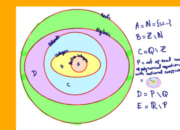
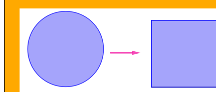
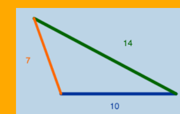
Advices for the paper

- Read the **rubric** (link in the syllabus)
- Note the importance of **citations and bibliography.**
- **Start writing the paper ASAP.** Today is a great day to start.
- Make sure you **understand** what you are writing about.
- Discuss your paper in the **Writing Center**
- The goal of the paper assignment is help you exercise independent reflection based on recognized scientific sources.
- Make sure that you **explain** clearly **any non-standard word** that you introduce.

MAT 336 Hellenic Mathematics before Euclid.







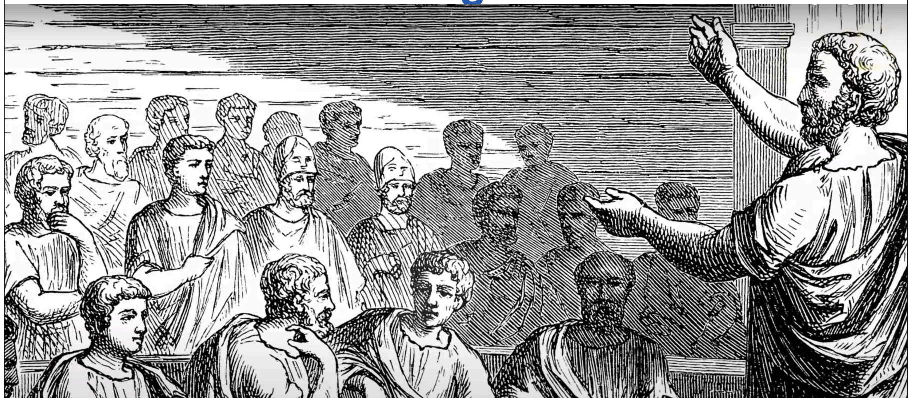
- Overview and timeline .
- The School of Athens
- Impossible problems of Antiquity.
- Thales
- Pythagoras, the Pythagoreans and Pythagoras' theorem
- Zeno's paradoxes
- Hippocrates
- Plato
- Aristotle

Introduction

Ancient Greece 101 I National Geographic, https://youtu.be/6bDrYTXQLu8

What was the form of government in Athens?

Sound arguments



The Greek city-states fostered an environment of intellectual competition and debate. In the agora (public square), scholars engaged in **discussions and debates**, which encouraged the refinement of ideas and the need for rigorous argumentation.

Greek or Hellenic Mathematics (Between ~600 BCE and 300 AD)

- No tablets or papyri or …
- Only copies of copies of copies of…
- Restored versions of texts of Euclid, Archimedes and Apollonius among others.
- To study how this mathematics was developed we rely on these copies and remarks of writers of the time.

Greek or Hellenic Mathematics (Between ~600 BCE and 300 AD)

- In mathematics, the question
  "**why?**"
- was now asked together with the older question
  "**how?**"

Greek or Hellenic Mathematics Two important aspects

1. Emphasis on proofs
2. Involvement with specific, challenging problems.
   - need to decide which new arguments counted as genuine proof
   - in this way, the problems led to
     - new discoveries
     - increasingly sophisticated ideas about proofs

"Recent studies of Babylonian sources have shown that we must revise former estimates of the extent to which the Greeks were indebted for the details of astronomy to the Babylonians. This debt proves to have been much greater than they have been imagined and further research may prove to be greater still" **Sir Thomas Heath,1932.**

"It is gradually beginning to be realized that many of the achievements of Greek culture in the fields of astronomy and mathematics did not spring, fully armed, from the Hellenic brain, but had their more remote origins in the civilizations of the ancient east."
Professor Filon.

(…) There is available now sufficient evidence to show that a great deal of the astronomical knowledge which has come down to us from the Hellenistic period (c. 500 B.C. to A.D. 150) was not initially discovered during that period; and such new empiric discoveries as were made in that time were not all due to Greeks, for important contributions were still being made by Babylonians during the Seleucid Era.

The Greeks founded a 'school' of theoretical astronomy and, with their highly developed mathematics, were able to go far with it; but their source-material was in very many cases not Greek.

"We may assume that whatever the Greeks take from the barbarians, they bring it to a finer perfection." Plato

Express, in a sentence of two, the content of the first two paragraphs.

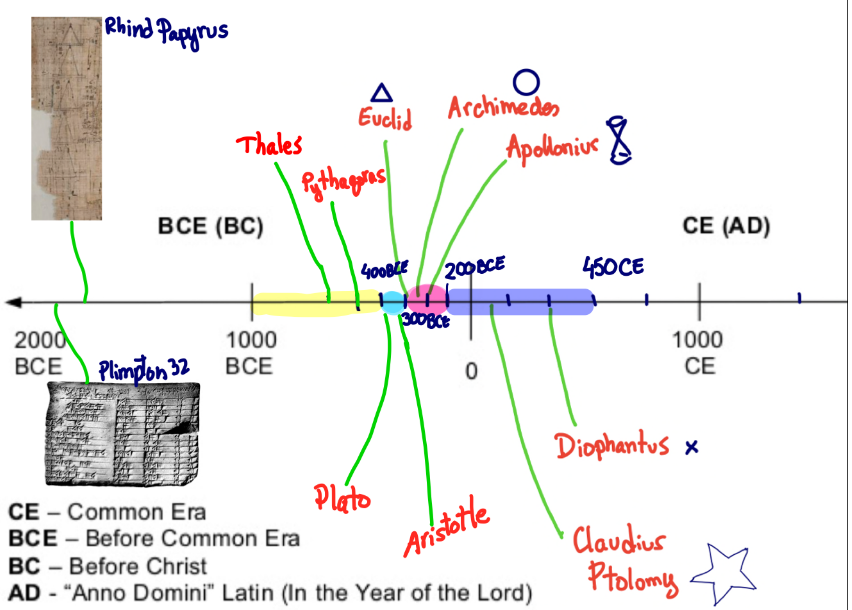
Adapted from GREEK ASTRONOMY AND ITS DEBT TO THE BABYLONIANS, LEONARD W. CLARKE The British Journal for the History of Science, Jun., 1962, Vol. 1, No. 1 (Jun.,1962), pp 65-77

