Hui's π inequality
Source: derivative work: Pbroks13 (talk) Gisling - Wikimedia Commons
π
Zu Chongzhi
Zu Gengzhi
500 CE
1000 BCE
Numerals from oracle bones, from 11-12th century BCE (Numbers between 1 to 30,000)
〇 一 二 三 四 五 六 七 八 九
0 1 2 3 4 5 6 7 8 9
十 廿 卅 百 千 万 亿 兆
10 20 30 100 1000 10000 10⁸ 10¹²
Rod Numerals
1 2 3 4 5 6 7 8 9
or
Not an original source
~200 BCE
The Nine Chapters Source Wikimedia Commons (1820 edition)
中文：清李潢撰《九章算術細草圖說》十卷。嘉慶二十五年（1820年）語鴻堂刻本，竹紙線裝，一函八冊。
200 CE
An animation showing a box-lid emerging from two intersecting cylinders. Credit: Van helsing Wikimedia (CC BY-SA 3.0)


Rough Chronology Math in Ancient China
Original Source
The Book of Numbers and Computations - Suan Shu Shu
Counting board
Computations Algorithms
200 BCE
Liu Hui
Diagram of Liu Hui's π inequality
Source: derivative work: Pbroks13 (talk) Gisling - Wikimedia Commons
π
Zu Chongzhi
Zu Gengzhi
500 CE
1000 BCE
Numerals from oracle bones, from 11-12th century BCE (Numbers between 1 to 30,000)
Rod Numerals
Not an original source
~200 BCE
The Nine Chapters Source Wikimedia Commons (1820 edition)
中文：清李潢撰《九章算術細草圖說》十卷。嘉慶二十五年（1820年）語鴻堂刻本，竹紙線裝，一函八冊。
200 CE
An animation showing a box-lid emerging from two intersecting cylinders. Credit: Van helsing Wikimedia (CC BY-SA 3.0)


Rough Chronology Math in Ancient China
Original Source
The Book of Numbers and Computations - Suan Shu Shu
Counting board
Computations Algorithms
200 BCE
Liu Hui
Diagram of Liu Hui's π inequality
Source: derivative work: Pbroks13 (talk) Gisling - Wikimedia Commons
π
Zu Chongzhi
Zu Gengzhi
500 CE
1000 BCE
Numerals from oracle bones, from 11-12th century BCE (Numbers between 1 to 30,000)
Rod Numerals
Not an original source
~200 BCE
The Nine Chapters Source Wikimedia Commons (1820 edition)
中文：清李潢撰《九章算術細草圖說》十卷。嘉慶二十五年（1820年）語鴻堂刻本，竹紙線裝，一函八冊。
200 CE
An animation showing a box-lid emerging from two intersecting cylinders. Credit: Van helsing Wikimedia (CC BY-SA 3.0)


Rough Chronology Math in Ancient China
Original Source
The Book of Numbers and Computations - Suan Shu Shu
Counting board
Computations Algorithms
200 BCE
Liu Hui
Diagram of Liu Hui's π inequality
Source: derivative work: Pbroks13 (talk) Gisling - Wikimedia Commons
π
Zu Chongzhi
Zu Gengzhi
500 CE
1000 BCE
200 CE
Write down a summary of the contents of the slide. Work in teams
An animation showing a box-lid emerging from two intersecting cylinders. Credit: Van helsing Wikimedia (CC BY-SA 3.0)



Oracle bone (~1200 BCE)

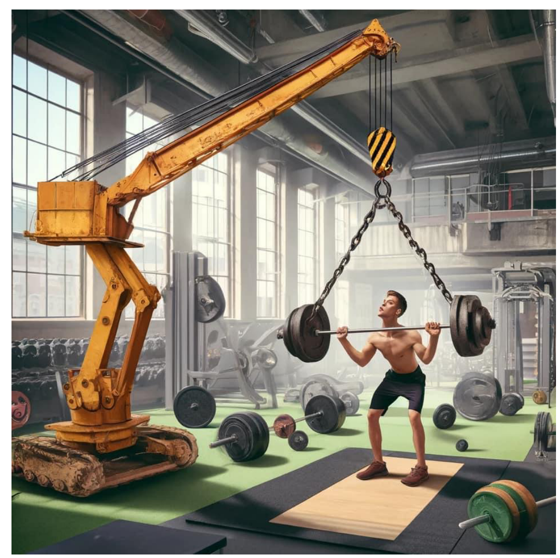
Inscribed tortoise carapace ("oracle bone"), Anyang period, late Shang dynasty, c. 1300–1050 B.C.E., tortoise shell, China, 6.5 high x 10.8 x 2.3 cm - Smithsonian Institution, Washington, D.C.

Complete the table, using the hints (the number in the row a is 72, the number in the row b is 26). Answer what are the numbers on rows c, d and e.

Rod numerals were used (approximately) between 500 BCE and 1500 CE.

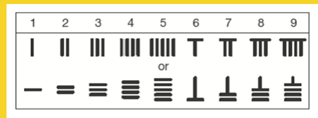
𝍯 𝍡	72	a
𝍪 𝍥	26	b
𝍬 𝍡		c
𝍣𝍩𝍡		d
𝍣 𝍡		e

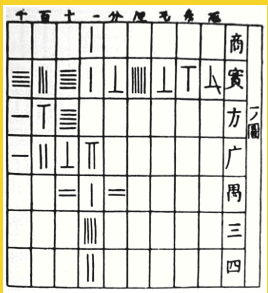
Image created by AI (DALL-E), illustrating something analogous to the use of AI to complete assignments



Counting rods and counting boards

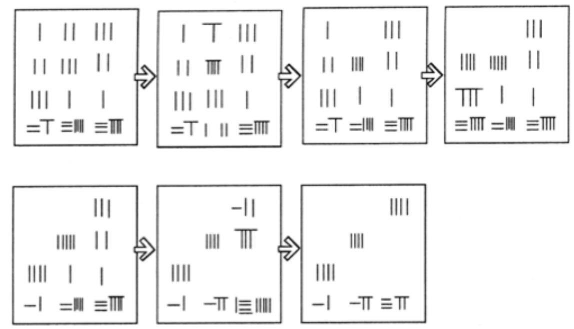
1. Rod numerals (see figure) were used approximately between 500 BCE and 1500 CE in China. The number system associated with the rod numerals was (choose the appropriate): additive, ciphered or alphabetic, multiplicative or positional?





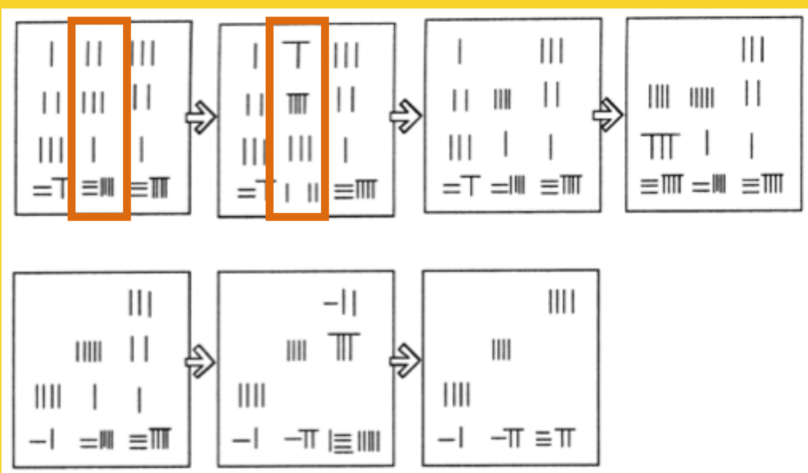
2. The Chinese counting board, with its grid of square cells, was also useful for storing and manipulating rows and columns of numbers. Much later, in the west, this grid of square cells was rediscovered and called: (choose the appropriate): lattice, matrix, magic square, spreadsheet?

