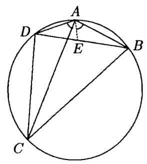


- Apollonius and conic sections
- Ptolemy and his table of chords.
- Archimedes
- Erathostenes and the measurement of the Earth
- Diophantus and Algebra

MAT 336 Hellenic Mathematics After Euclid

Review: What do you remember about Euclid's Elements?

Euclid's Elements. Select one true statement.

A. All the logical consequences of the postulates and common notions in Euclid's Elements are stated and proven in the Elements. Hence, no **new propositions were proven after Euclid.**

B. All the logical consequences of the postulates and common notions in Euclid's Elements were either proven in the Elements or before the fifteenth century. **Since the fifteenth century, no new propositions were proved.**

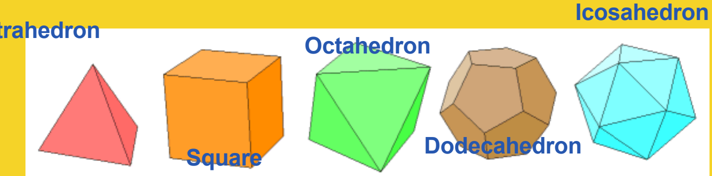
C. There are logical consequences of the postulates and common notions in Euclid's Elements **which not have been proven.**

D. All the above statements are **false**.

Euler Characteristic (Polyhedral Formula) In a polyhedron, the following equation holds: V-E+F=2, where V is the number of vertices, E is the number of edges and F is the number of faces. **Descartes (~1600); Euler (~1700)**

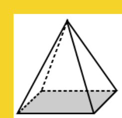
Choose one of the following polyhedra, and compute the number of vertices, edges and faces. Check that the Euler characteristic is two. (State which polyhedron you choose)

Tetrahedron Icosahedron



1

2



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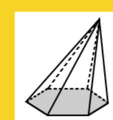
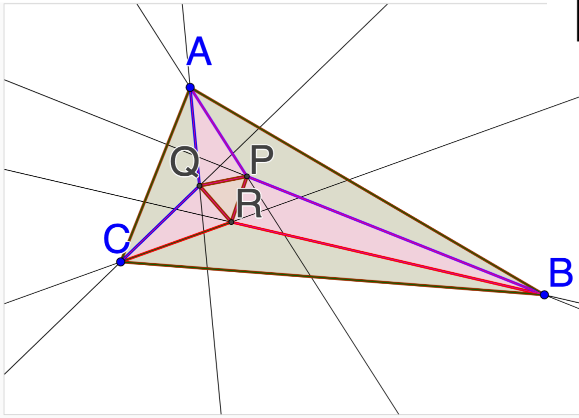
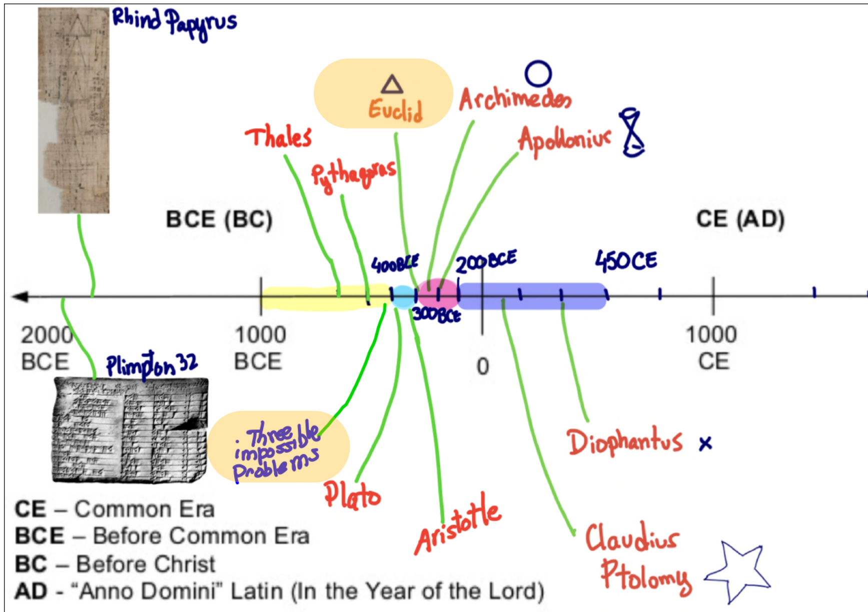


4

Morley's trisector theorem, 1899

https://ggbm.at/FRZ9Nfec

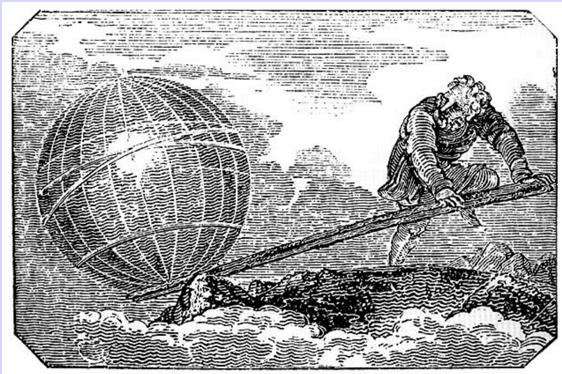




Birds are fed by their parents in their infancy. When the time comes to feed themselves, there can be some confusion when the food does not go into their mouth by itself.
https://twitter.com/fasc1nate/status/1582504199015002112?s=48&t=6rkCZIkmqBgzNYl_WQLd4Q

Archimedes

Archimedes (~250 BCE)

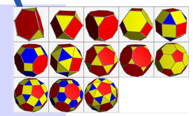


GIVE ME A PLACE TO STAND AND I WILL MOVE THE EARTH

Archimedes (~250BCE)

Derived and computed approximations to:

- Area of the circle
- Surface area and volume of the sphere
- Area of an ellipse.
- Area under a parabola

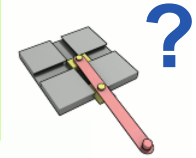


Ideas of calculus

- Infinitesimals
- Method of exhaustion

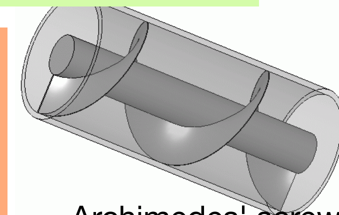
Dealt with infinity!!!

- Approximation of π
- Created a number system to deal with **arbitrary** large numbers



?

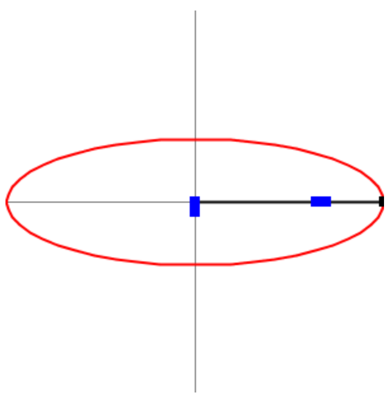
- Applied mathematics to physics
- Explanation of principle of the lever
- Innovative machines



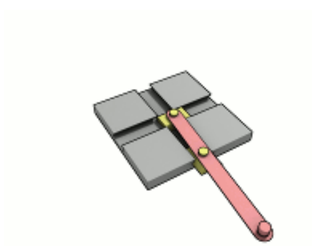
?

Archimedes' screw

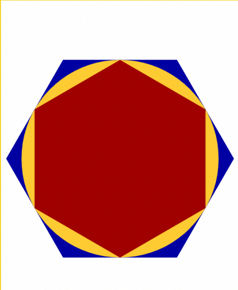
Trammel of Archimedes



Animation of the trammel of Archimedes by Alastair Rae for Wikimedia Commons

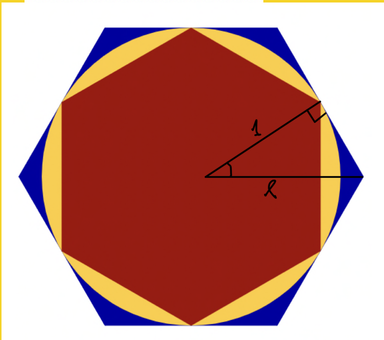


Archimedes and π

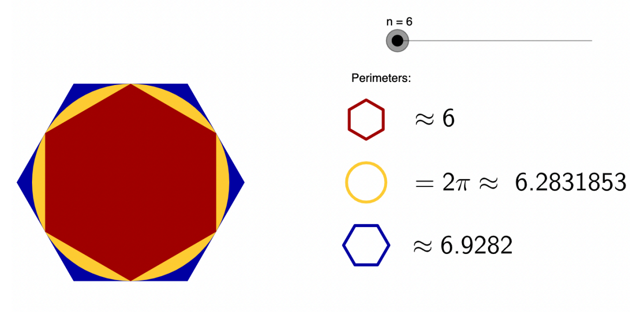


What is the length of the perimeter of the inscribed hexagon?
What is the length of the perimeter of the circumscribed hexagon?