Mesopotamian Influence

We possess a great number of texts from all periods which contain lists of reciprocals, square and cubic roots, multiplication tables, etc., but these tables rarely go beyond two sexagesimal places (i.e., beyond 3600). **A reverse influence of astronomy on mathematics can be seen in the fact that tables needed for especially extensive numerical computations** come from the Seleucid period; tables of reciprocals are preserved with seven places (corresponding to eleven decimal places) for the entry and up to seventeen places (corresponding to twenty-nine decimal places) for the result. It is clear that numerical computations of such dimensions are needed only in astronomical problems.

The superiority of Babylonian numerical methods has left traces still visible in modern times. The division of the circle into 360 degrees and the division of the hour into 60 minutes and 3600 seconds reflect the unbroken use of the sexagesimal system in their computations by medieval and ancient astronomers. But though the base 60 is the most conspicuous feature of the Babylonian number system, this was by no means essential for its success. The great number of divisors of 60 is certainly very useful in practice, but the **real advantage of its use in the mathematical and astronomical texts lies in the place-value notation**, which is consistently employed in all scientific computations. This gave the Babylonian number system the same advantage over all other ancient systems as our modern place-value notation holds over the Roman numerals. The importance this invention can well be compared with that of the alphabet. Just eliminates as the alphabet eliminates the concept of writing as an art to be acquired only after long years of training, notation eliminates mere computation as complex art in itself.

THE HISTORY OF ANCIENT ASTRONOMY : PROBLEMS AND METHODS
Author(s): O. Neugebauer
Source: *Publications of the Astronomical Society of the Pacific*, February 1946, Vol. 58, No. 340 (February 1946), pp. 17-43
Published by: Astronomical Society of the Pacific



60: Behind Every Second, Millenniums of History

Share full article

By Roni Jacobson
July 8, 2013

The number of seconds in a minute — and minutes in an hour — comes from the base-60 numeral system of ancient Mesopotamia. Developed around 3100 B.C., the sexagesimal system, as it is known, has fallen out of favor but is still used (with slight adjustments) to measure time and angles.

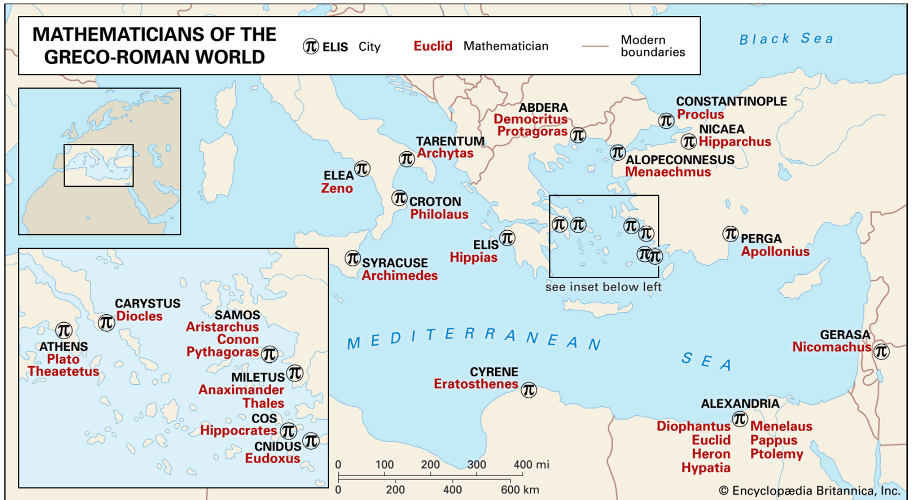
Most modern societies use the base-10 system (also called decimal) of Hindu-Arabic numerals. That system probably comes from counting on our two five-fingered hands, but because it has a limited number of divisors, 10 is actually an inefficient number on which to build a numeral system. Sixty, on the other hand, is eminently divisible — in mathematics, it is considered a "highly composite number" because it has more divisors than any smaller positive integer.

Georges Ifrah, a 20th-century French mathematician, proposed that the sexagesimal system grew out of an alternative method of counting known as the duodecimal system, common throughout Asia. Instead of counting the five digits on each hand, the thumb is used as a pointer, touching each of the four fingers on the right hand, beginning with the pinkie. When the count reaches 12, a digit on the left hand is lowered to mark the place — making "60" when all five digits are balled into a fist.

The Babylonians adopted the base-60 system from the Sumerians. In Babylonian astronomy, a year is 360 days, which is divided into 12 months of 30 days each. By 2000 B.C. the base-60 system had largely disappeared from common use, but it survives in our measures of months, days, hours, minutes and seconds, so called because they are the second division of 60 from the hour. Another vestige of Babylonian mathematics endures in the 360-degree circle.