"Matrices" already appeared in the Nine chapters
The following illustration shows how the above problem **would** be solved on a traditional Chinese counting board.



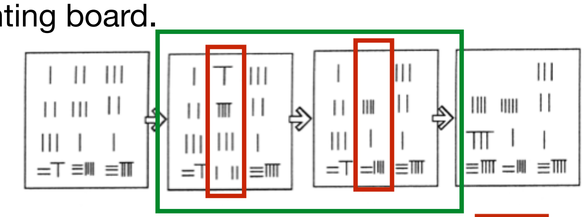
https://www.maa.org/press/periodicals/convergence/a-classic-from-china-the-nine-chapters-matrices

What is the relation between the two highlighted columns?

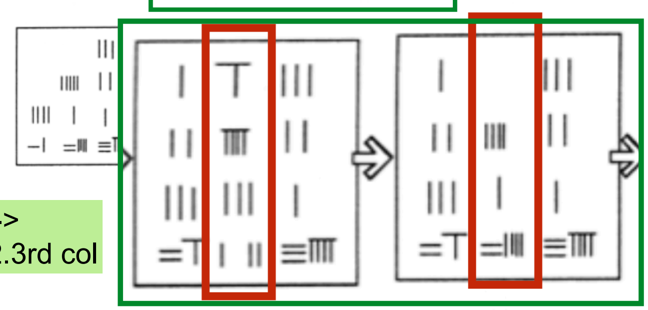


https://www.maa.org/press/periodicals/convergence/a-classic-from-china-the-nine-chapters-matrices

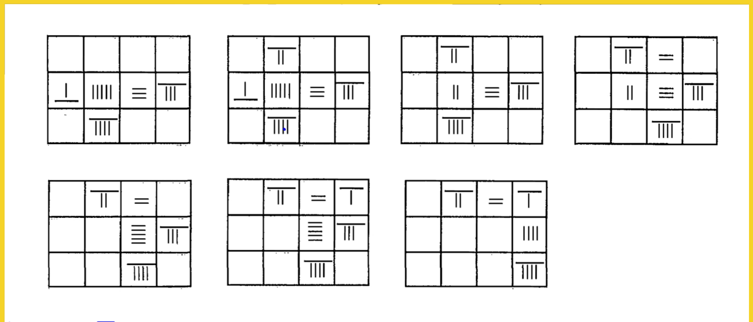
"Matrices" already appeared in the Nine chapters
The following illustration shows how the above problem **would** be solved on a traditional Chinese counting board.



2nd col. ->
2nd col-2.3rd col



Divide 6538 by 9 (by hand, no gadgets) and compare the steps with the ancient Chinese algorithm below



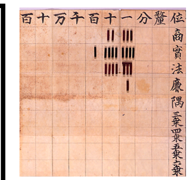
Note ⊥ = T = 6

From Straffin Jr, Philip D. "Liu Hui and the first golden age of Chinese mathematics." *Mathematics Magazine* 71.3 (1998): 163-181.

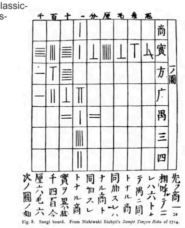
Explain how this sequence of counting boards relates to the division algorithm

- in use by **400 BCE,**
- made of polished wood with rulings that formed a grid of square cells
- **positional** number system: columns represented numbers according to their units, tens, hundreds
- To "write" a number on the counting board, its numerals were placed one per cell, on one row of the grid. (A blank cell stood for zero.)
- **highly developed set of algorithms** for
  - multiplication, division,
  - computation of square and cubic roots
  - Solving linear equations
  - Solution of higher degrees with multiples unknowns.
- "Some" use of negative numbers

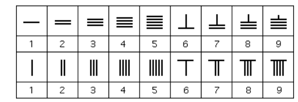
Counting boards

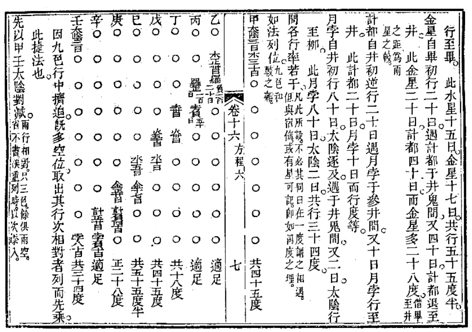


paper facsimile a counting. board https://www.maa.org/press/periodicals/convergence/a-classic-from-china-the-nine-chapters-numbers-and-units



Counting Board (Sangi in Japanese). Source: Smith & Mikami 1914 https://kartsci.org/kocomu/computer-history/history-abacus-ancient-computing/



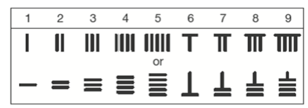


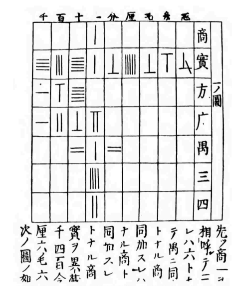
$$\begin{bmatrix} 3 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 45 \\ 0 & 30 & -120 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 30 & 30 & 0 & 0 & 0 & 0 & 18 \\ 0 & 0 & 0 & 0 & 50 & 10 & 0 & 0 & 0 & 45 \\ 0 & 0 & 0 & 0 & 0 & 15 & 17 & 0 & 0 & \frac{111}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 20 & -40 & 0 & 28 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20 & -10 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 80 & 34 \end{bmatrix}$$

Source: http://rhart.org/algebra/

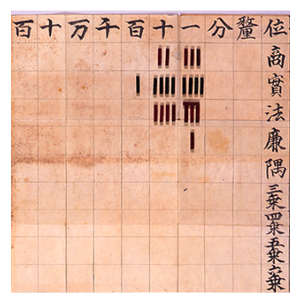
A *fangcheng* problem with 9 conditions in 9 unknowns of the form of the generalized "well problem," from Mei Wending's 梅文鼎 (1633–1721) *On* Fangcheng (*Fangcheng lun* 方程論, c. 1674)

Counting boards and rod numerals





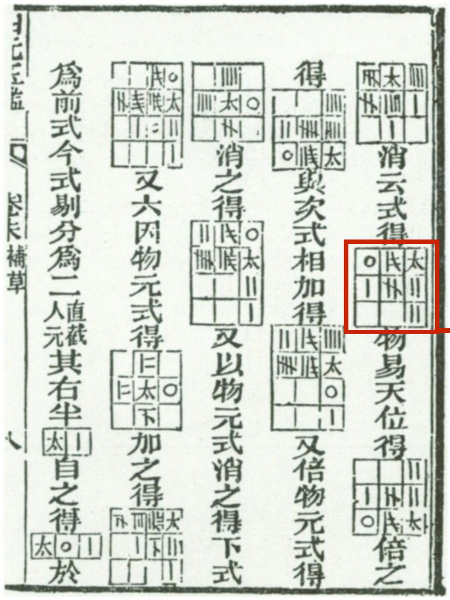
Counting Board (Sangi in Japanese). Source: Smith & Mikami 1914 https://kartsci.org/kocomu/computer-history/history-abacus-ancient-computing/



Paper facsimile is shown above were known to have been used in Edo Period Japan (1603-1867) - No old drawings of counting boards survive from China. https://www.maa.org/press/periodicals/convergence/a-classic-from-china-the-nine-chapters-numbers-and-units

Fig. 9. From Miyake Kenryū's work of 1795.

Accounting with Rod Numerals on a Counting Board. Source : Smith & Mikami 1914.

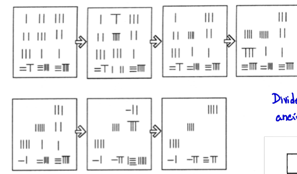


A page from a 19th century edition of the *Ssu Yuan Yii Chien of* Chu Shih-Chieh (**1303 a.D.**) showing the 'matrices' of the Men Yuan algebraic notation.

