Presentations

Incas

Inca Quipu
Claus Ableiter nur hochgeladen aus enWiki - enWiki, hochgeladen von User Lyndsaruel

Incas

- 1400-1500 AD
- Trivia: Gave us beef jerky!

Inca empire in 1500

By https://clevelandart.org/art/1957.136, CC0, https://commons.wikimedia.org/w/index.php?curid=78063088

Early Morning in Machu-Plchu
Pedro Szekely at https://www.flickr.com/photos/pedrosz/

About the Chasquis (or Chaskis)

- A **chasqui** was a messenger of the Inca empire.
- The chakis carried quipus, oral information and small packets.
- Chaskis were short-distance relay runners. Young men, especially those with superior running skills, were chosen for this job.
- Since the Incas had no written language, messages were memorized and repeated to the next runner during the relay.
- Runners traveled up to 10 to 15 kilometers (6 to 9 miles) until they reached a a small house where another chaski was waiting to run the next segment of the relay.

A chaski. Felipe Guaman Poma de Ayala (Quechua, 1535–1616). Pen and ink drawing published in The First New Chronical and Good Government (1615). Royal Library, Copenhagen GKS 2232 4°

Stretch of Inka road through the upper Amazon. Near Chachapoyas, Peru, 2014. Photo:Inge Schjellerup

Source https://americanindian.si.edu/nk360/inka/pdf/inka-teachers-guide.pdf

About the Chasquis (or Chaskis)

A chasqui was a messenger of the Inca empire.

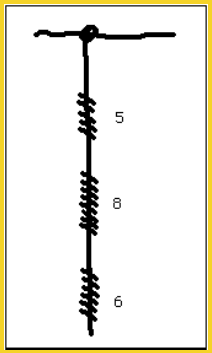
These 'chasques' were employed in this kingdom. They were the sons of curacas [local chiefs] who were loyal. Each runner had a white feather sunshade on his head which he wore so that the next chasque would see him at a distance. The chasque also carried a trumpet called 'putoto' [shell] so that the next runner would be ready. The arms they carried were the 'champi' [a star-headed club] and a 'uaraca' [sling].

These chasques were paid by the Inca, and they got their food from his storehouses. Each 'churo chasque' [chasqui carrying a shell] was stationed at intervals of one-half league [2.5 kilometres (1.6 mi)] so he could run the distance quickly. They say that they could bring a snail alive from the New Kingdom of Granada [northern part of south America] to the Inca in Cuzco.

—Felipe Guaman Poma de Ayala, 1615

Threads That Speak: How The Inca Used Strings to Communicate | National Geographic
https://youtu.be/AmPyz1kCbOw

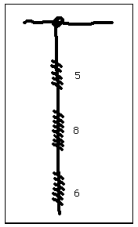
Incas used a positional number system and record their numbers in chords like the one on the photo. What do you think is the number represented in the drawing on the left?



Portion of a quipu
https://mathshistory.st-andrews.ac.uk/HistTopics/Inca_mathematics/

Quipu
By Pi3.124 - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=69539294

Quechua was the main language of the Inca Empire. Quipu (or khipu) is the Quechuan word for “knot”.



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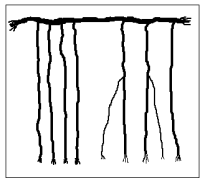
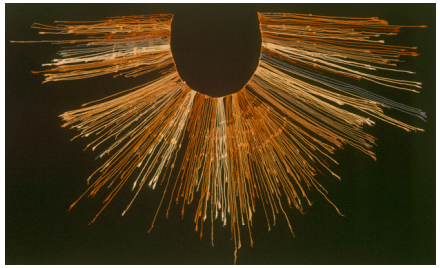
Quipu
By Pi3.124 - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=69539294

A quipu is a collection of colored knotted cords, in which the colors, the placement of the cords, the knots on the individual cords, the placement of the knots, and the spaces between the knots all contribute to the meaning of the recorded data.
Every quipu has a main cord, thicker than the others, to which are attached other cords, called pendant cords, to which may be attached further cords, called subsidiary cords. Sometimes there is a top cord, a cord placed near the center of several pendant cords and tied so that when the quipu lies flat it falls in a direction opposite the pendant cords. Data is recorded on the cords (other than the main cord) by a system of knots. The knots are clustered together in groups separated by spaces and represent numbers using a base-10 place value system with the highest value place closest to the main cord.