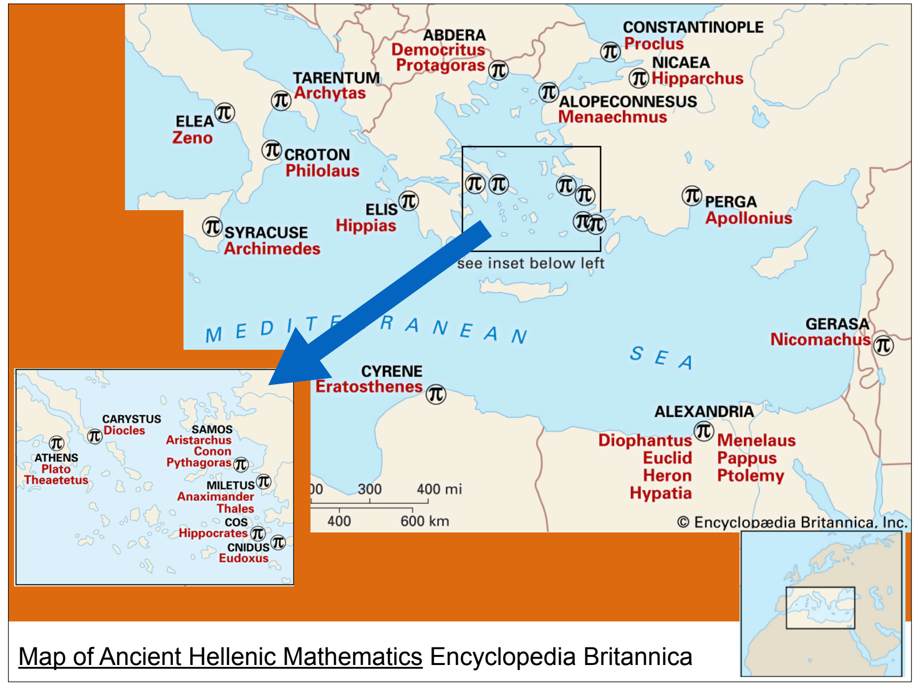
Groups like the Dozenal Society of America advocate converting to numeral systems based on divisors of 60 because of their comparative ease with fractional computations. Champions of 12 refuse to give in to the hegemony of 10, calling themselves "dozenalists" rather than "duodecimalists," — but even they don't expect the conversion any time soon.

A version of this article appears in print on July 9, 2013, Section D, Page 2 of the New York edition with the headline: Take a Number. Order Reprints | Today's Paper | Subscribe



[Map of Ancient Hellenic Mathematics](#)

Encyclopedia Britannica



[Map of Ancient Hellenic Mathematics](#) Encyclopedia Britannica

[Mathigon timeline - Hellenic](#)

Some of the characters in this painting (The school of Athens) are doing something mathematical. Write down their names. If you have time, write down also the name of mathematicians or mathematics-related characters in the painting who are not doing mathematics. (Google is allowed and encouraged for this question)

[The school of Athens(~1500), by Raffaele](#)

https://commons.wikimedia.org/wiki/File:%22The_School_of_Athens%22_by_Raffaello_Sanzio_da_Urbino.jpg

The school of Athens, by Raffaele

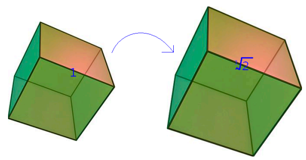
The school of Athens, by Raffaele

Impossible problems of the Antiquity

If a cube C has side length 1 inch, what is the length of the cube whose volume is the double than that of C.

Three "impossible" problems

Doubling a cube: Given a cube, **construct** another cube with twice its volume.



"Construct" means "construct using only straightedge and compass

These problems originated around 400 BCE

What reason gave Plato to the Delians that for the oracle of the god?

In his work entitled *Platonicus* Eratosthenes says that, when the god announced to the Delians by oracle that to get rid of a plague they must construct an altar double of the existing one, their craftsmen fell into great perplexity in trying to find how a solid could be made double of another solid, and they went to ask Plato about it. He told them that the god had given this oracle, not because he wanted an altar of double the size, but because he wished, in setting this task before them, to reproach the Greeks for their neglect of mathematics and their contempt for geometry.
Theon of Smyrna

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Theon of Smyrna

knew Plato and tutored Alexander the Great, is thought to have discovered conic sections (often called simply "conics"). He found them while trying to solve the famous Delian problem of "doubling the cube." According to legend, terrified citizens of the Greek island of Delos were reassured by an oracle that plague would depart only after they had doubled the size of Apollo's cubical altar. Assuming that the altar had unit volume, the task of doubling it amounted to constructing a new edge of length precisely equal to the cube root of 2. Although the legend is doubtful, the Delian problem was certainly studied in Plato's Academy. Plato insisted on an exact solution accomplished using only ruler and compass.

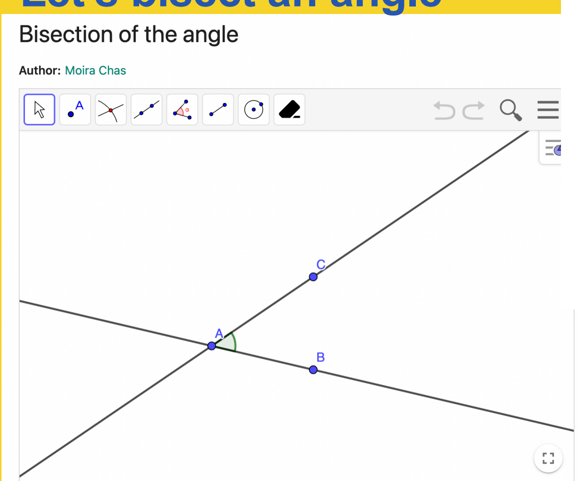
Recall annotated bibliography

Early Greek mathematics

Early Greek mathematics was not one but many*; there were* ***various levels of practice,*** *from calculations on the abacus to indirect proofs concerning incommensurable lines, and varying attitudes, from laughing off attempts to square the circle to using attempts to square the circle as examples in a second-order discussion about the nature of demonstration. In sum,* ***different forms of mathematics were used for different purposes by different groups of people****. Perhaps one common feature is clearly distinguishable:* ***mathematics was a public activity,*** *it was played out in front of an audience, and it fulfilled functions that were significant at a communal level, be they counting revenues, measuring out land or exploring the limits of persuasive speech.*

Serafina Cuomo - Ancient Mathematics - Roudledge 2005.