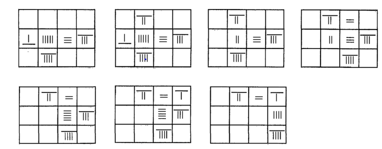
0	−120	·
1	−2	2
		2

$xy^2 - 120y - 2xy + 2x^2 + 2x$

Source: On ancient Chinese mathematics by D. Struik, The Mathematics Teacher , Vol. 56, No. 6 (OCTOBER 1963), pp. 424-432

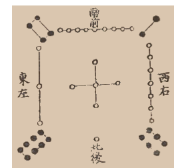


Divide 6538 by 9 (by hand, no gadgets) and compare the steps with the ancient Chinese algorithm below

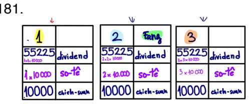


Note ⊥ = ⊤ = 6

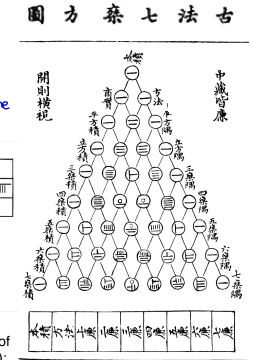
From Straffin Jr, Philip D. "Liu Hui and the first golden age of Chinese mathematics." *Mathematics Magazine* 71.3 (1998): 163-181.



Magic square



Algorithm for computing square roots



Drawing of Pascal's Triangle published in 1303. It was called Jia Xian triangle or Yanghui Triangle by the Chinese, after the mathematician Jia Xian & Yang Hui.
Note rod numerals!

The Chinese counting board is a good example of how a technological invention can influence how science develops, and even how people think.

Randy K. Schwartz- *A Classic from China: The Nine Chapters -*
https://www.maa.org/press/periodicals/convergence/a-classic-from-china-the-nine-chapters-numbers-and-units

Nine Chapters of the Mathematical Art

Book of Numbers and Computations

The Book of Numbers and Computations - Suan Shu Shu

1. **235 BCE** (found in a 186 BCE tomb)
2. **original source** (not a copy)
3. Possibly from a variety of sources.
4. 68 problems
5. 200 bamboo strips
6. Elementary calculations with fractions, Rule of False Position and volumes of various solid shapes.

算數書

The Nine Chapters of the Mathematical Arts

1. **~200 BCE**
2. Earliest copy made in 1213 CE.
3. Most likely from a variety of sources.
4. 246 problems
5. Some topics taken from the Book of Numbers.
6. Organized in chapters by topics.
7. Calculation with specific numbers with general explanation.
8. Often sophisticated and difficult in its treatment of algorithms.
9. Example: Treatment of simultaneous equations (3 by 3)
10. Later commentators made **important contributions**

The Book of Numbers and Computations - Suan Shu Shu

A fox, raccoon, and hound go through customs, and (together) pay tax of 111 qian. The hound says to the raccoon, and the raccoon says to the fox: since your fur is worth twice as much as mine, then the tax you pay should be twice as much! How much should each one pay?

Bamboo strips - The Book of Numbers and Computations
Suan Shu Shu

The Book of Numbers and Computations - Suan Shu Shu

A fox, raccoon, and hound go through customs, and (together) pay tax of 111 qian. The hound says to the raccoon, and the raccoon says to the fox: since your fur is worth twice as much as mine, then the tax you pay should be twice as much! How much should each one pay?

The result says: the hound pays 15 6/7 qian, the raccoon pays 31 5/7 qian, and the fox pays 63 and 3/7 qian.

The method says: let each one double the other; adding them together (1 + 2 + 4), 7 is the divisor; taking the tax, multiplying by each (share) is the dividend; dividing the dividend by the divisor gives each one's (share).

The Book of Numbers and Computations - Suan Shu Shu

(The tax on) 3 (square) bu of millet is 1 dou; (on) 4 (square) bu of wheat is 1 dou; (and on) 5 (square) bu of small beans is 1 dou. If the combined tax (on all of them together) is 1 shi (capacity), then how much is the tax (on each one)?

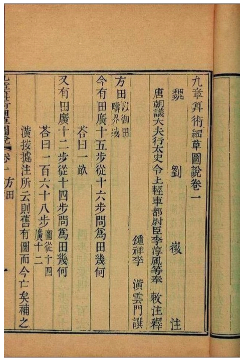
The result says: the tax on millet is 4/12 dou; the tax on wheat is 3 9/47 dou; (and) the tax on beans is 2 26/47 dou.

The method says: put down (**on the counting board** the amount of) millet 3 bu, wheat 4 bu, and beans 5 bu; let the product of the millet and wheat be the dividend for the beans; the product of the beans and the millet be the dividend for the wheat; (and the product of the wheat and the beans be the dividend for the millet); for each of the different (amounts) put down (on the counting board) one shi multiplied by each (of the amounts for beans, wheat, and millet) as the dividends; (taking) 47 as the divisor gives the result in dou.

The Book of Numbers and Computations - Suan Shu Shu

- The geometrical problems are much more varied and some are very difficult (difficult for mathematics around the world in that era!)
- Example
  - the correct method for finding the **volume of a cone** (on the tacit assumption that the circumference is 3 times the diameter) and from this a method for finding the **volume of a frustum of a cone** is given.

The Nine Chapters on the Mathematical Art, around 200BC.



Source Wikimedia Commonts (1820 edition)
中文：清李潢撰《九章算術細草圖說》十卷。嘉慶二十五年（1820年）語鴻堂刻本，竹紙線裝，一函八冊。

- Practical handbook of mathematics intended to provide methods to be used to solve everyday problems of engineering, surveying, trade, and taxation.
- Played a fundamental role in the development of mathematics in China, (analogous to the role of Euclid's Elements in Western Mathematics.)
- Many important commentators!

Chinese mathematicians were clearly concerned about justifying their methods and establishing the validity of their results. Their proofs were not axiomatic proofs, but they were proofs nevertheless, and they were clearly able to establish the truth of correctness of the solutions they proffered. Joseph Dauben

Liu Hui (~250) 劉徽

Liu Hui ~ 200 CE

- Little is known of his life
- He was a mathematician of great power and creativity.
- Liu's ideas arrived to us in
  - commentary, ~250 CE of the Nine Chapters on the Mathematical Art.
  - work on mathematics for surveying, Sea Island Mathematical Manual

Liu Hui Introduction to his commentary of the Nine Chapters

I read the Nine Chapters as a boy, and studied it in full detail when I was older. [I] observed the division between the dual natures of Yin and Yang [the positive and negative aspects] which sum up the fundamentals of mathematics. **Thorough investigation shows the truth therein, which allows me to collect my ideas and take the liberty of commenting on it.** Things are known to belong to various classifications. Just as the branches of a tree are to its trunk, so are a multitude of things to an archetype. Therefore I have tried to explain the whole theory as concisely as possible, with spatial forms shown in diagrams, so that the reader should have a reasonably good all-around understanding of it.