(You can assume the circle has radius 1) If yo have time, make the same computation for the circumscribed hexagon.



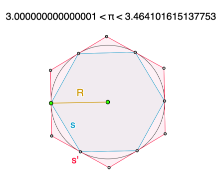
The inscribed hexagon is the red one (its vertices are on the circle)
The circumscribed hexagon is the (blue) one with its six sides tangent to the (yellow) circle.
All hexagons here are regular.

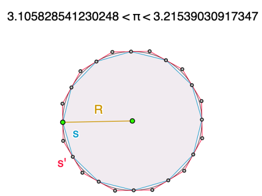


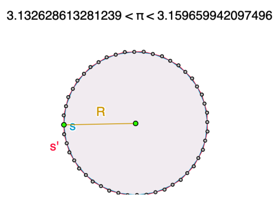
What is the exact length of the perimeter of the circumscribed hexagon?

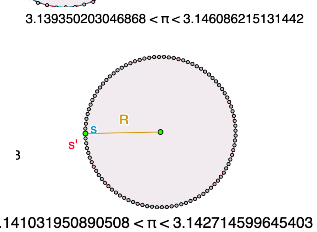
https://www.geogebra.org/m/ajamkye2

Archimedes, 300BCE

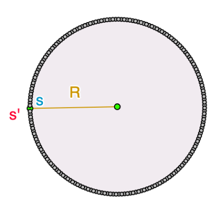




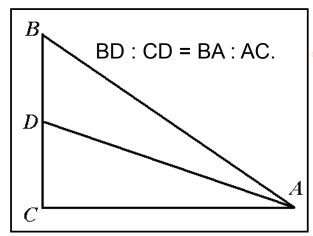


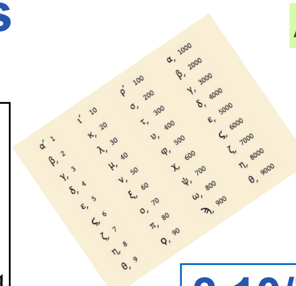


Method of exhaustion



Archimedes (~250BCE)



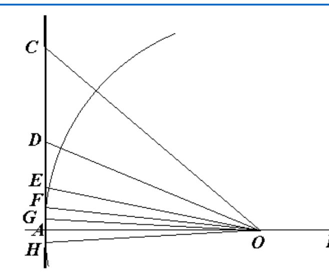


Approximation of π

Computed the perimeter of a regular polygon of 96 sides!!!!!!

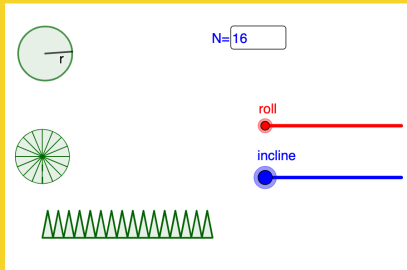
3 10/71< π < 3 1/7

Throughout this proof, Archimedes uses several rational approximations to various square roots. Nowhere does he say how he got those approximations.



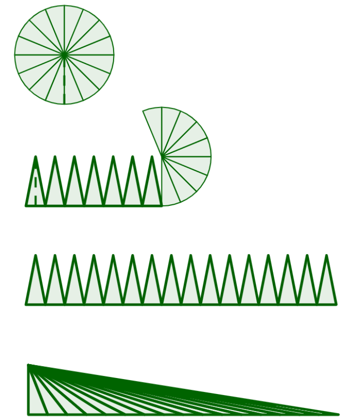
Images and text from: https://itech.fgcu.edu/faculty/clindsey/mhf4404/archimedes/archimedes.html https://www.geogebra.org/m/fg9d4ueb

1. What are the lengths of the legs of the right triangle (In the limit, when N goes to infinity)
2. *The area of a circle of radius R is equal to the area of a right triangle with legs of length A and B.*(Replace A and B by the appropriate words)



Archimedes (~250BCE)

For I realized that just as every circle equals a triangle having as its base the circumference of the circle and altitude equal to the [distance] from the center to the circle [that is, the radius]…



Archimedes and large numbers

Archimedes (~250BCE)

- Created a number system to deal with **arbitrary** large numbers

Archimedes wrote an essay on the **number of grains of sand that would fill a sphere whose diameter was equal to the distance from earth to the fixes stars.**

Since he had to work with very large numbers, he imagined a “doubled class” of numbers of eight numerals (instead of the four the the Greek ciphered system)

1. 1 to 99,999,999
2. 100,000,000 to 10^{16}-1
3. etc

α′	1	ι′	10	ρ′	100	α,	1000
β,	2	κ,	20	σ,	200	β,	2000
γ,	3	λ,	30	τ,	300	γ,	3000
δ,	4	μ,	40	υ,	400	δ,	4000
ε,	5	ν,	50	φ,	500	ε,	5000
ς,	6	ξ,	60	χ,	600	ς,	6000
ζ,	7	ο,	70	ψ,	700	ζ,	7000
η,	8	π,	80	ω,	800	η,	8000
θ,	9	ϙ,	90	ϡ,	900	θ,	9000

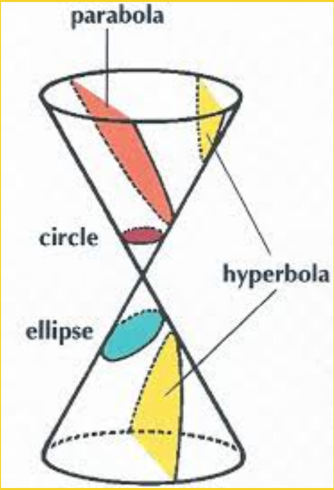
Some of the numbers to which I have given a name [...] surpass not only the number of grains of sand that could fill the Earth [...] but even the number of grains of sand that could fill the universe itself.

Archimedes, Sand-Reckoner

Conic Sections

Conic sections

https://www.geogebra.org/m/pgu4vvtj

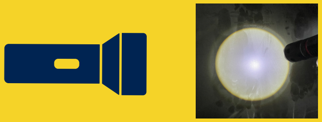


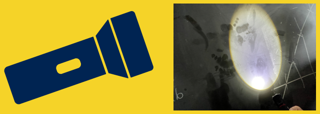
http://jwilson.coe.uga.edu/emt725/Conics%20Images/Conics.html

1. When α is 40 and β is 35 the intersection of the plane and the (double) cone is a hyperbola. Find all the pairs (α, β) with this property, (that is, the intersection of the plane and the cone is a hyperbola)
2. For which values of (α, β) the intersection of the plane and the cone is an ellipse?
3. For which values (α, β) of the intersection of the plane and the cone is a circle?
4. For which values of (α, β) the intersection of the plane and the cone is a parabola?
5. Can you find any other curve as the intersection of the plane and the cone?

Find all the values or as many as you can.

What curves do you see in the demonstration? Explain why we see those curves. (Hint: there should be a cone in the explanation)

1.. 

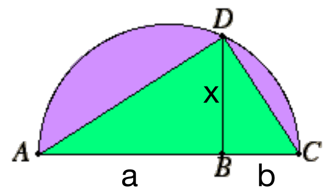
2.. 

3.. 

4.. 

Proposition VI.16 of Euclid's elements

To find a mean proportional to two given straight lines.

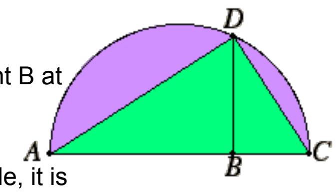


A positive number x is the **mean proportional** of two (positive) numbers a and b if a/x=x/b.

Proposition VI.16 of Euclid's elements

To find a mean proportional to two given straight lines.

- Let AB and BC be the two given straight lines.
- It is required to find a mean proportional to AB and BC.
- Place them in a straight line, and describe the semicircle ADC on AC. Draw BD from the point B at right angles to the straight line AC, and join AD and DC. (I.11)
- Since the angle ADC is an angle in a semicircle, it is right. (III.31)
- Since, in the right-angled triangle ADC, BD has been drawn from the right angle perpendicular to the base, therefore BD is a mean proportional between the segments of the base, AB and BC. (VI.8,Corollary)
- Therefore a mean proportional BD has been found to the two given straight lines AB and BC



A positive number x is the **mean proportional** of two (positive) numbers a and b if a/x=x/b.