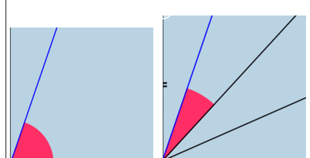
Let's bisect an angle



https://www.geogebra.org/m/av94xc8u

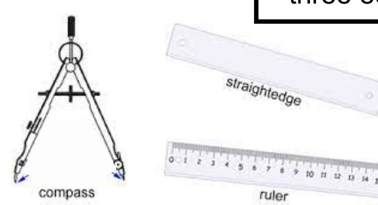
What is the conclusion? (Complete the sentence) Given an angle, using a compass and a straight edge, it is possible to construct another angle such that...

Three "impossible" problems



Trisecting an angle: Given an angle between two straight lines, **construct** two lines that divide the angle into three equal parts.

"Construct" means "construct using only straightedge and compass

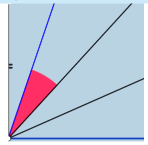


These problems originated around 400 BCE

Trisecting an angle

Constructing an angle α from the angle 3α ⟺ equivalent ⟺ Constructing a segment of length $\cos(\alpha)$ from a segment of length $\cos(3\alpha)$.

Also, $\cos(3\alpha) = 4\cos^3(\alpha) - 3\cos(\alpha)$.
If we set $\cos(\alpha)=x$ and $\cos(3\alpha)=a$ then
$4x^3 - 3x - a = 0$



Thus, trisecting an angle is equivalent to solving certain cubic equation

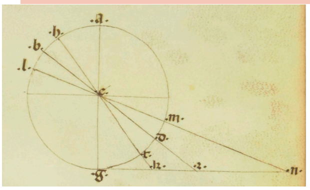


Diagram from 14th-century manuscript copy of Ptolemy's Almagest, folio 22 recto. Manuscript owned and digitized by gallica.bnf.fr / Bibliothèque nationale de France.

It is likely that the question of trisection of angles arose when trying to construct a table of chords for astronomical purposes. (A chord for the angle 3° can be constructed so it would have been natural to try to get a chord for the angle 1° from the chord for the angle 3°).

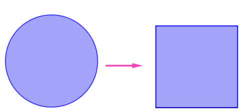
If a circle has radius 1 inch, what is the side length of a square that has the same area as the circle?

Three "impossible" problems

Squaring a circle is closely related to finding its area

"Construct" means "construct using only straightedge and compass

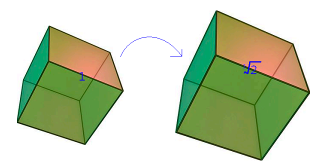
Squaring a circle: Given a circle, **construct** a square with the same area.

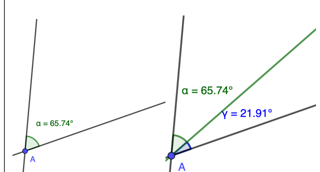


These problems originated around 400 BCE

Three "impossible" problems

Doubling a cube: Given a cube, **construct** another cube with twice its volume.

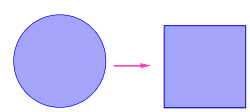




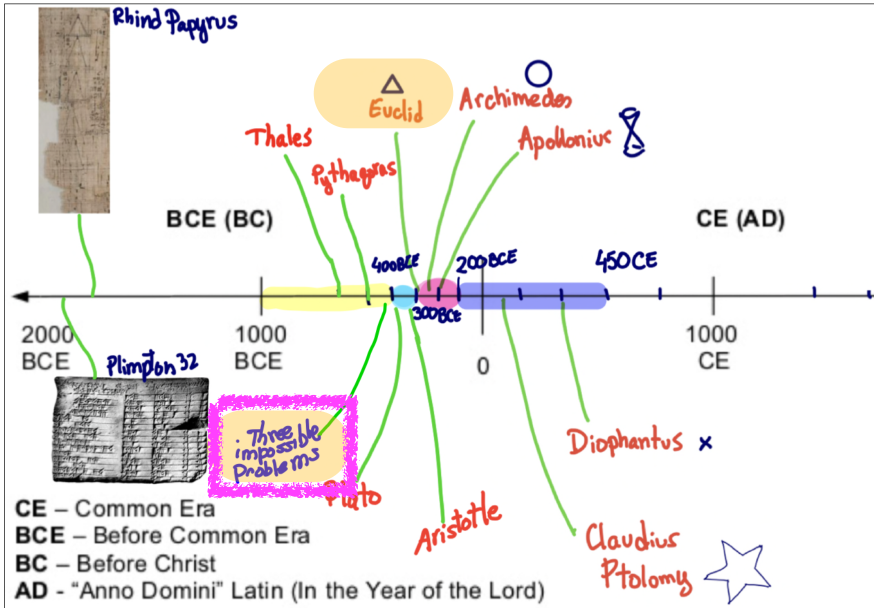
Trisecting an angle: Given an angle between two straight lines, **construct** two lines that divide the angle into three equal parts.

"Construct" means "construct using only straightedge and compass

Squaring a circle: Given a circle, **construct** a square with the same area.