More about incas and quipus



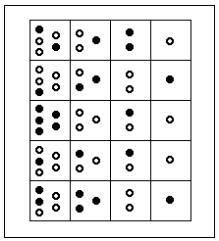


Quipu
By Claus Ableiter nur hochgeladen aus enWiki - enWiki, hochgeladen von User Lyndsaruell; siehe http://en.wikipedia.org/wiki/Image:Inca_Quipu.jpg, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=2986739

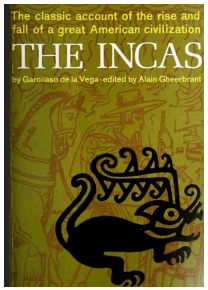
The largest number so far discovered on a quipu is **97,357**.

The ordinary judges gave a monthly account of the sentences they imposed to their superiors, and they in turn reported to their immediate superiors, and so on finally to the Inca or those of his Supreme Council. The method of making these reports was by means of knots, made of various colours, where knots of such and such colours denote that such and such crimes had been punished. Smaller threads attached to thicker cords were of different colours to signify the precise nature of the punishment that had been inflicted.

Inca Garcilaso de la Vega ~ 1600



yupana and is presumed to be the counting board of the Incas.
https://mathshistory.st-andrews.ac.uk/HistTopics/Inca_mathematics/

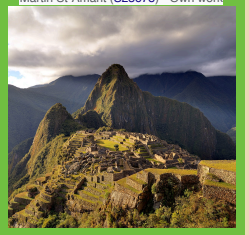


The Incas also had an excellent knowledge of arithmetic, and the way they counted was quite remarkable. Throughout the Empire, all levies and taxes, both exemptions and claims, were recorded by means of knots made in colored cords. **They could add, subtract, and multiply with these knots, and in order to determine what was owed to each village, made their allotments on the basis of little pebbles and grains of corn, in such a way that there could be no mistakes in their calculations.** Special bookkeepers were appointed for each section, whether it had to do with war or peace, such as taking a census of the vassals, or of levies, herds, laws, and ceremonies, and they were able to give an accounting at any moment, their collections of cords and knots being as clear and exact for them as any books. The head accountants, who were in charge of several things at a time, worked with these same means, quite as easily as the others.

Gómez Suárez de Figueroa, (Inca Garcilaso de la Vega) 1600

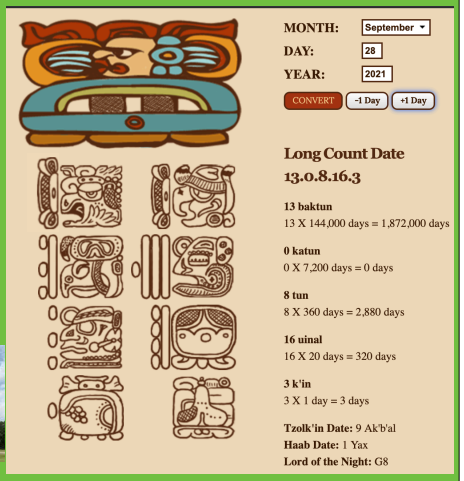
Mayans

Maya
ProtoplasmaKid - Own work



Machu Pichu
Martin St-Amant (S23678) - Own work

Daniel Schwen - Own work



https://maya.nmai.si.edu/calendar/maya-calendar-converter

Search for images on Google related to "Maya culture" or "Maya script." Based on those images, what conclusions can you draw about the Maya civilization?

Introduction and Mayan script

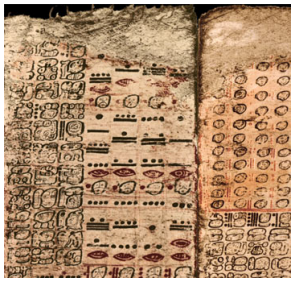
Photograph of a Maya Vase. Unknown author - Art from late Classic c. 600 - 900 AD, per book "The Blood of Kings, Dynasty and Ritual in Maya Art" by Linda Schele, Mary Ellen Miller, Justin Kerr, Kimball Art Museum, Fort Worth, 1986, plate 115 Mechanical reproduction of art more than 1,000 years old.
https://commons.wikimedia.org/wiki/File:Maya_vase.jpg

What information does this documentary provide about Mayan mathematics?

Ancient Maya 101 | National Geographic
https://youtu.be/Q6eBJjdca14

The Mayas

- had a **written language**, which help us learn about their mathematics.
- records were preserved in **bark paper** or carved in **stone**.
- priestly class who studied mathematics and astronomy.
- Remarkable development between 250 to 900 of our era.