Conic sections history, according to tradition

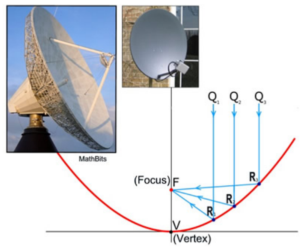
- Hippocrates of Chios (~400BCE) reduced the problem of doubling the cube to the equivalent problem of "two mean proportionals" (In modern terms, these are the solutions x and y of the equations **a/x=x/y and x/y=y/(2b)**, where **a** and **b** are given).
  - Consider two numbers **a** and **b**. The side of a cube of volume **(b/a)** times the volume of a cube of side a is **x**.
- Menaechmus (~350BCE) discovered conic sections while attempting to solve the problem of doubling the cube.
  - (We can "see" some of the equations of the conics from the two mean proportionals.
    1. $x^2 = ay$
    2. $y^2 = bx$
    3. $xy = ab$)

A story about Archimedes' and mirrors

Apollonius of Perga (~200BCE) Studied parabolas. Current applications of properties parabolas

A parabolic dish is a surface with a cross-sectional shape of a parabola -paraboloid- used to direct light or sound waves. This concept is used by

- Radio telescope antennas
- satellite dishes
- parabolic reflector,
- car's headlights and in spotlights.

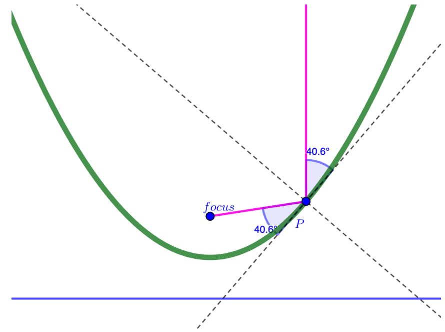


https://mathbitsnotebook.com/Algebra2/Quadratics/QDParabolaApplied.htm

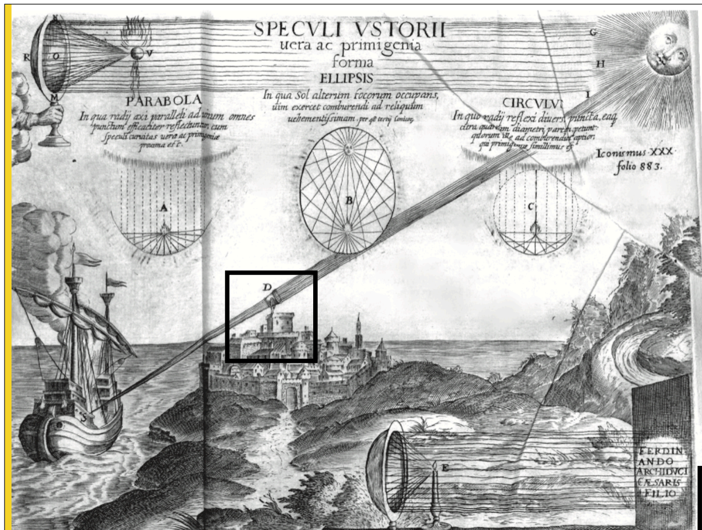
A parabolic mirror

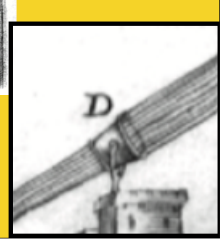
https://www.geogebra.org/m/fkqe4v5m

See also https://www.geogebra.org/m/hSdBYsG9



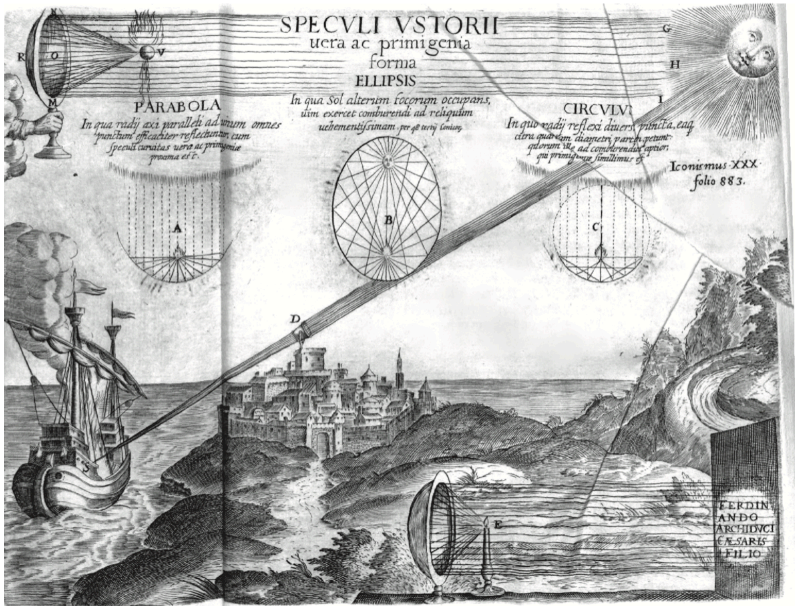
See https://www.geogebra.org/m/jeqqdnkh for spherical mirrors





What do you think is happening in this engraving? (The boat on the left belongs to the enemy)

Archimedes and a mirror



Setting fire to a ship using the sun and a mirror. Engraving from a book by our buddy Athanasius Kircher, 1671. (Wellcome Collection)

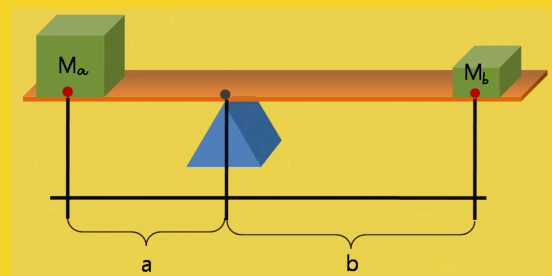
One of the many stories about Archimedes tells that he manage destroyed the Roman fleet at the siege of Syracuse in 213 BC by the application of directed solar radiant heat.

Many studies debunked this…

Archimedes and the Law of the Lever

Archimedes Law of the Lever

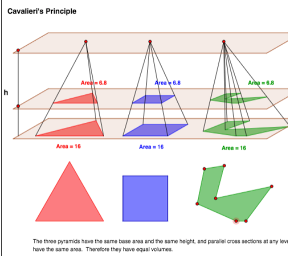
Magnitudes are in equilibrium at distances reciprocally proportional to their weights.



Source: Wikimedia. https://commons.wikimedia.org/wiki/File:Lever_Principle_3D.png

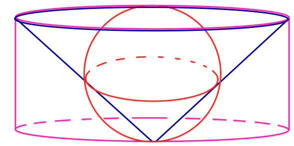
Write down a formula for Archimedes Law of the Lever in terms of the weights M_a and M_b and the distances a and b

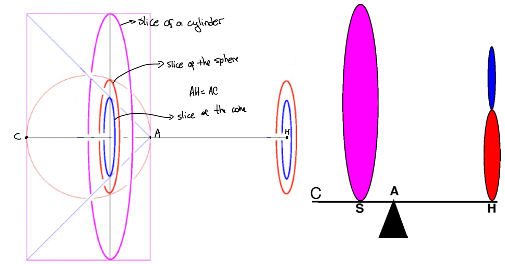
Cavalieri's principle (although was known by Chinese mathematicians before Cavalieri stated it)



Two solids such that the section of one by each horizontal plane bears a fixed ratio to the section of the other by the same plane have volumes in that same ratio.

Vol(Sphere) + Vol(Cone) = (1/2)Vol(Cylinder)





- Applied Cavalieri's principle, imagining the sections of a region balanced about a fulcrum
- If each pair of corresponding sections balance at distances a and b, then the bodies themselves will balance at these distances
- The balancing force of the cylinder is the same if all of its mass is concentrated at its center of mass, which the midpoint of AC.

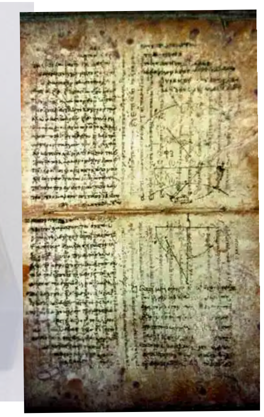
Example of Archimedes' Method

Archimedes Law of the Lever: Magnitudes are in equilibrium at distances reciprocally proportional to their weights.