Liu Hui Introduction to his commentary of the Nine Chapters

Some of the material in the Nine Chapters predates the great book-burning and burial-alive of scholars of 213 B.C., ordered by emperor ShihHuang-ti of the Qin dynasty. Indeed, Liu Hui writes in the preface of his commentary:

In the past, the tyrant Qin burnt written documents, which led to the destruction of classical knowledge ... Because of the state of deterioration of the ancient texts, Zhang Cang and his team produced a new version ...filling in what was missing

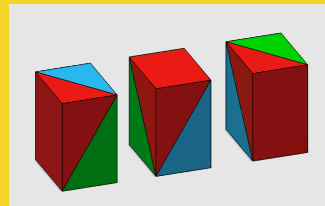
Liu Hui and the First Golden Age of Chinese Mathematics, Philip D. Straffin, Jr.,Mathematics Magazine, Jun., 1998, Vol. 71, No. 3 (Jun., 1998)

Liu Hui on the volume of yangma (rectangular pyramid)

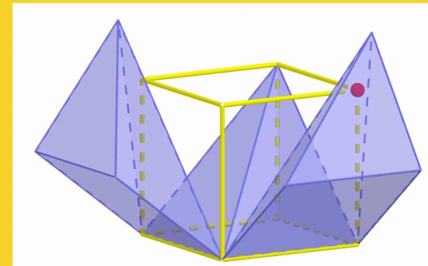
https://www.geogebra.org/m/mpfx3dt8

- A ***yangma*** is a pyramid with rectangular base and one of its lateral edges perpendicular to the base
- Liu Hui (among many other results) studied the volume of the Yangma.

By sliding the red vertex, the cube becomes a rectangular prism.
The blue vertex opens up the prims into three "yangmas". Are these yangmas congruent? Educated guess: Do the have the same volume?



https://www.maa.org/book/export/html/842724



https://www.geogebra.org/m/ugxEMM5D

Liu Hui on the volume of yangma (rectangular pyramid)

- A **yangma** (阳马) is a pyramid with rectangular base and one of its lateral edges perpendicular to the base

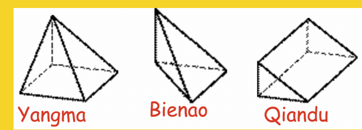
  It probably has its origin in architecture. In Japanese explanation, yangma is called a "sunshine-carrying horse," which in fact conveys part of its literal meaning in Chinese (四角錐, n.d.).

- A **qiandu** (堑堵), (meaning literally an embankment beside a trench), is a a right triangular prism.
- A **bie'nao** (鳖臑), (meaning literally a turtle's foreleg bone,) is a pyramid with base a right triangle, and two of the lateral faces are right triangles that share a side with the base. The third lateral face is an isosceles triangle formed between the two right triangular faces opposite the base.

http://donwagner.dk/Pyramid/Pyramid-4.html

https://maa.org/book/export/html/842718

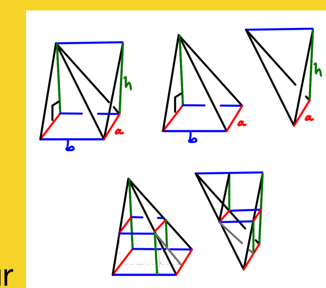
Liu Hui on the volume of yangma (rectangular pyramid)



1. "Add" a bienao to the yangma to make a prism (or qiandu)
2. Show that the volume of the yang is 2/3 of the volume of the qiandu.

Write down the volume of the yangma (in terms of a, b and h)

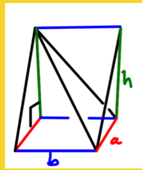
Next slide

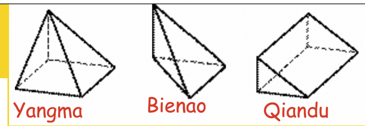


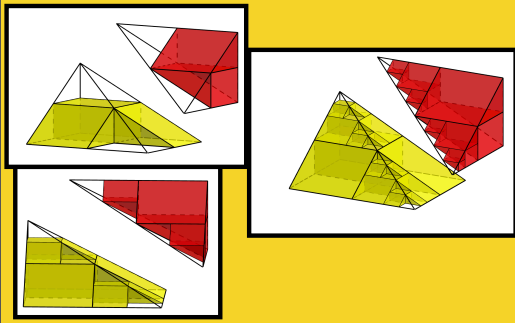
https://www.geogebra.org/m/vwvuznur

Liu Hui on the volume of yangma (rectangular pyramid)

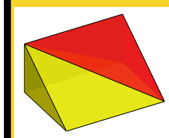
A ***yangma*** is a pyramid with rectangular base and one of its lateral edges perpendicular to the base volume of a yangma is, where **a** and **b** are the sides of the rectangular base and **h** is the height.







Since
Red Volume =(1/2)Yellow volume
And the leftover gets smaller and smaller,
Red Volume =(1/2)Yellow volume below



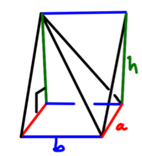
Then Red=(1/3) of qiandu
Yellow=(2/3) of qiandu

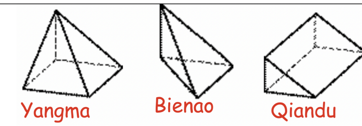
Write down the volume of the yangma (in terms of a, b and h)

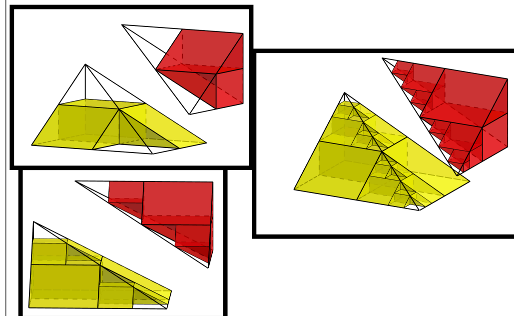
http://donwagner.dk/Pyramid/Pyramid-4.html

Liu Hui on the volume of yangma (rectangular pyramid)

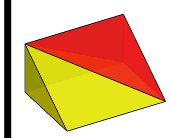
A ***yangma*** is a pyramid with rectangular base and one of its lateral edges perpendicular to the base Liu Hui showed that the volume of a yangma is **(1/3)a.b.h** , where **a** and **b** are the sides of the rectangular base and **h** is the height.







Since
Red Volume =(1/2)Yellow volume
And the leftover gets smaller and smaller,
Red Volume =(1/2)Yellow volume below



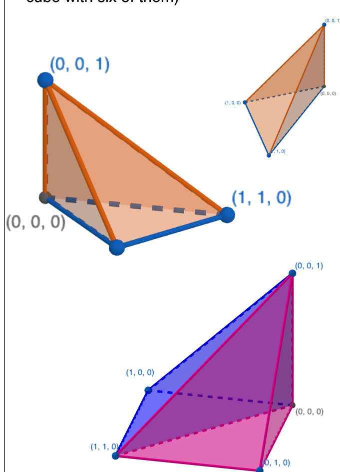
Then Red=(1/3) of qiandu
Yellow=(2/3) of qiandu

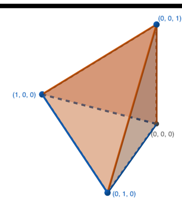
Vol(Quiandu)=(1/2)Vol(Rect. prism)=(1/2)a.b.h=(3/2)Vol(Yangma)
Then **Vol(Yangma)=(1/3)a.b.h**

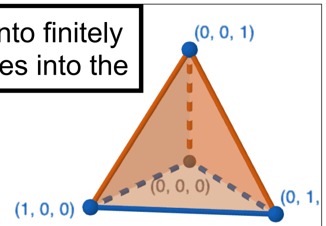
http://donwagner.dk/Pyramid/Pyramid-4.html

Fast forward (Dehn, 1900): One cannot cut into finitely many pieces (with planes) one of these shapes into the

Vertices of "quadrirectangular tetrahedron (0,0,0), (1,0,0), (1,1,0) and (1,1,1).
Dehn invariant 0 (because we can make a cube with six of them)



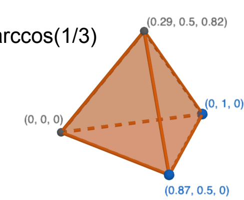




Vertices of right tetrahedron (0,0,0), (1,0,0), (0,1,0) and (0,0,1).

The dihedral angles through the edges with no endpoint at (0,0,0) is not rational (its cosine is 1/√3).