Mayan page in tree bark
From "Cracking the Mayan code"
https://www.pbs.org/wgbh/nova/mayacode/

Mayan writing carved on stone
From "Cracking the Mayan code"
https://www.pbs.org/wgbh/nova/mayacode/

The Monuments of Yaxchilan: Lintel 14 - photograph by Jorge Pérez de Lara

Breaking the Maya Code #2: The Maya Script

https://youtu.be/1t5FALvtMQg

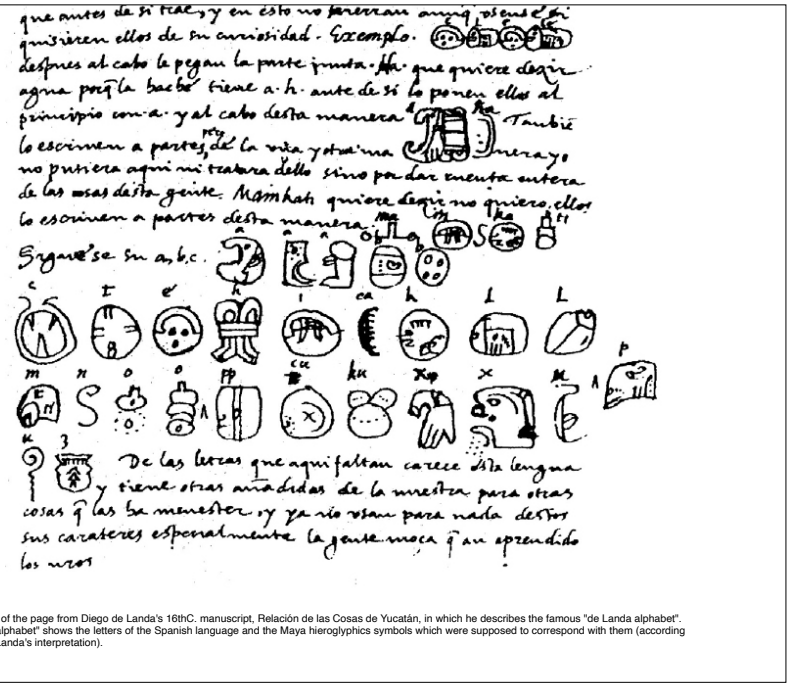
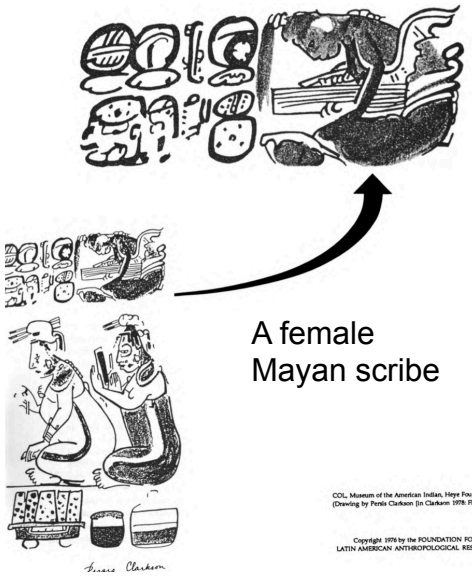


Image of the page from Diego de Landa's 16thC. manuscript, Relación de las Cosas de Yucatán, in which he describes the famous "de Landa alphabet". This "alphabet" shows the letters of the Spanish language and the Maya hieroglyphics symbols which were supposed to correspond with them (according to de Landa's interpretation).

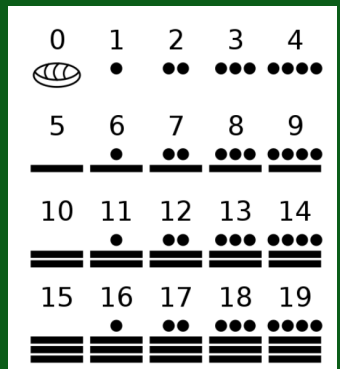
Codex-Style Vessel with Two Scenes of Pawahtun Instructing Scribes; c. A.D. 550–950; Possibly Mexico or Guatemala, Maya culture, Late Classic period (A.D. 600–900); Ceramic with monochrome decoration; height 8.9 cm Kimbell Art Museum, Fort Worth, Texas; AP 2004.04 see https://www.kimbellart.org/collection-object/codex-style-vessel-two-scenes-pawahtun-instructing-scribes



A female Mayan scribe

COL. Museum of the American Indian, Heye Foundation (Drawing by Ferris Clarkson [in Clarkson 1978: Fig. 5])