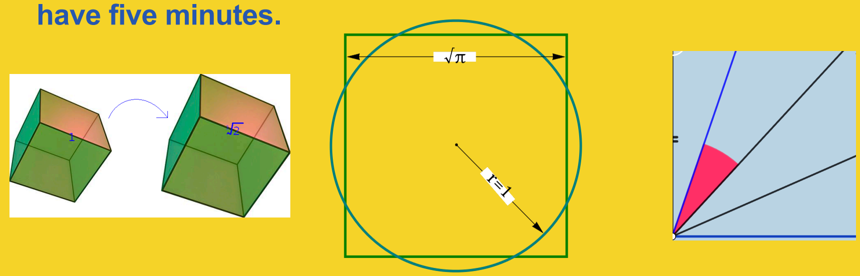


These problems originated around 400 BCE



1.Which of the three classical problems of antiquity was the hardest to tackle and why?
2.The three classical problems were proved impossible many centuries after they were posed. Find out who proved this impossibility in each case and the year it was proved it.

The three classical problems are``squaring the circle", ``trisecting an angle", and ``doubling a cube”. Google is allowed in this question.You have to discuss this question in teams of three. Try to find an answer that all members of the team agree. Each member must answer in Slido. You have five minutes.



Thursday

1. Zeno’s Paradoxes
2. Pythagoras and the Pythagoreans
3. Plato
4. Aristotle
5. Platonic solids
6. Incommensurable magnitudes

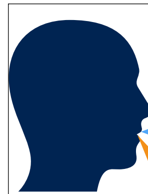
Zeno (~450BCE) and Paradoxes

Paradox

Greek
- **para**: distinct from
- **doxa**: opinion, belief.

A statement contrary to common belief or expectation

A statement or proposition from an acceptable premise and following sound reasoning that yet leads to an illogical conclusion



I am a barber.
I shave anyone in my town who does not shave himself, and no one else.

Do I shave myself?

Barber paradox

Analogous to Russell's paradox

Analogous the paradoxes "*This sentence is false*" and the one in the Star Trek clip

Liar paradox

I am a liar

Is my previous sentence true of false

Zeno's Paradox Achilles and the Tortoise
Do you think Achilles reach the tortoise? Why or why not?

Video from Open University https://www.youtube.com/watch?v=skM37PcZmWE

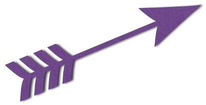
"In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead."

Description of Achilles and the Tortoise paradox by

Aristotle, Physics, VI:9, 239b15

Zeno's Arrow Paradox (adapted from Wikipedia)

- For motion to occur, an object must change the position which it occupies.
- for an example of an arrow in flight in any one (duration-less) **instant** of time, the arrow is **neither moving to where it is, nor to where it is not**.
    - It cannot move to where it is not, because no time elapses for it to move there;
    - it cannot move to where it is, because it is already there.



- In other words, **at every instant of time there is no motion occurring.** If everything is motionless at every instant, and time is entirely composed of instants, **then motion is impossible**

Is movement impossible?

- *Nude descending a staircase*, by Marcel Duchamp 1912
- Provoked an scandal when first shown.
- The object at each moment is at a fixed position (according to Zeno at rest) how can it then move?

More about Zeno's paradoxes

Simplicius has Zeno saying "it is impossible to traverse an infinite number of things in a finite time". This presents Zeno's problem not with finding the sum, but rather with finishing a task with an infinite number of steps: how can one ever get from A to B, if an infinite number of (non-instantaneous) events can be identified that need to precede the arrival at B, and one cannot reach even the beginning of a "last event"

What is the point of Zeno's paradoxes? What can we learn from them?

Paradoxes are important in mathematics, and, more generally, in the pursuit of knowledge.
Why? Try to find reasons.
(Hint: Many natural philosophers and mathematicians gave explanations trying to resolve them)

Pythagoras(~600 BC?) - The Pythagoreans

Pythagoras (~600BC?) - The Pythagoreans

- We know almost nothing about Pythagoras; some scholars even doubt he existed. Most believed he as a Holy Man.
- The Pythagoreans form a very secretive sect.
- The mathematical and the mystical were merged.

Doll by UneekDollDesigns

All things are number

No primary sources 🙁

Pythagoras (~600BC?) - The Pythagoreans

Aristotle in the Metaphysics writes (bullet points and bold fonts are added for emphasis)

> **They [the Pythagoreans]were the first to advance this study [of mathematics]**, and having been brought up in it **they thought its principles were the principles of all things.** Since of these principles numbers are by nature the first, and **in numbers they seemed to see many resemblances to the things that exist and come into being**—… —and similarly **almost all other things being numerically expressible**; since, again, they saw that **the attributes and the ratios of the musical scales were expressible in numbers**; since, then, **all other things seemed in their whole nature to be modeled after numbers, and numbers seemed to be the first things in the whole of nature**, **they supposed the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and a number**.

Educated guess: do you think that the Pythagorean ideas have impact in today's science? Why or why not?

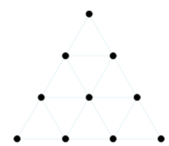
The Pythagoreans

Number rules the universe. ⟷ Great influence on scientists from then on

Examples of mystical properties Pythagoreans attributed to numbers

- The number one represents the origin of all things
- The number seven was sacred because it was the number of planets and the number of strings on a lyre, and because Apollo's birthday was celebrated on the seventh day of each month.
- 1 is not even, nor odd.
- Odd numbers are masculine, that even numbers are feminine, and that the number five represents marriage, because it was the sum of two and three.
- Ten is the "perfect number" and the Pythagoreans honored it by never gathering in groups larger than ten.
- Pythagoras was credited with devising the tetractys, the triangular figure of four rows which add up to the perfect number, ten.