Regular tetrahedron:
The dihedral angle is arccos(1/3)



Liu Hui (~250 CE)

- Commentary on the proof the Pythagorean theorem.
- Volumes of plane and solid figures.
- Solution of linear equation with two unknowns.
- Algorithm to compute π.

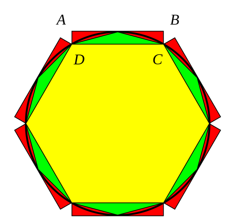


Diagram of Liu Hui's π inequality
Source: derivative work: Pbroks13 (talk)
Gisling - Wikimedia Commons

- Outstanding and original mathematician with a deep understanding of difficult concepts
- Familiar with the literary and historical classics of China.
- Never claimed results of which he was not fully confident. He wrote:-
  Let us leave the problem to whoever can tell the truth.
- Cared about the conditions of people and about the economy of the country
  (From https://mathshistory.st-andrews.ac.uk/Biographies/Liu_Hui/)

Computation of π
Liu Hi - Zu Chongzhi

Liu Hui approximation of π (**~200 CE**)

- Using Gougu (Pythagorean) theorem , computed the **perimeter** of regular polygons of 6,12, 24, ...,**96**. (3.2, 3.2^2, 3.2^3, 3.2^4, 3.2^5) sides to approximate the circumference.

Mathematician Zu Chongzhi (**~500CE**)

- same method as Liu Hui
- used a regular polygons of 6,12, 24, ..., **24,576**. (3.2, 3.2^2, 3.2^3,… 3.2^{13}) sides

Page from a sixteenth century edition of the Nine Chapters on the Mathematical Art.

https://www.maa.org/press/periodicals/convergence/mathematical-treasures-jiuzhang-suanshu

- Calculated with counting rods
- Duplicating the number of sides
- Gougu (Pythagorean) theorem

Later

Describe (in words, not in equations) the work of Liu Hui, Zu Chongzhi and Zu Geng computing the volume of the sphere

---

Liu Hui (~250) 劉徽

Liu Hui on the volume of the sphere

Problem from the Nine Chapters: There is a sphere of volume 16441866437500 chi. Find the diameter.

Answer: 14300 chi.

Method: Put down the volume in chi, multiply by 16 and divide by 9.

Extract the cube root of the result to get the diameter of the sphere.

Find a formula for the volume V of the sphere (according to this problem) in terms of the diameter d. (of the form V=…). If one assumes that π=3, is the formula correct?

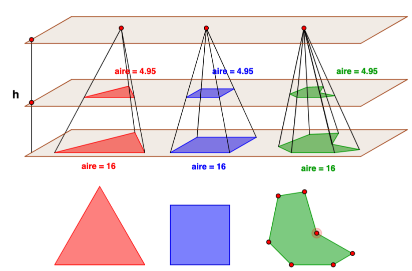
Note: 1 chi is approximately 1 foot (0.3 m.)

Zu Geng's (~450 CE) statement of his basic assumption

疊棋成立積
緣冪勢既同
則積不容異

If blocks are piled up to form volumes,
And corresponding areas are equal,
Then the volumes cannot be unequal

Source: http://donwagner.dk/SPHERE/SPHERE.html



This is called now the "Cavalieri principle". ~1600

Is it fair?

Trattato della sfera e prattiche per vso di essa, Roma, 1682

Problem from the Nine Chapters: There is a sphere of volume 16441866437500 chi. Find the diameter.

Answer: 14300 chi.

Method:

Put down the volume in chi, multiply by 16 and divide by 9.

Extract the cube root of the result to get the diameter of the sphere.

1,644,866,437,500

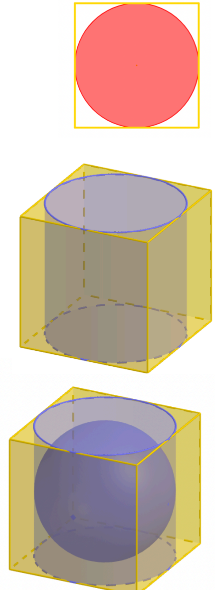
A commentary (of the Nine Chapters) attributed to Liu Hui 劉徽

- Recall: the volume V of the sphere of diameter d is $(\pi/6)d^3$
- According to a problem in the Nine Chapters $V=(9/16)\ d^3$.
- Liu Hiu says that this should be interpreted as $V=(4/3)^2\ d^3$.
- And gives the following argument

The Nine Chapters and the volume of the sphere

Liu Hui's Argument

- It is known:Vol(cube)/Vol(cylinder)= 4/3 (i.e.,4/π)
- It is believed: Vol(cylinder)/Vol(sphere)=4/3 (i.e. 4/π)
- It was accepted π=3 (but there are different possibilities)
- **If so, Vol(sphere)=(9/16) Vol(cube)**
- **Then Vol(sphere)=(9/16)d^3 where d is the diameter (or V=(π/4)^2d^3)**



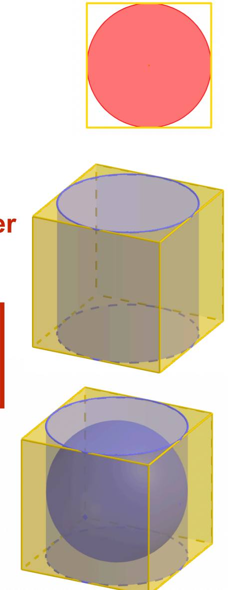
Wagner, Donald Blackmore. "Liu Hui and Tsu Keng-chih on the volume of a sphere." Chinese Science 3 (1978): 59-79.
http://donwagner.dk/SPHERE/SPHERE.html#Heading18

https://www.geogebra.org/m/nkdqansv#material/hnCyRFV3

The Nine Chapters and the volume of the sphere

- It is known:Vol(cube)/Vol(cylinder)= 4/3 (4/π)
- It is believed: Vol(cylinder)/Vol(sphere)=4/3 (4/π)
- It was accepted π=3
- **If so, Vol(sphere)=(9/16) Vol(cube)**
- **Then Vol(sphere)=(9/16)d^3 where d is the diameter 4)^2d^3)**

- (9/16)D^3 = 0.5625 D^3
- The volume is(4/3) π $(D/2)^3$ ~ 0.5235988D^3

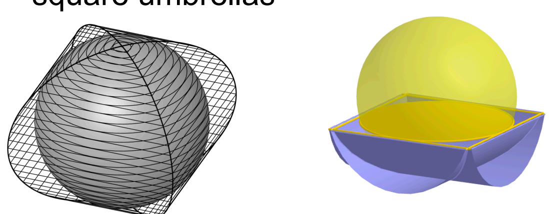


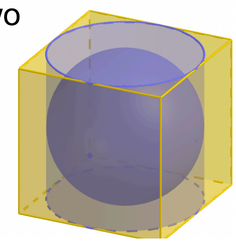
Wagner, Donald Blackmore. "Liu Hui and Tsu Keng-chih on the volume of a sphere." Chinese Science 3 (1978): 59-79.
http://donwagner.dk/SPHERE/SPHERE.html#Heading18

https://www.geogebra.org/m/nkdqansv#material/hnCyRFV3

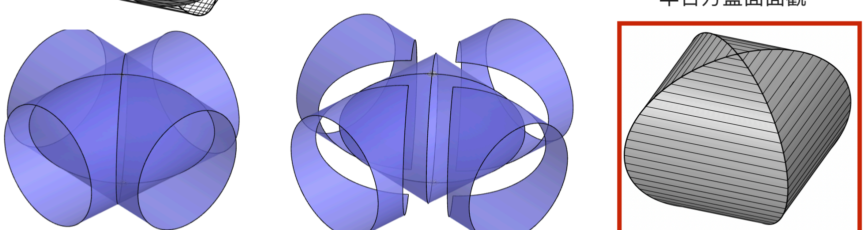
Liu Hui and the volume of the sphere

- Liu -200 CE- discovered the Mouhefanggai (牟合方蓋面面觀) -the solid formed by two square umbrellas