Number system



Mayan numerals - Wikipedia

Breaking the Maya Code #3: The Maya Books
https://youtu.be/GgCf3Q7qooQ

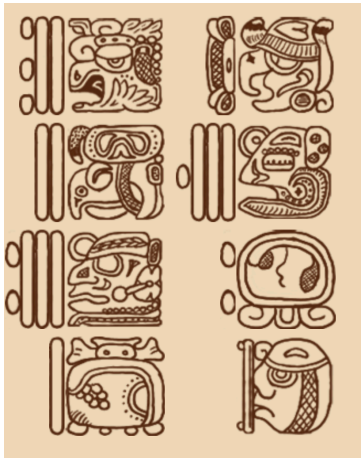
What type of Mayan hieroglyphs were deciphered first?

Mayan Calendars

The ancient Maya civilization had two (or three, depending of how we count) calendars

- Round calendar, composed of
  - Tzolkin or sacred calendar
  - Haab or farmer's calendar
- Long count calendar (for very long periods of time)

Mayan Calendars

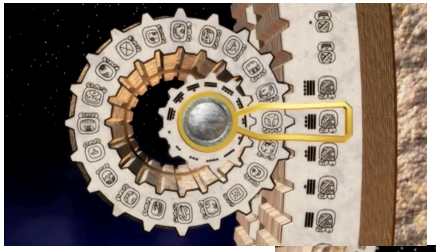


https://maya.nmai.si.edu/calendar/maya-calendar-converter

Please describe the Mayan calendars discussed in this clip. Additionally, if possible, identify any mathematical structure to which the Mayan calendar system is connected.

Breaking the Maya Code #4: The Maya Calendar https://youtu.be/qhWItvjk9Yg

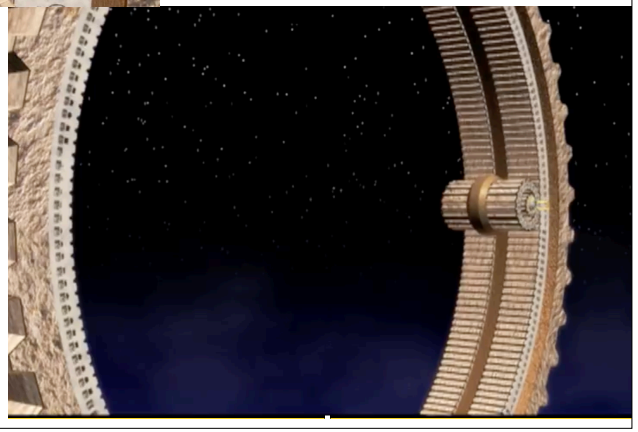
Breaking the Maya Code by Night Fire films (extract) https://vimeo.com/ondemand/115559/223534703

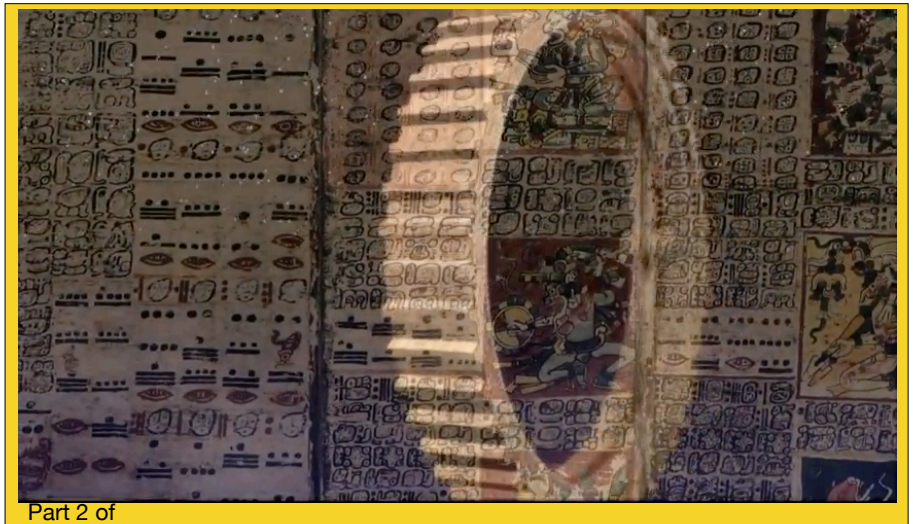


Round calendar
- **The two smaller wheels represent the Tzolkin or sacred calendar.**
- **The largest wheel represents the farmers or Haab calendar**

Modular arithmetic!