The Archimedes' Palimpsest

The Archimedes' Palimpsest

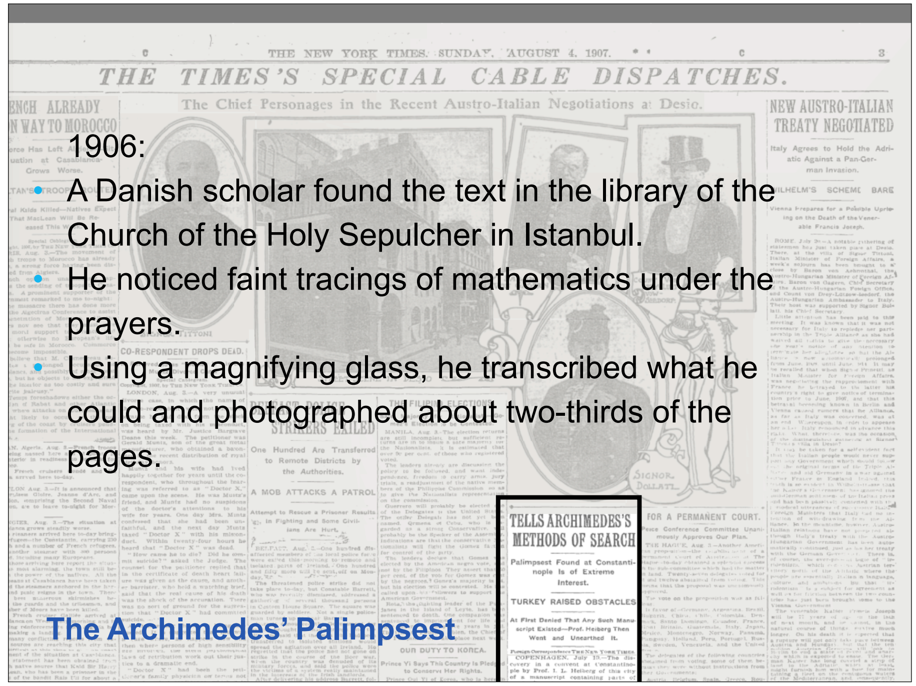
- Several of Archimedes treatises (which had been copied in 10th-century Constantinople) were found on a Byzantine prayer book from the 13th century (written on vellum)
- Discovered in 1906 by the Danish scholar Johan Ludwig Heiberg.
- Disappeared in the 1920s
- The text had been scraped away to make room for the prayer book.
- The book then went missing until it was auctioned – in a much more damaged state – at Christie's in New York in 1998.
- Bought by an anonymous American collector for $2m, it was deposited at Baltimore's Walters Art Museum, where scientists, conservators, classicists and historians have been working on uncovering the secrets of oldest surviving copy of Archimedes' works



http://www.archimedespalimpsest.org/about/

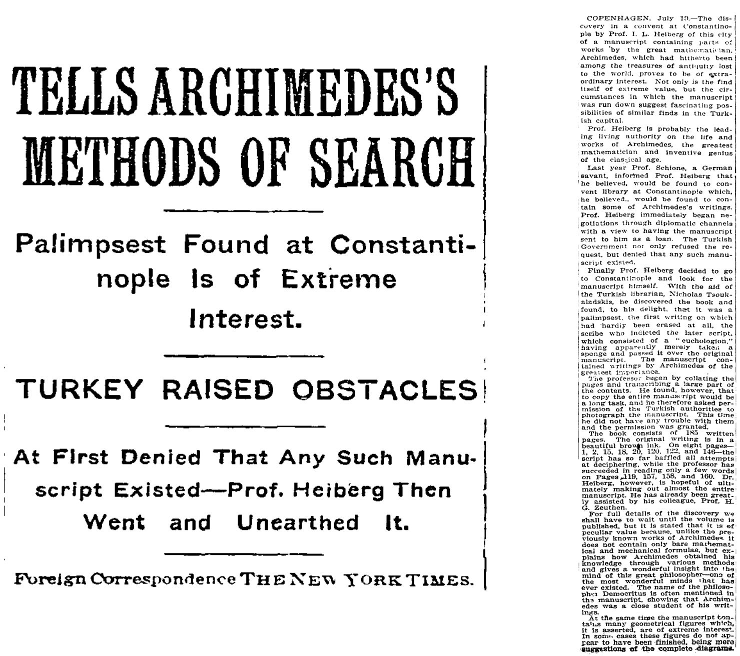
The Archimedes' Palimpsest

- For Archimedes, as for all others from antiquity, we do not have the original works but copies of copies of copies.
- Probably written in the **second half of the tenth century** a scribe made a copy of a few text of Archimedes. Most likely at Constantinople, which was the one place with a continued tradition of copying and preserving ancient texts from antiquity through the Middle Ages.
- In the **13th century** Christian monks, needing vellum for a prayer book, ripped the manuscript apart, washed it, folded its pages in half and covered it with religious text.
- The prayer book ended up in a monastery in Constantinople.
- The prayer book is known as a palimpsest, because it contains text that is written over.



1906:
- A Danish scholar found the text in the library of the Church of the Holy Sepulcher in Istanbul.
- He noticed faint tracings of mathematics under the prayers.
- Using a magnifying glass, he transcribed what he could and photographed about two-thirds of the pages.

The Archimedes' Palimpsest



TELLS ARCHIMEDES'S METHODS OF SEARCH

Palimpsest Found at Constantinople Is of Extreme Interest.

TURKEY RAISED OBSTACLES

At First Denied That Any Such Manuscript Existed—Prof. Heiberg Then Went and Unearthed It.

Foreign Correspondence THE NEW YORK TIMES.

COPENHAGEN, July 19.—The discovery in a convent at Constantinople by Prof. I. L. Heiberg of this city of a manuscript containing parts of works by the great mathematician, Archimedes, which had hitherto been among the treasures of antiquity lost to the world, proves to be of extraordinary interest. Not only is the find itself of extreme value, but the circumstances in which the manuscript was run down suggest fascinating possibilities of similar finds in the Turkish capital.

Prof. Heiberg is probably the leading living authority on the life and works of Archimedes, the greatest mathematician and inventive genius of the classical age.

Last year Prof. Schione, a German savant, informed Prof. Heiberg that he believed, would be found to convent library at Constantinople which, he believed., would be found to contain some of Archimedes's writings. Prof. Heiberg immediately began negotiations through diplomatic channels with a view to having the manuscript sent to him as a loan. The Turkish Government not only refused the request, but denied that any such manuscript existed.

Finally Prof. Heiberg decided to go to Constantinople and look for the manuscript himself. With the aid of the Turkish librarian, Nicholas Tsoukaladskis, he discovered the book and found, to his delight, that it was a palimpsest, the first writing on which had hardly been erased at all, the scribe who indicted the later script, which consisted of a "euchologion," having apparently merely taken a sponge and passed it over the original manuscript. The manuscript contained writings by Archimedes of the greatest importance.

The professor began by collating the pages and transcribing a large part of the contents. He found, however, that to copy the entire manuscript would be a long task, and he therefore asked permission of the Turkish authorities to photograph the manuscript. This time he did not have any trouble with them and the permission was granted.

The book consists of 185 written pages. The original writing is in a beautiful brown ink. On eight pages—1, 2, 15, 18, 20, 120, 122, and 146—the script has so far baffled all attempts at deciphering, while the professor has succeeded in reading only a few words on Pages 119, 157, 158, and 160. Dr. Heiberg, however, is hopeful of ultimately making out almost the entire manuscript. He has already been greatly assisted by his colleague, Prof. H. G. Zeuthen.

For full details of the discovery we shall have to wait until the volume is published, but it is stated that it is of peculiar value because, unlike the previously known works of Archimedes, it does not contain only bare mathematical and mechanical formulae, but explains how Archimedes obtained his knowledge through various methods and gives a wonderful insight into the mind of this great philosopher—one of the most wonderful minds that has ever existed. The name of the philosopher Democritus is often mentioned in the manuscript, showing that Archimedes was a close student of his writings.

At the same time the manuscript contains many geometrical figures which, it is asserted, are of extreme interest. In some cases these figures do not appear to have been finished, being mere suggestions of the complete diagrams.