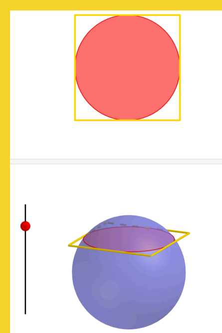


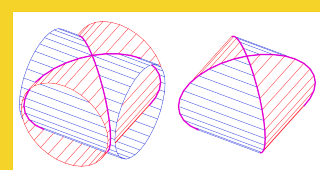


牟合方蓋面面觀



What is the relation of the volume of the Mouhefanggai 牟合方蓋 (M) and the volume of the sphere (S)?





https://www.geogebra.org/m/nkdqansv#material/WWpP9bGD

The Nine Chapters and the volume of the sphere

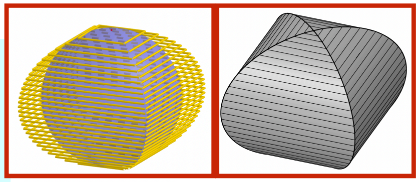
Recall: That in the Nine Chapters, it stated that Vol(sphere)=(9/16)D^3 (D is the diameter)

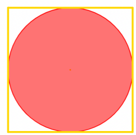
Liu Hui proves that the assumption is incorrect by showing that this relation

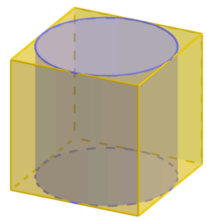
Vol(cylinder)/Vol(sphere)= $4/\pi$

Is false by finding an object **M**, strictly included in the cylinder such that

Vol(**M**)/Vol(sphere)= $4/\pi$



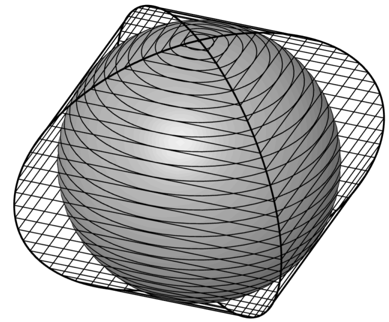


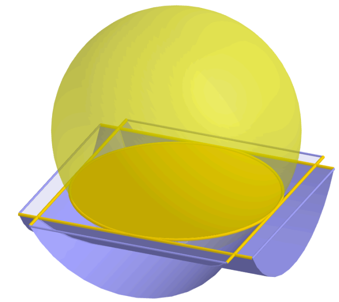


Wagner, Donald Blackmore. "Liu Hui and Tsu Keng-chih on the volume of a sphere." Chinese Science 3 (1978): 59-79.
http://donwagner.dk/SPHERE/SPHERE.html#Heading18

https://www.geogebra.org/m/nkdqansv#material/hnCyRFV3

Liu Hui and the volume of the sphere





S(r) sphere of radius r

M(r) Mouhefangai around S(r)

$$\frac{\text{Volume(S(r))}}{\text{Volume(M(r))}} = \pi/4$$

Math Poem

The geometer's frustration by Liu Hui

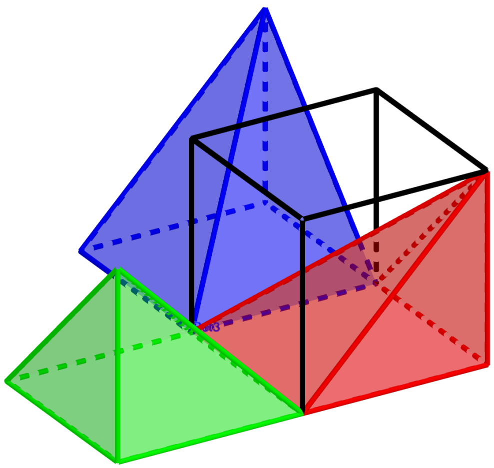
Liu Hui had solved part of the problem. The difficulty that remains is to find the volume of the box-lid. He concludes with the following bit of doggerel.

Look inside the cube
And outside the box-lid;
Though the diminution increases,
It doesn't quite fit.
The marriage preparations are complete;
But square and circle wrangle,
Thick and thin make treacherous plots,
They are incompatible.
I wish to give my humble reflections,
But fear that I will miss the correct principle;
I dare to let the doubtful points stand,
Waiting for one who can expound them.

Source: http://donwagner.dk/SPHERE/SPHERE.html

Zu Chongzhi and Zu Geng on the Volume of a Sphere

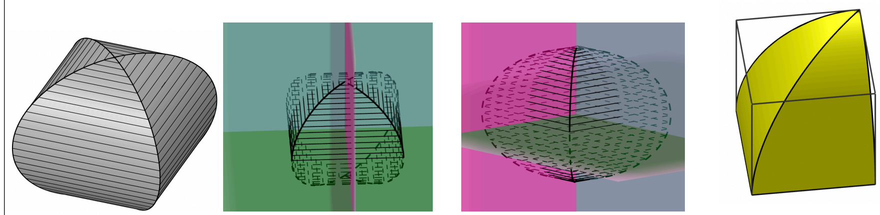
Volume of the Pyramid



$$\frac{\text{Volume(Pyr)}}{\text{Volume(Cube)}} = 1/3$$

https://www.geogebra.org/m/ky84nh9g

Zu Chongzhi and Zu Geng on the Volume of a Sphere



Divide M(r) into eight congruent pieces.

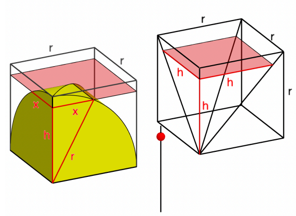
Each of the eight blocks is “like” a rounded *yangma* and “fits” in C(r).

https://www.geogebra.org/m/bprtgwh8#material/gevzbyfd
https://www.geogebra.org/m/bprtgwh8

Zu Chongzhi and Zu Geng on the Volume of a Sphere

https://www.geogebra.org/m/nkdqansv#material/kQxCtQbq

C(r) denotes cube of side r
P(r) denotes slanted pyramid in cube of side r.
S(r) sphere of radius r
M(r) Mouhefangai around S(r)



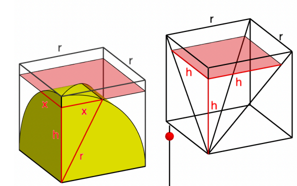
Volume(M(r))/8 = Vol(C(r))-Vol(P(r)) = (2/3)Vol(C(r))

Volume(M(r)) = = (16/3)Vol(C(r))

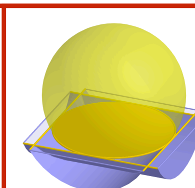
Zu Chongzhi and Zu Geng on the Volume of a Sphere

https://www.geogebra.org/m/nkdqansv#material/kQxCtQbq

C(r) denotes cube of side r
P(r) denotes slanted pyramid in cube of side r.
S(r) sphere of radius r
M(r) Mouhefangai around S(r)

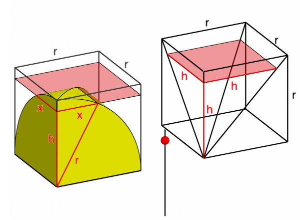


$$\frac{\text{Volume(S(r))}}{\text{Volume(M(r))}} = \pi/4$$



Zu Chongzhi and Zu Geng on the Volume of a Sphere

https://www.geogebra.org/m/nkdqansv#material/kQxCtQbq



C(r) denotes cube of side r
P(r) denotes slanted pyramid in cube of side r.
S(r) sphere of radius r
M(r) Mouhefangai around S(r)

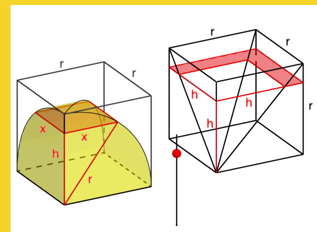
Volume(M(r)) = (16/3)Vol(C(r))

$$\frac{\text{Volume}(S(r))}{\text{Volume}(M(r))} = \pi/4$$

Volume(S(r)) = (π/4) Volume(M(r))

= (π/4) (16/3)Vol(C(r)) = (4/3)π r^3

What is the relation of the volume of the Mouhefanggai 牟合方蓋 (M) and the volume of the cube (C)?



https://www.geogebra.org/m/nkdqansv#material/kQxCtQbq

Math Poems

The geometer's frustration by Liu Hui

Liu Hui had solved part of the problem. The difficulty that remains is to find the volume of the box-lid. He concludes with the following bit of doggerel.