The wheels are a modern way of representing the calendars, not a Mayan representation.





Part 2 of

Breaking the Maya Code #4: The Maya Calendar https://youtu.be/qhWItvjk9Yg

What do the large numbers found in the Dresden Codex represent? Could you explain the concept of the Long Count in the Mayan calendar system?

Based on astronomical observations, the Maya developed an elaborate system of calendars.

- Round calendar
  - Tzolkin calendar
  - Haab calendar
- Long count

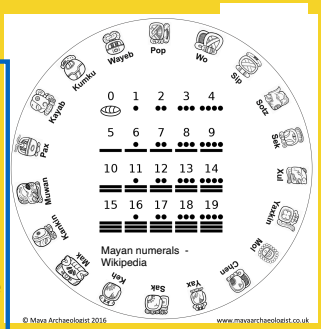
Mayan Calendars

https://maya.nmai.si.edu/calendar/maya-calendar-converter

Round calendar (Composed of the union of Haab and Tzolkin)

Haab Calendar

- Used by farmers
- **18 "months"** plus **five unlucky days**.
- Each "month" lasted 20 days and was name after a god.
- This calendar works similarly to ours: First date is 0 Pop, followed by 1 Pop, 2 Pop … 19 Pop, 1 Wo and so on. (Pop and Wo are months)

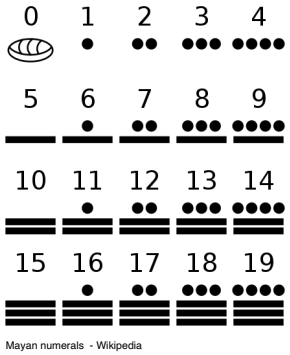


Educated guess: How many days are there on the Haab or farmer's calendar ? Hint: Use common sense, what does a farmer need to know?

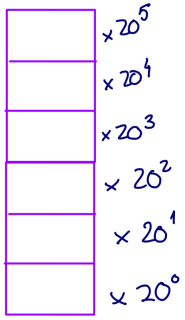
- The unlucky month was named Uayeb (which translates to "That which has no name".)

Mayan number system

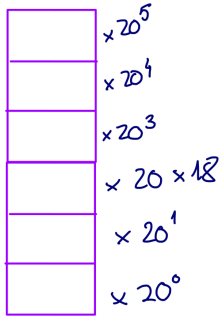
Write down 365 in the Mayan number system. (numbers are written vertically, lower units at the bottom)



Base 20



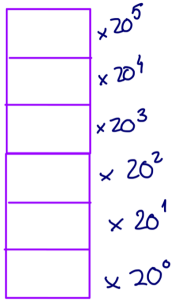
Modified base 20



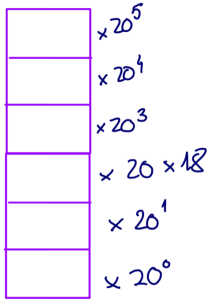
Translate the Mayan number below to Hindu-Arabic numerals in two ways:

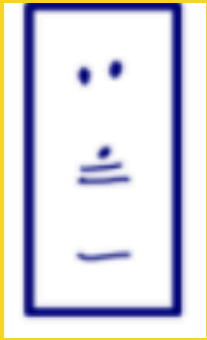
1. **Assuming the Mayan number system was an "honest" base 20.**
2. **Assuming the "calendar way", that is, the third play unit instead of begin 20x20 is 20x18.**

Base 20



Modified base 20





Haab Calendar

- Used by farmers
- **18 "months"** plus **five unlucky days**.
- Each "month" lasted 20 days and was name after a god.
- This calendar works like ours: First date is 0 Pop, followed by 1 Pop, 2 Pop … 19 Pop, 1 Wo and so on. (Pop and Wo are months)

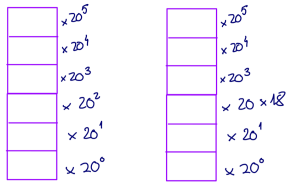
When days were counted, the third-place unit, instead of being 20.20 was 20.18.
This was probably because there are 360 days in the "regular" part of the 365 days calendar know as the *Haab* and the other 5 days were considered "unlucky" (and so best not included in the count)

Translate a Mayan number on to Hindu-Arabic in two ways (units at the bottom):

1. Assuming Mayan number system was "honest" base 20

2. Assuming the "calendar way" (that is days were counted, the third-place unit, instead of being 20x20 was 20x18).

Base 20 | Modified base 20



Tzolkin or Sacred Calendar

- 20 names of gods: Imix, Ik, Akbal, Kan, Chicchan, Cimi, Manik, Lamat, Muluc, Oc, Chuen, Eb, Ben, Ix, Men, Cib, Caban, Etznab, Cauac, and Ahau.
- 13 days.