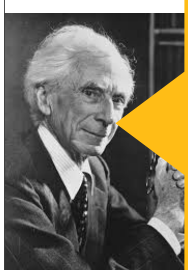
They believed Pythagoras discovered that musical notes could be translated into mathematical equations when heard the sound of blacksmiths' hammers clanging against the anvils.



Pythagoras - The Pythagoreans

Most scholars will agree that there was a Pythagorean school of philosophy from the sixth until probably the fourth century BC, that they were involved in politics and that they had certain beliefs about life and the universe, including perhaps the tenet that 'everything is number', or that number holds the key to understanding reality. **But most scholars today also think, for instance, that Pythagoras never discovered the theorem that bears his name.**

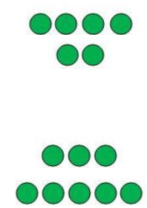
Cuomo, S. 2001. Ancient Mathematics, Science of Antiquity, Routledge.



Pythagoras, as everyone knows, said that **"all things are numbers."** This statement, interpreted in a modern way, is logical nonsense, but what he meant was not exactly nonsense. **He discovered the importance of numbers in music and the connection which he established between music and arithmetic survives in the mathematical terms "harmonic mean" and "harmonic progression."** **He thought of numbers as shapes, as they appear on dice or playing cards. We still speak of squares or cubes of numbers, which are terms that we owe to him.** He also spoke of **oblong numbers, triangular numbers, pyramidal numbers,** and so on. These were the numbers of pebbles [or calculi] (or as we would more naturally say, shot) required to make the shapes in question.

Bertrand Russell, in A History of Western Philosophy (1945), Book One, Part I, Chapter III, Pythagoras, p. 35

The Pythagoreans



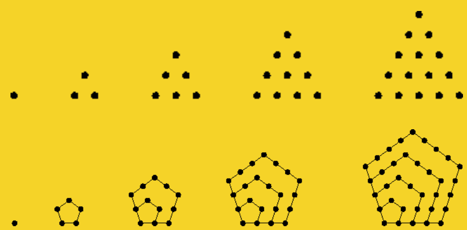
- Dichotomy between odd and even.
- Pythagoreans probably represented numbers with pebbles.
  - A number is even if it can be represented by a configuration of pebbles that than be divided into two equal parts. Otherwise is odd. (Well, 1 was not considered odd, nor even)
  - Proof: 4+2 is even.
  - Proof: An even sum of odd numbers is even.

The Pythagoreans - Figurate numbers

- Triangular numbers
- Square numbers
- Pentagonal numbers
- …

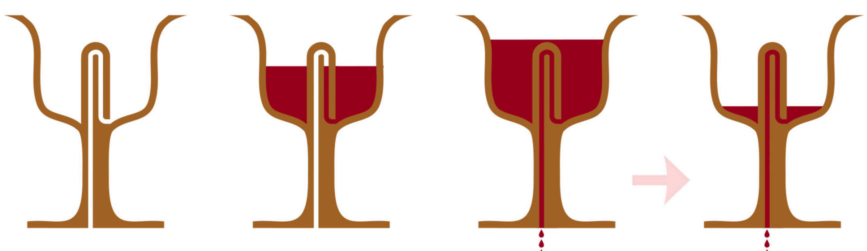
The *gnomon* is the piece of the figure that needs to be added to a given figurate numbers in order to get the next greater figurate number.

Find the 6th triangular number and the sixth pentagonal number. If you have time, give a formula for the n-th triangular number and one for the n-th pentagonal number.



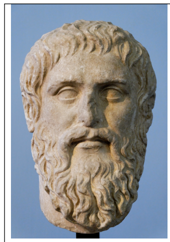
The so-called Pythagoreans, who were the first to take up mathematics, not only advanced this subject, but saturated with it, they fancied that the principles of mathematics were the principles of all things.
Aristotle - Metaphysica 1-5

Cross section of a Pythagorean cup being filled: at B, it is possible to drink all the liquid in the cup; but at C, the siphon effect causes the cup to drain



https://commons.wikimedia.org/wiki/File:Physagorian_Pythagoras_Greedy_Tantalus_cup_05.svg

Plato ~ 400 BC

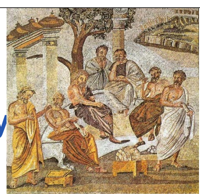


Plato (~400 BC)

- Proclus (400 AD) wrote that Plato '**greatly advanced mathematics in general, and geometry in particular, because of his zeal for these studies**'
- was an **important influence** upon the mathematicians of his time, by inspiration and direction.
- the works of Plato are some of our fullest and **best source of information about the mathematical developments at that time**.
- many scholars agree that defined the complex of general ideas forming the imperishable origin of Western thought

A *regular polygon* is a plane figure bounded by segments with equal sides and equal interior angles, such as the square.
Find two other examples of regular polygons and find how many regular polygons there are.
If you have time, discuss the analog to the regular polygons in dimension 3 (what they are, how many there are).

Plato's Academy



Plato (~400 BC)

- Distinction between pure math and "merely useful"
- Math education is essential for the mind (of an elite)
- Thaetus, a friend of Plato, may have been the first to recognize there are only 5 regular solids. Plato wrote about these solids which are now called "Platonic solids"

Some outliers notwithstanding, almost all Roman dodecahedrons were found in Britain, Gaul, and Roman Germany. IMPERIUM ROMANA

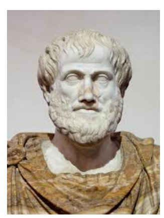
Two ancient Roman bronze dodecahedrons and an icosahedron (3rd c. AD) in the Rheinisches Landesmuseum in Bonn, Germany.
The dodecahedrons were excavated in Bonn and Frechen-Bachem; the icosahedron in Arloff.