Look inside the cube
And outside the box-lid;
Though the diminution increases,
It doesn't quite fit.
The marriage preparations are complete;
But square and circle wrangle,
Thick and thin make treacherous plots,
They are incompatible.
I wish to give my humble reflections,
But fear that I will miss the correct principle;
I dare to let the doubtful points stand,
Waiting for one who can expound them.

The geometers triumph by Zu Gengzhi

The proportions are extremely precise
And my heart shines.
Zhang Heng copied the ancient,
Smiling on posterity;
Liu Hui followed the ancient,
Having no time to revise it.
Now what is so difficult about it?
One need only think.

The quotation from Zu Gengzhi probably ends here.

Source: http://donwagner.dk/SPHERE/SPHERE.html

Describe (in words, not in equations) the work of Liu Hui, Zu Chongzhi and Zu Geng computing the volume of the sphere

Magic Squares

Can you find a pattern?

These arrangements of numbers are magic squares.

Write down the the definition of magic square

2	9	4
7	5	3
6	1	8

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

4	14	15	1
9	7	6	12
5	11	10	8
16	2	3	13

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

A ***magic square*** is an nxn matrix, such that

1. The entries are the numbers 1,2, …, n^2.
2. The sum of columns, rows and diagonals is constant.

2	9	4
7	5	3
6	1	8

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

4	14	15	1
9	7	6	12
5	11	10	8
16	2	3	13

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

Write down the magic constant of an n x n magic square in terms of n.

The sum of the rows (or columns, or diagonals) of a magic square is called the ***magic constant***.

8	1	6
3	5	7
4	9	2

4	3	8
9	5	1
2	7	6

2	9	4
7	5	3
6	1	8

6	7	2
1	5	9
8	3	4

6	1	8
7	5	3
2	9	4

2	7	6
9	5	1
4	3	8

4	9	2
3	5	7
8	1	6

8	3	4
1	5	9
6	7	2

(2 minutes) These eight magic squares are *equivalent* in some sense. Can you explain how are they equivalent?

4	14	15	1
9	7	6	12
5	11	10	8
16	2	3	13

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Note: These two magic squares are *not* equivalent

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

***not* equivalent**

4	14	15	1
9	7	6	12
5	11	10	8
16	2	3	13

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

***not* equivalent**

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

2	9	4
7	5	3
6	1	8

***3x3 all* equivalent**

Order	Number of magic squares
1	1
2	0
3	1
4	880
5	275305224
6	?

It is an unsolved problem to determine the number of magic squares of an arbitrary order, but the number of distinct magic squares (excluding those obtained by successive rotations and reflections)

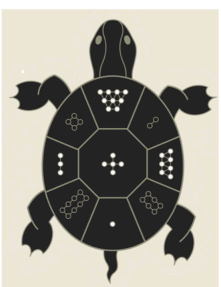
Order	Number of magic squares
1	1
2	0
3	1
4	880
5	275305224
6	?

$\sim 1.775399 \cdot 10^{19}$

If you want to keep reading about magic squares

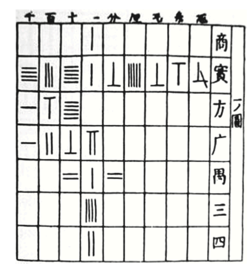
http://www.mathematische-basteleien.de/magsquare.htm

The Mythological Emperor Yu, (~2000BCE) received a divine gift from a Lo river tortoise. The gift, a diagram called the Lo shu, is believed to contain the principles of Chinese Mathematics.

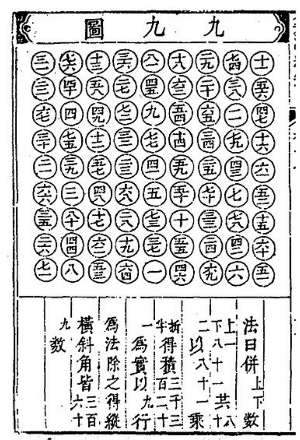


4	9	2
3	5	7
8	1	6

Lo Shu from "The Astronomical Phenomena" (*Tien Yuan Fa Wei*). Compiled by Bao Yunlong in 13th century, published during the Ming dynasty, 1457–1463.



Chinese counting board



A page displaying 9×9 magic square from Cheng Dawei's Suanfa tongzong (1593)

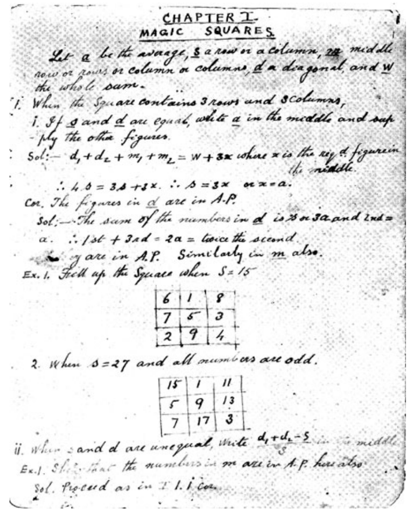
Melencolia I

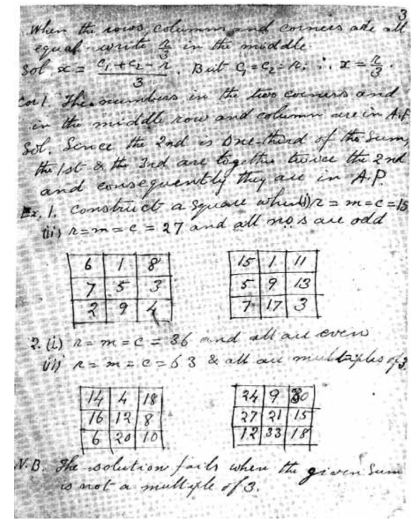
1514

Albrecht Dürer German

https://www.metmuseum.org/en/art/collection/search/336228

Two Pages from Ramanujan's Notebooks





https://www.imsc.res.in/~rao/ramanujan/introindex.html

Square root computations

step 1 置積[1]爲實[2].

$x^2 = 55225$. Find x

Put the (known) square (of a certain unknown number) (in the second row from the top of the counting-board) to be the *Shih* 實, dividend.

55225	dividend

step 2 借[3]) 一算

Make use of one counting-rod (and put it in the bottom row of the counting-board in the furthest right-hand digit column) [This one counting-rod is to be called the preliminary *Chieh-suan* 借算].

55225	dividend
1	preliminary chieh-suan

step 3 步[1]) 之，超一等.

This one counting-rod is moved forward (from right to left) by steps of two places each (as far as it can go without transgressing the furthest left digit of the dividend) [This one counting-rod, with its new place-value, is to be called the *Chieh-suan* 借算].

55225

1 → 10000 (2, 2)

55225	dividend
10000	chieh-suan

Move this counting-rod to the left by 2 steps of 2 places

step 4 議[3]) 所得.

(The first figure of the root is selected through trial, taking 1, 2, 3, one after another). Discuss the *So-tê*, 所得[3]). (The *So-tê* is the product of the first root figure under trial multiplied by the *Chieh-suan*). (What is meant by 'discussion' is that when the selected number has multiplied the *So-tê* once, the product must not be greater than the dividend; and at the same time the largest possible root figure must be selected)[4]). [The selected figure is placed in the top row of the countingboard. This is the *Fang* 方 row which will ultimately contain the answer.]