The Archimedes' Palimpsest

- It reappeared in the 1970's, in the hands of a French family that had bought it in Istanbul in the early 20's and held it for five decades before trying to sell it.
- They had trouble finding a buyer, however, in part because there was some question of whether they legally owned it.
- also, the manuscript looked terrible. It had been ravaged by mold in the years the family kept it, and it was ragged and ugly.
- In 1998, an anonymous billionaire bought it for $2 million and lent it to the Walters Art Museum in Baltimore, where it still resides.

The Stomachion

- With the manuscript in hand, a small group of scholars set out to reconstruct the original Greek text. It was not easy
- Used ultraviolet light, computer imaging, software to to pick out writing from the "noise" around it, old photographs (from the Danish scholar)

The Stomachion

- The Stomachion was far ahead of its time: a treatise on combinatorics, a field that did not come into its own until the rise of computer science.
- Among all of Archimedes' works, the Stomachion has attracted the least attention, ignored or dismissed as unimportant or unintelligible.
- Only a tiny fragment of the introduction survived, and it seemed to be about an ancient children's puzzle -- also known as the Stomachion.
- Archimedes was trying to see how many ways the 14 irregular strips could be put together to make a square.
- The answer -- 17,152 -- required a careful and systematic counting of all possibilities. '
- How many ways can you put the pieces together to make a square?

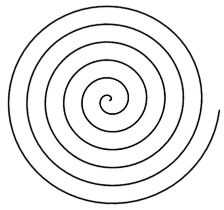
Archimedes' Spiral

https://www.geogebra.org/m/rVx87NXP

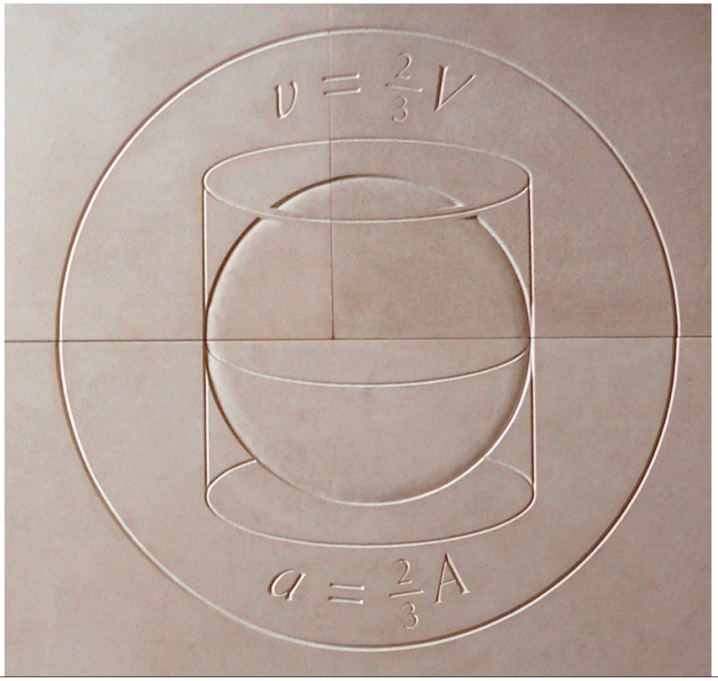
Archimedean spiral

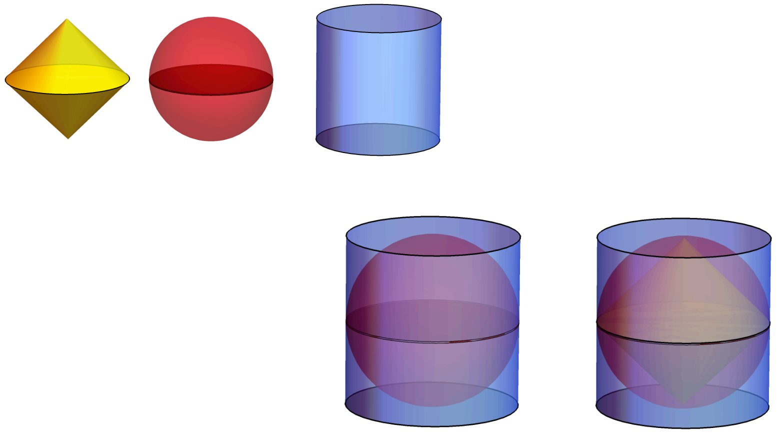
If a straight line drawn in a plane revolve at a uniform rate about one extremity which remains fixed and return to the position from which it started, and if, at the same time as the line revolves, a point move at a uniform rate along the straight line beginning from the extremity which remains fixed, the point will describe a spiral (ελιξ) in the plane.

(ελιξ) Helix



Archimedes' favorite result



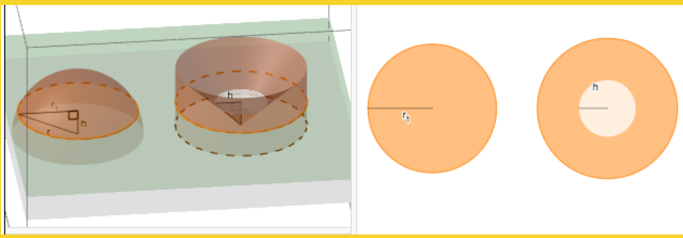


1. By intersecting horizontal "green" plane at height h with a sphere of radius r we obtain a circle. Write down the area of this circle in terms of with h and r.
2. By intersecting horizontal "green" plane at height h with a cylinder with a cone removed we get an annulus (that is, a ring) Write down the area of this annulus in terms of with h and r.
3. What is the relation between S and C?

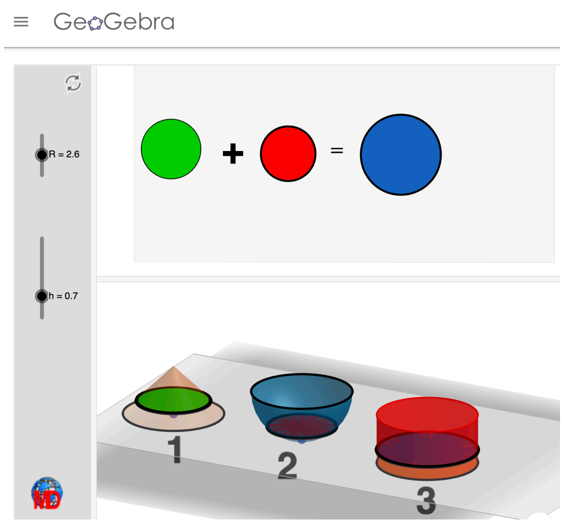
C= Volume of cylinder of height r and base a circle of radius r.
S=Volume of a sphere of radius r
0= Volume of a cone of height r and base a circle of radius r.
Recall that 0=C/3

Archimedes (~250BCE)

https://www.geogebra.org/m/dnv3fph3



Archimedes Computation of the Volume of the Sphere



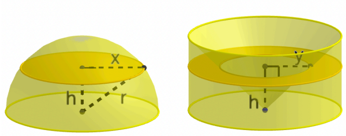
And although he made many excellent discoveries, he is said to have asked his kinsmen and friends to place over the grave where he should be buried a cylinder enclosing a sphere, with an inscription giving the proportion by which the containing solid exceeds the contained.