Why did Plato put the sign "Let no one ignorant of geometry enter" at the door of his academy?

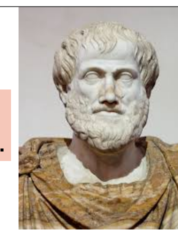
Aristotle ~ 400 BC

Aristotle (~300BC)

- An axiom is a statement worthy of acceptance
- By applying logic, one deduces new propositions.



Aristotle (~300BC)



Posterior Analytics: Two sorts of starting points for demonstration, **axioms** and **posits**.

An **axiom** (axiôma) is a statement worthy of acceptance and is needed prior to learning anything. (for instance: when equals taken from equals the remainders are equal.)

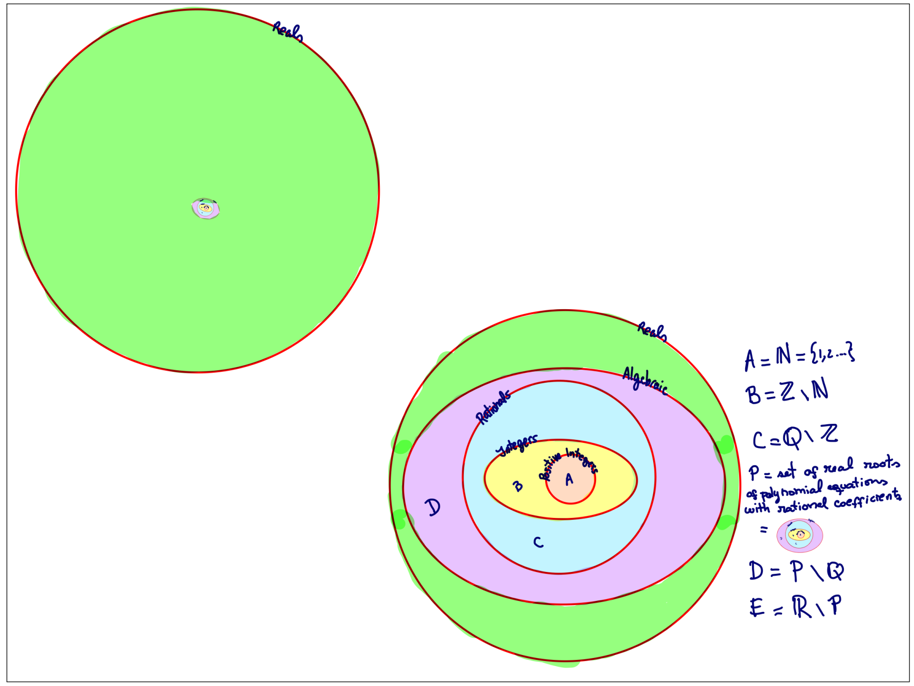
Example of **syllogism**
All men are mortal,
Socrates is a man,
therefore Socrates is mortal.

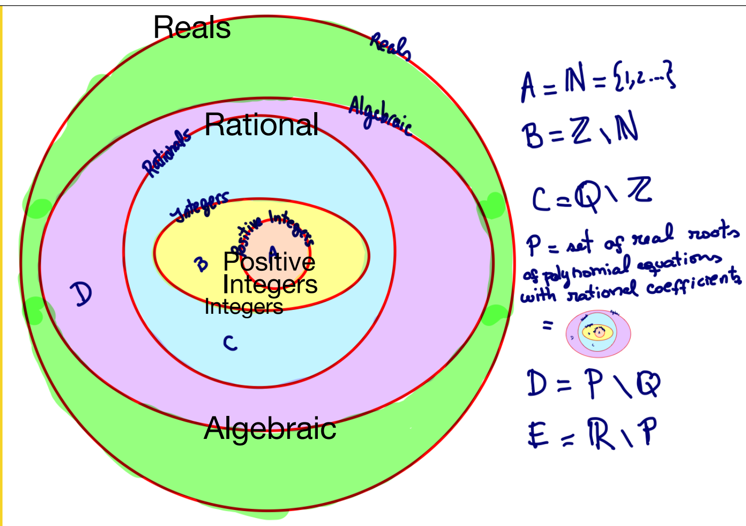
Posits are either:
Hypothesis (in math) the stipulation of objects at the beginning of a typical proof.
Definition expressions which are equivalent in some way to the defined term.

True proofs should be built out of **syllogisms** (“*a discourse in which , certain things being stated, something other than what is stated follows of necessity from their being so*.” *Prior Analytics*)

https://plato.stanford.edu/entries/aristotle-mathematics/

Incommensurable magnitudes





Write down a number in each of the different sets (C, D, and E) and explain why the number is in that set. (Example for B, “-1 is a negative integer, so it is in B; -1 is not in A because -1 is negative)

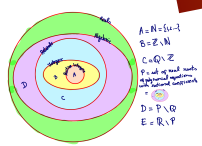
Commensurability

√2 is not rational

Does it have a "nice" expression in terms of whole numbers?

No pattern!

Commensurability



https://cosmosmagazine.com/mathematics/the-square-root-of-2/

Holy beard of Zeus! It's expanding irrationally and infinitely! Credit: Jeffrey Phillips

π is not rational

π is not even algebraic.