Recall, we are finding a positive integer a such that $(100a)^2 = 10000a^2 < N = 55225$

Too small ↓

1	
55225 ($1 \times 1 \times 10000$)	dividend
1×10000	so-tê
10000	chieh-suan

↓ OK

2	Fang
55225 ($2 \times 2 \times 10000$)	dividend
2×10.000	so-tê
10000	chieh-suan

Too big ↓

3	
55225 ($3 \times 3 \times 10000$)	dividend
3×10000	so-tê
10000	chieh-suan

Find a

step 5 以一[2]) 乘所借一算爲法[1]).

The *Chieh-suan* is multiplied by the (selected) first figure of the root [2]). The product is the divisor, *Fa* 法 (which is put in the third row from the top). [It should be noted that in this square root series, but not in the cube root series, the values of *So-tê* and *Fa* are identical.]

2	Fang
55225 ($2 \times 2 \times 10000$)	dividend
2×10.000	so-tê
10000	chieh-suan

$100^2 a^2$

2	Fang
	dividend
20000	divisor
10000	chieh-suan

step 6 而以除.

This divisor, *Fa*, is used to divide the dividend (and the remainder is put in the second row from the top of the counting-board). [This is to be called the first remainder].

a=2

$55225 - (2 \cdot 100)^2$

	2	
55225	15225	first remainder
20000		divisor, Fa
10000		chieh-suan

$$55225 = 2 \times 20000 + 15225$$

step 7 除已, 倍法爲定法, 其復除, 折法. NOTE!

a) After the division has been made, the divisor, *Fa*, is doubled to form the *Ting-fa*,
b) The *Ting-fa*[1]) is cut short (i.e. moved back by one digit) [and this is the (first) fixed divisor, $Ting\text{-}fa_1$] in preparation for the next division operation.

$4 \cdot 10^3 \cdot b$

	2	
55225	15225	first remainder
20000	4000	divisor, Fa
10000	100	chieh-suan

step 8 而下復置借算步之如初.

Again the counting-rod (which took up its position in step 3) in the bottom row is moved (backward from left to right by one step of two places) as before[1]). [This counting-rod, with its new place-value, is to be called the $Chieh\text{-}suan_1$.]

step 8 而下復置借算步之如初.

Again the counting-rod (which took up its position in step 3) in the bottom row is moved (backward from left to right by one step of two places) as before[1]). [This counting-rod, with its new place-value, is to be called the $Chieh\text{-}suan_1$.]

2	
15225	first remainder
4000	divisor
10000	chieh-sua

→

2	
15225	first remainder
4000	divisor
100	chieh-suan

Second Phase:

step 9[2])

(Again, the second figure of the root is selected through trial and discussion. The discussion aims to find the $Ting\text{-}fa_2$ by the process given in step 10. The product of the $Ting\text{-}fa_2$ multiplied by the second figure of the root under trial must not be greater than the first remainder. The largest figure which does not violate this condition is selected).

step 10 以復議一乘之, 所得副以加定法. The $Chieh\text{-}suan_1$ is multiplied by the second figure of the root[1]). (The product is the $So\text{-}t\hat{e}_2$). The $so\text{-}t\hat{e}_2$ is added to the $Ting\text{-}fa_1$. (The result is called $Ting\text{-}fa_2$, which is put in the third row from the top.)

21	
15225 1×4100	first remainder
4000 + 1×100	divisor
100	chieh-sua

22	
15225 2×4200	first remainder
4000 + 2×100	divisor
100	chieh-sua

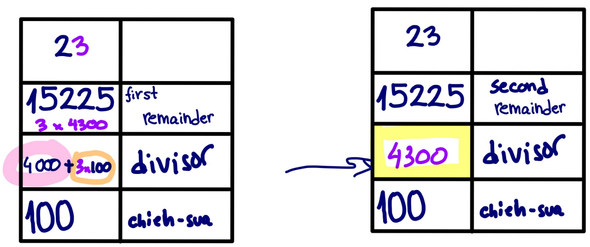
OK →

23	
15225 3×4300	first remainder
4000 + 3×100	divisor
100	chieh-sua

24	
15225 4×4400	first remainder
4000 + 4×100	divisor
100	chieh-sua

step 10 以復議一乘之，所得副以加定法。

The *Chieh-suan*₁ is multiplied by the second figure of the root [1]). (The product is the *So-tê*₂). The *so-tê*₂ is added to the *Ting-fa*₁. (The result is called *Ting-fa*₂, which is put in the third row from the top.)

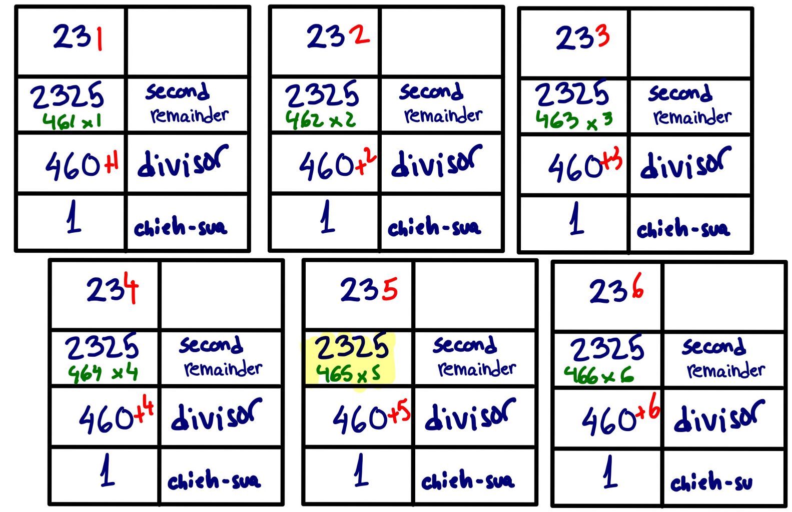


$(b=3)\quad 15225 - 900 - 4\cdot 10^3 \cdot 3$

Third Phase:

Steps 14, 15, and 16.

(will be necessary only if the root comes to three figures; in which case they will follo[w] steps 9, 10, and 11 precisely).

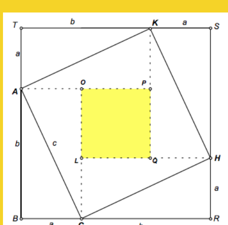


Pythagorean or *Gou-Gu* Theorem

- Let us cut a rectangle (diagonally), and make the width **A** (units) wide, and the length **B** (units) long.
- The diagonal between the (two) corners will then be **C** (units) long.
- Now, after drawing a square on this diagonal, circumscribe it by half-rectangles like that which has been left outside, so as to form a (square) plate.
- Thus the (four) outer half-rectangles, of width **A**, length **B** and diagonal **C**, together make two rectangles (of area **ANSWER 1**); then (when this is subtracted from the square plate of area **ANSWER 2**) the remainder is of area **ANSWER 3**. This (process) is called "piling up the rectangles."

(translation by Needham,1959)

Write down ANSWER 1, 2 and 3 in terms of A, B and C



https://cd1.edb.hkedcity.net/cd/maths/en/ref_res/material/MSS_e/Exemp21.pdf

1st century BC

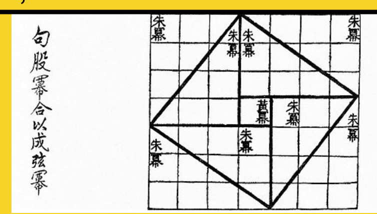


Diagram added by Zhao Shuang to the Zhoubi Suanjing that can be used to prove the Pythagorean Theorem

https://en.wikipedia.org/wiki/Zhoubi_Suanjing#Background_behind_Pythagorean_derivation