Plutarch (AD 45-120), Parallel Lives: Marcellus

A tomb in Syracuse in the Necropolis of Grotticelli referred to affectionately (or deceptively) as "Archimedes' Tomb", but known to be of Roman origin dating at least two centuries after the death of Archimedes. https://www.math.nyu.edu/~crorres/Archimedes/Tomb/TombIllus.html

https://www.geogebra.org/material/edit/id/b95bd3m8

C= Volume of cylinder of height r and base a circle of radius r.
S=Volume of a sphere of radius r
0= Volume of a cone of height r and base a circle of radius r.
Recall that 0=C/3

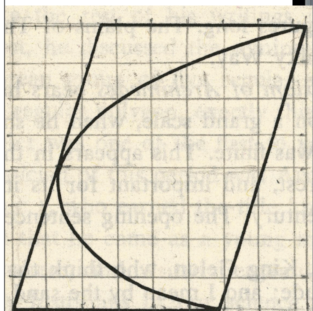


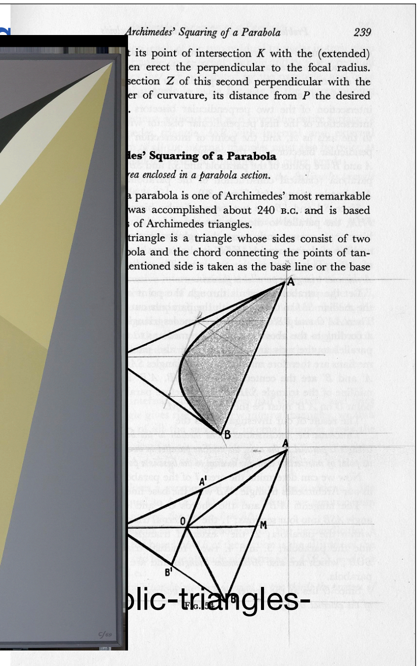
Archimedes (~250BCE)

Archimedes and the area under the parabola

- Bullet

Heading





Archimedes' Squaring of a Parabola 239

t its point of intersection *K* with the (extended)
en erect the perpendicular to the focal radius.
section *Z* of this second perpendicular with the
er of curvature, its distance from *P* the desired

des' Squaring of a Parabola

rea enclosed in a parabola section.

a parabola is one of Archimedes' most remarkable
was accomplished about 240 B.C. and is based
of Archimedes triangles.
triangle is a triangle whose sides consist of two
bola and the chord connecting the points of tan-
entioned side is taken as the base line or the base

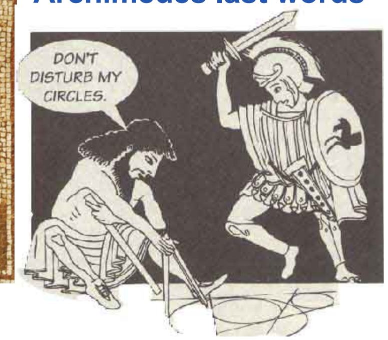
https://www.si.edu/ ... plic-triangles-archimedes:nmah_...

Stories about Archimedes

Eureka!

Death of Archimedes (Mosaic, Municipal Gallery Francfort)

Archimedes last words



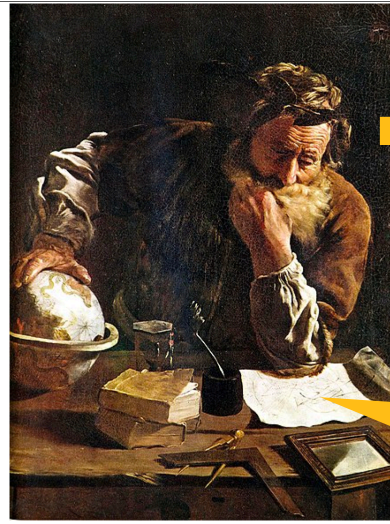
Cassus: "*The head and not the lines*"

Georgios Pachymeris: "*heat me (my head) do not destroy the lines* "(drawing)

Tzetzes: "*Stay away man from my diagram*"

Unknown: "*Don't disturb my circles*"

Eratostenes, the librarian of Alexandria (~200 BCE)



Archimedes Thoughtful (also known as Portrait of a Scholar) by Domenico Fetti, 1620

By Bernardo Strozzi - Musée des beaux-arts de Montréal, Public Domain, https://commons.wikimedia.org/w/index.php?curid=42582581

"The proofs then of these theorems I have written in this book and now send to you (that is, Eratosthenes)"

A well at Kom Ombo, 50 km north of Syene

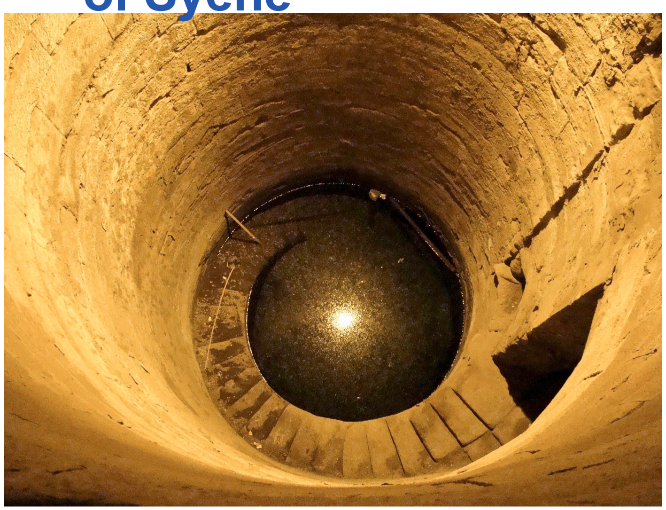
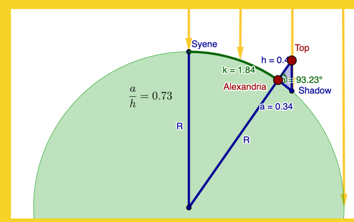
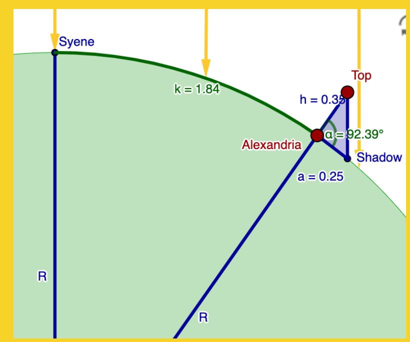


Image credit: https://medium.com/roaming-physicist/looking-down-a-well-in-egypt-to-measure-the-size-of-the-earth-5e12f2df0b2

Eratosthenes on the circumference of the earth

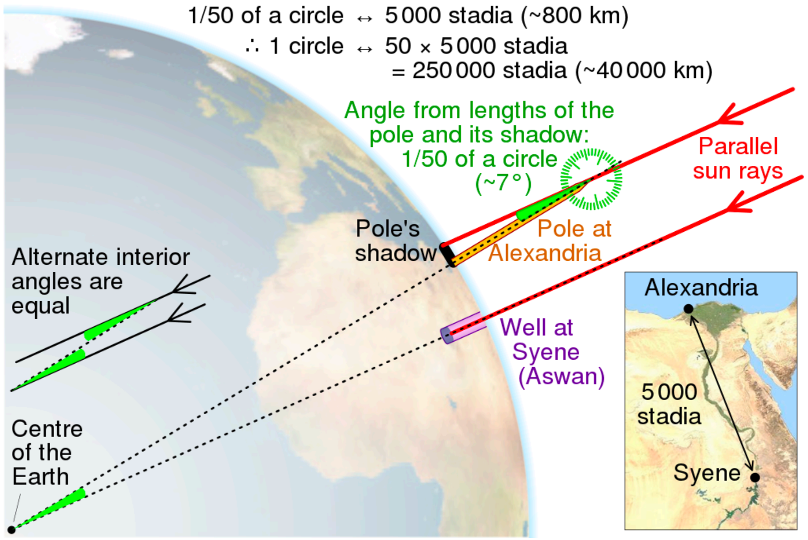




Discussion in groups.
Questions in Geogebra.
Answer individually in Wooclap.

https://www.geogebra.org/m/ty5ych75

Illustration of the method Eratosthenes used to calculate the circumference of the Earth



Apollonius (~200 BCE)

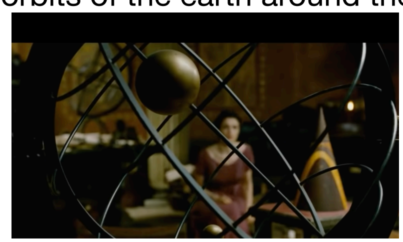
Apollonius of Perga (~200BCE)

- Alexandria
- worked out more fully and generally than in the writings of others
- studied and later taught under the followers of Euclid

Conics - 8 books

*“**the most and prettiest of these theorems are new,** and it was their discovery which made me aware that Euclid did not work out the syntheses of the locus with respect to three and four lines, but only a chance portion of it, and that not successfully; for it was not possible for the said synthesis to be completed without the aid of the additional theorems discovered by me.”*

Hypatia (~400 CE) was a philosopher, astronomer, and mathematician who lived in Alexandria.
In the clip below from movie Agora (a fictionalized version of her life), she stars by discussing the assumption that the orbits of the earth around the sun area circles.



Hypatia arrives to the conclusion that the orbits follow another curve and not a circle. What is that curve? What is the property of that curve that she describes?

https://youtu.be/bUUB1u8xJA4

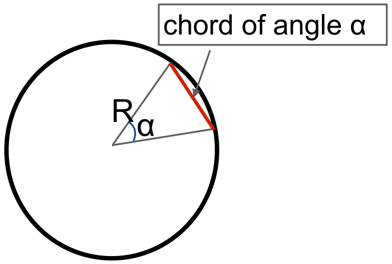
What can I do to make the lectures more interesting? Is there any special topic you would like to be covered? Maybe something closer to the present? Any other suggestion?