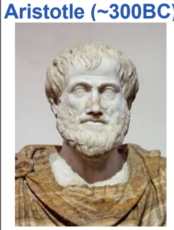
Proof of the irrationality of √2 discussed by Aristotle

It is clear then that the ostensive syllogisms are effected by means of the aforesaid figures; these considerations will show that reductiones ad also are effected in the same way. For all who effect an argument per impossibile infer syllogistically what is false, and prove the original conclusion hypothetically when something impossible results from the assumption of its contradictory; **e.g.** **that the diagonal of the square is incommensurate with the side, because odd numbers are equal to evens if it is supposed to be commensurate. One infers syllogistically that odd numbers come out equal to evens, and one proves hypothetically the incommensurability of the diagonal, since a falsehood results through contradicting this**. For this we found to be reasoning per impossibile, viz. proving something impossible by means of an hypothesis conceded at the beginning. Consequently, since the falsehood is established in reductions ad impossibile by an ostensive syllogism, and the original conclusion is proved hypothetically, and we have already stated that ostensive syllogisms are effected by means of these figures, it is evident that syllogisms per impossibile also will be made through these figures. Likewise all the other hypothetical syllogisms: for in every case the syllogism leads up to the proposition that is substituted for the original thesis; but the original thesis is reached by means of a concession or some other hypothesis. But if this is true, every demonstration and every syllogism must be formed by means of the three figures mentioned above. But when this has been shown it is clear that every syllogism is perfected by means of the first figure and is reducible to the universal syllogisms in this figure.

Aristotle - The Organon ANALYTICA PRIORIA Book 1 Part 23- ~350 BCE

- There are two distinct types of "quantities":
  - the continuous (magnitude)

Aristotle (~300BC)



- *"A **magnitude** is that what which is divisible into divisible that are infinitely divisible."*
- Example: lines, surfaces, bodies and time.
  - the discrete (number)
    - *A "**number**" quantities that is composed of distinct, separate units or parts that can be counted. (Examples 1, 2, 3, ...)*

Proposition VII.2: To find the greatest common measure of two given numbers not relatively prime.

–Let AB and CD be the two given numbers (which are) not relatively prime. So it is required to find the greatest common measure of AB and CD.

–In fact, **if CD measures AB, CD is thus a common measure of CD and AB**, (since CD) also measures itself. And (it is) manifest that (it is) also the greatest (common measure). For nothing greater than CD can measure CD.

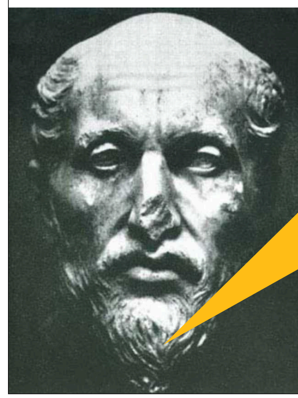
–But if CD does not measure AB then some number will remain from AB and CD, the lesser being continually subtracted, in turn, from the greater, which will measure the (number) preceding it. For a unit will not be left. But if not, AB and CD will be prime to one another [Prop. VII.1]. The very opposite thing was assumed. **Thus, some number will remain which will measure the (number) preceding it.**

Euclidean algorithm

Thales ~ 600 BC

About Thales de Miletus ~ 600 BC

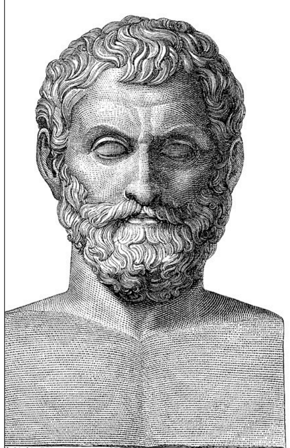
No primary sources 🙁



Hey, I am Proclus and I wrote:
"Thales was the first to go to Egypt and this study [geometry] bring back to Greece
He himself discovered many propositions and disclosed the underlying principles of many others to his successors, in some case his method being more general, in others more empirical..."

- *Proclus's Summary*
- ***written around 450 CE***
- *of Eudemus' History of Geometry*
- ***written around 320 BC***

Thales de Miletus ~ 600 BC



- We know very little.
- Earliest Greek mathematical investigations (and first proof!) that we know of
- According to Proclus
  - "*Thales travelled to Egypt and was the first to introduce mathematics into Greece*".
  - was the first to demonstrate that
    - *"the circle is bisected its diameter".*
    - *"A triangle inscribed on a circle, with a diameter as one of its sides is a right triangle."*

No primary sources 🙁

Ideas of Math in the Ancient Greek World

Prometheus at Rockefeller Center, sculpture by Paul Manship, 1933

Paul Manship, Public domain, via Wikimedia Commons

Prometheus Bound By Aeschylus

PROMETHEUS: ..hear what wretched lives people used to lead, how babyish they were – until I gave them intelligence, **I made them masters of their own thought**. [...] they knew nothing of making brick-knitted houses the sun warms, nor how to work in wood. They swarmed like bitty ants in dugouts in sunless caves. They hadn't any sure signs of winter, nor spring flowering, nor late summer when the crops come in. All their work was work without thought, until I taught them to see what had been hard to see: where and when the stars rise and set. **What's more, I gave them the numbers, chief of all the stratagems.** And the painstaking, putting together of letters: to be their memory of everything, to be their Muses' mother, their handmaid! [...] In a word: listen! All the arts are from Prometheus.

Birds, by Aristophanes performed in 414 BC

A new city has to be founded from scratch. The main character, Peisthetaerus, is visited by various people who offer their services.

METON : With the straight rod I measure out, that **so the circle may be squared**; and in the centre a market-place; and streets be leading to it straight to the very centre; just as from a star, though circular, straight rays flash out in all directions.
PEISTHETAERUS : **Why, the man's a Thales!**

Translation by Serafina Cuomo - Ancient Mathematics