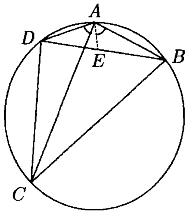
Claudius Ptolemy (100 – 170 CE)

Claudius Ptolemy ~100AD

Mathematical Collection (Almagest) - Mathematical Astronomy

- Astronomer and physicist (hence, mathematician of his era)
- Trigonometry - Table of chords
  - Solved plane triangles
  - Solved spherical triangles





Claudius Ptolemy ~100AD

I know that I am mortal and the creature of a day; but when I search out the massed wheeling circles of the stars, my feet no longer touch the Earth, but, side by side with Zeus himself, I take my fill of ambrosia, the food of the gods.

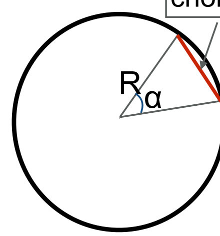
(Ptolemy's Almagest)

Claudius Ptolemy Table of Chords

The idea of ***angles*** seems to have appeared in Greece between 6th and 5th century BCE.

Trigonometry is the branch of mathematics that studies relationships between side lengths and angles of triangles.

Ptolemy made a table of correspondence between angles and their chords. (This is the earliest form of trigonometry we know of)



Ptolemy's treatise on astronomy, written around 150 CE, explains how to construct the table.

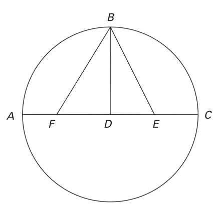
Claudius Ptolemy ~100AD

- Worked with a circle of radius 60 (because he did calculations in base 60) **Question: Why base 60?**
- Computed chord(36°). How?

Main idea: computed chord tables of certain angles using

- propositions from Euclid's elements
- square roots (or approximations)

In the Almagest, Ptolemy announces that he is going to employ in general the base 60 system, in order to avoid "**the embarrassment of the fractions"**



Example: Suppose that E bisects DC

Find F so EF=EB

Then (prove it!) DF is the side of a decagon inscribed on a circle with radius DC.

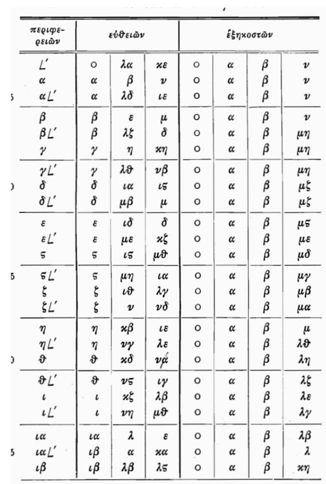
Ptolemy computed DF=chord(36°):

$DF^2=(EF-ED)^2=(EB-ED)^2=((BD^2+ED^2)^{1/2} - ED)^2=$

$((60^2+30^2)^{1/2} - 30)^2=(37;4;55)_{60}$

Claudius Ptolemy ~100AD

- regular dodecagon -> chord(30°)
- regular decagon-> chord(36°)
- chord(36°) and chord(30°) ->chord(6°)
- Bisecting chord(3°)->chord(1° 30')
- Upper and lower values of the chord(1°)
- Finally, a table of 360 entires, with chords of at 30f' increments up to 180°.

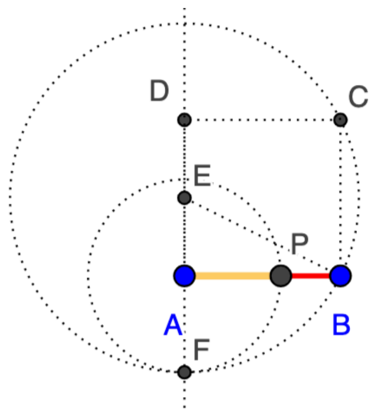


περιφερειῶν	εὐθειῶν			ἑξηκοστῶν			
∠′	ο	λα	κε	ο	α	β	ν
α	α	β	ν	ο	α	β	ν
α∠′	α	λδ	ιε	ο	α	β	ν
β	β	ε	μ	ο	α	β	ν
β∠′	β	λζ	δ	ο	α	β	μη
γ	γ	η	κη	ο	α	β	μη
γ∠′	γ	λθ	νβ	ο	α	β	μη
δ	δ	ια	ις	ο	α	β	μζ
δ∠′	δ	μβ	μ	ο	α	β	μζ
ε	ε	ιδ	δ	ο	α	β	μς
ε∠′	ε	με	κζ	ο	α	β	με
ς	ς	ις	μθ	ο	α	β	μδ
ς∠′	ς	μη	ια	ο	α	β	μγ
ζ	ζ	ιθ	λγ	ο	α	β	μβ
ζ∠′	ζ	ν	νδ	ο	α	β	μα
η	η	κβ	ιε	ο	α	β	μ
η∠′	η	νγ	λε	ο	α	β	λθ
θ	θ	κδ	νά	ο	α	β	λη
θ∠′	θ	νς	ιγ	ο	α	β	λζ
ι	ι	κζ	λβ	ο	α	β	λε
ι∠′	ι	νη	μθ	ο	α	β	λγ
ια	ια	λ	ε	ο	α	β	λβ
ια∠′	ιβ	α	κα	ο	α	β	λ
ιβ	ιβ	λβ	λς	ο	α	β	κη

Beginning of Ptolemy table of chords

https://www.wilbourhall.org/pdfs/HeibergAlmagestComplete.pdf

Euclidean Construction of the Golden Ratio



Two positive numbers a and b are in the ***golden ratio*** if

$(a+b)/a = a/b$ and $a>b$

or

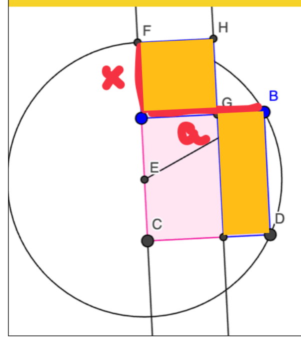
$(a+b)/b = b/a$ and $b>a$

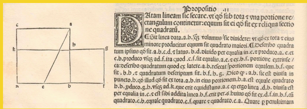
https://www.geogebra.org/m/fyassfgv

proof http://www.u.arizona.edu/~sreyes/archived/GoldenSection.htm

A straight line (segment AB) is given. Set a=AB and AH=x. Write down an equation relating a and x (Hint: x is the point such that the two yellow areas are equal.)

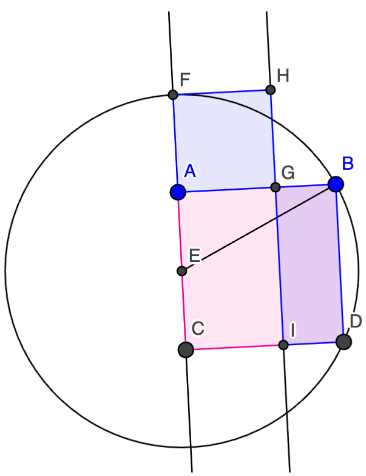
Proposition II.11: To cut a given straight line so that the rectangle contained by the whole and one of the segments equals [the area of] the square on the remaining segment.





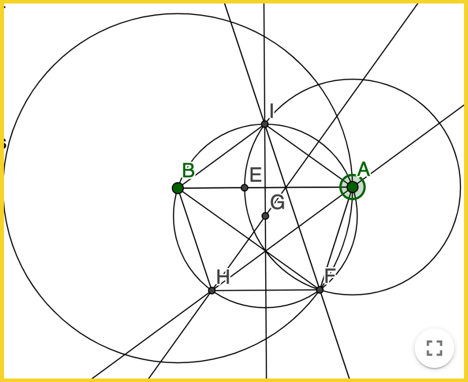
Extract from Euclid's "Elements" (1482 edition)

Golden ratio (although this name came much later)



- Used in the construction of regular pentagon and decagon in **Euclid's** Elements.
- **Hypsicles**, (~150 BCE) used golden ratio to construct regular polyhedra inscribed on spheres.
- **Heron** (50 CE)approximate values for the ratio of the area of the pentagon to the area of the square of one side.
- **Ptolemy** (100 CE) side of a regular pentagon in terms of the radius of the circumscribed circle.
- **Luca Pacioli** (1500 CE) gave the name, *Divina proportione*
- **Cardano, Bombelli, Kepler**
- Relation to Fibonacci numbers!

Construction of the regular pentagon using the Golden ratio

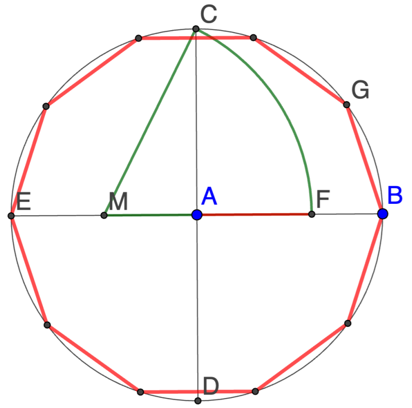


Give a definition of Golden triangle (Hint: Recall the equation $x^2=x(a-x)$)

https://www.geogebra.org/m/xjtepsde

https://www.geogebra.org/m/pCwXXjbj

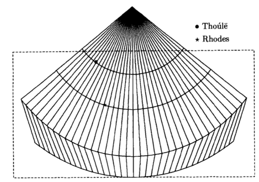
Construction of Regular Decagon



See also https://www.geogebra.org/m/zrmskt9k

Claudius Ptolemy ~100AD

Ptolemy's world map, reconstituted from Ptolemy's Geography (circa 150) in the 15th century, indicating "Sinae" (China) at the extreme right, beyond the island of "Taprobane" (Ceylon or Sri Lanka, oversized) and the "Aurea Chersonesus" (Southeast Asian peninsula).



Ptolemy method of making (The History of MathematicsA Brief Course - Roger Cooke · 2011)

One of the first scholars to look at the problem of representing large portions of the Earth's surface on a flat map.

Diophantus (~200 CE) and The birth of literal algebra

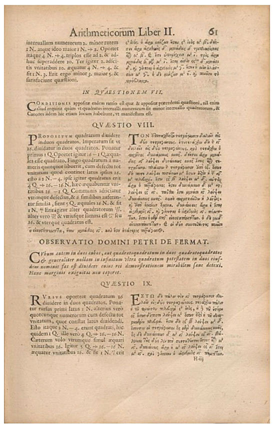
Diophantus - (possibly ~250AD) -Alexandria

Arithmetica

- Major advance in the solution of equation: Introduction to symbolism.
  - Linear and quadratic equations
  - Higher degree equations
  - The method of false position

$$K^{\Upsilon}\alpha\varsigma\gamma\pitchfork\Delta^{\Upsilon}\gamma\mathring{M}\alpha$$

$$x^3 - 3x^2 + 3x - 1$$


Arithmeticorum Liber II.

QVÆSTIO VIII.

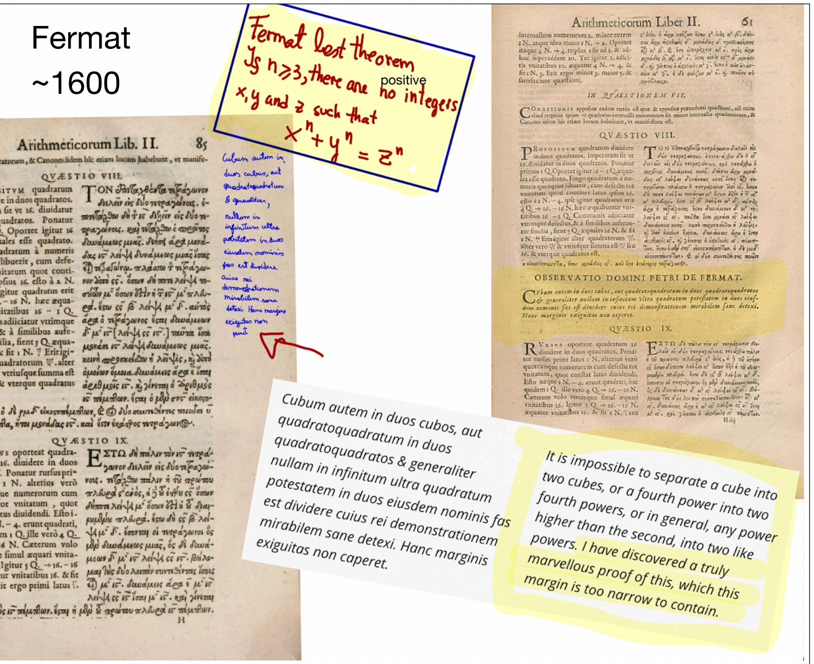
OBSERVATIO DOMINI PETRI DE FERMAT.

QVÆSTIO IX.

Problem II.8 in the Arithmetica (edition of 1670), annotated with Fermat's comment which became Fermat's Last Theorem. Wikipedia

Fermat
~1600

Fermat last theorem
Is $n \geq 3$, there are no positive integers x, y and z such that
$x^n + y^n = z^n$



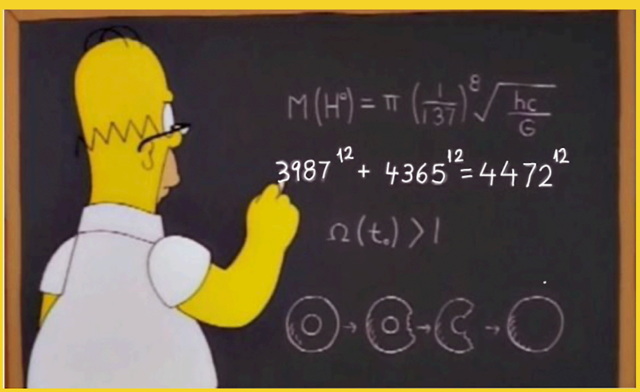
Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos & generaliter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est dividere cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.

It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. I have discovered a truly marvellous proof of this, which this margin is too narrow to contain.

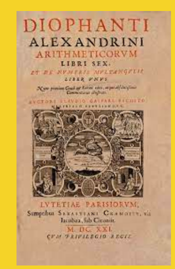
Fermat last theorem
Is $n \geq 3$, there are no positive integers x, y and z such that
$x^n + y^n = z^n$

Andrew Wiles
presented a proof in 1993 - 1994

Use a gadget (phone, computer..) that can only use numbers of 10 digits to compute first $3987^{12}+4365^{12}$ and second, 4472^{12}. Have we (jointly with Homer Simpson) disproved Fermat Last Theorem? Why or why not?



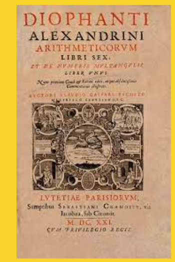
Solve the following problem by Diophantus. To split a given number (100) in two parts having a given difference (40).



Solve the following problem by Diophantus. To split a given number (100) in two parts having a given difference (40).

In fact, let the given arithmos be 100, and the excess M̊ 40. To find the arithmoi. Let the smaller be assigned, ϛ 1. Therefore, the larger will be ϛ 1 M̊ 40. Therefore, together they become ϛ 2 M̊ 40. But M̊ 100 are given. Therefore, M̊ 100 are equal to ϛ 2 M̊ 40. And from similars are similars. I subtract from 100, M̊ 40 [and from the 2 arithmoi and of 40 units similarly 40 units]. Remainders ϛ 2 equal M̊ 60. Therefore each ϛ becomes M̊ 30. For the actualities. The smaller will be M̊ 30 and the larger M̊ 70, and the demonstration is obvious.

To split a given number (60) in two parts having a given ratio (3:1).



To split a given number (60) in two parts having a given ratio (3:1).

2. To divide the prescribed arithmos into two arithmoi in a ratio that's given.

In fact, let it be prescribed to divide 60 into two arithmoi in a ratio 3-times. Let the smaller be assigned ϛ 1. Therefore, the larger will be ϛ 3, and the larger is triple the smaller. It is required the two be equal to M̊ 60. But the two added are ϛ 4. Therefore, ϛ 4 are equal to M̊ 60. Therefore, ϛ is M̊ 15. Therefore, the smaller will be M̊ 15 and the larger M̊ 45.

To find two numbers in a given ratio and such that their difference is also given.

Given ratio 5 : 1, given difference 20.

- Illustration by Rubens for "Opticorum libri sex philosophis juxta ac mathematicis utiles", by François d'Aguilon. It demonstrates the principle of a general perspective projection, of which the stereographic projection is a special case. Scan of a work published originally in 1613

Peter Paul Rubens - Opticorum libri sex philosophis juxta ac mathematicis utiles