Image from Breaking the Maya Code

Each day is specified by a pair (a, D_b), where

- $1 \le a \le 13$
- $1 \le D_b \le 20$ (we will use numbers instead of gods names. Imix = 1, Ik=2, etc)
- The Tzolkin year begins with (1, 1), continues with (2,2), (3,3) and so on. (That is 1 Imix, 2 Ik, 3 Akbal,…)

How many different dates (that is, pairs (a, D_b)) can be formed in this calendar?

A Maya representation of the Tzolk'in from the Madrid Codex. Time is represented by 260 dots marking a path or a journey of twenty days and thirteen numbers.

Source: Smithonian, National Museum of the American Indian
https://maya.nmai.si.edu/sites/default/files/resources/The%20Maya%20Calendar%20System.pdf

Suppose the Haab and Tzolkin calendar start at a given date (say, 0 Pop, 1 Imix). How many days would it take to go back to that date?

- A Haab year contains 365 days.
- A Tzolkin year contains 260 days.

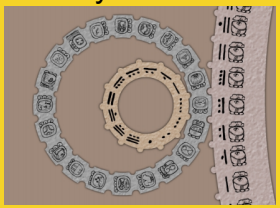


Image from Breaking the Maya Code

A contemporary representation of the Calendar Round, interlocking the Tzolk'in (left) with the Haab (right).
Source: Smithonian, National Museum of the American Indian
https://maya.nmai.si.edu/sites/default/files/resources/The%20Maya%20Calendar%20System.pdf

The Haab and Tzolkin together form the *round calendar*

- Merging the **Tzolkin** and the **Haab** calendars Mayans obtained a cycle of 18,980 days.
- A Calendar Round date, with both Tzolk'in and Haab components, can uniquely mark an event within a 52 year period.
- Any event spanning more than 52 years required an additional calendar, the **Long Count calendar.**

Round calendar

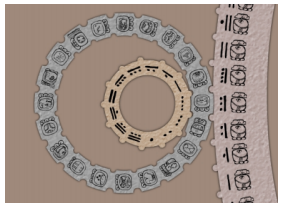


Image from Breaking the Maya Code

The Maya believe that when a person reaches 52 years of age, they attain the special wisdom of an elder.

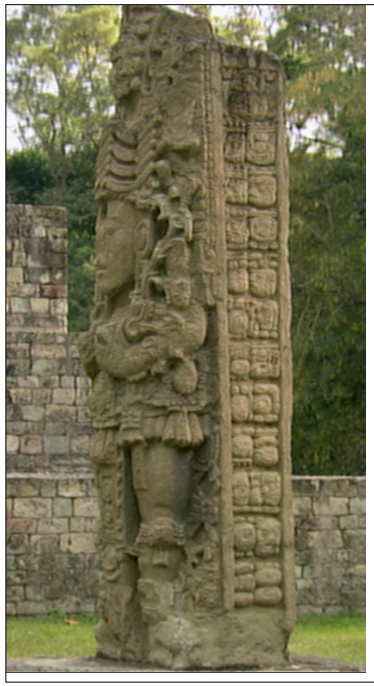
Long count calendar

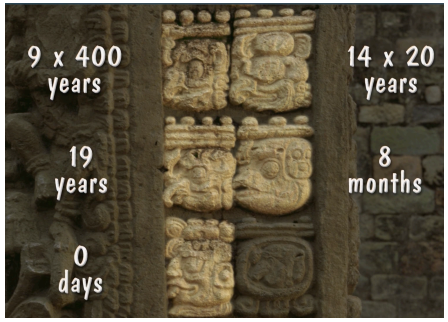
What are the numbers on the top of the glyphs a, b, c and d?

Images from the documentary

Breaking the Mayan Code

https://nightfirefilms.org/breaking-the-maya-code/





Images from the documentary

Breaking the Mayan Code

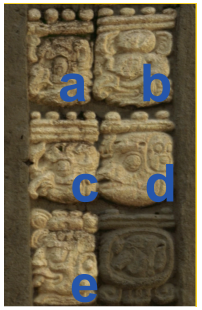
https://nightfirefilms.org/breaking-the-maya-code/

The long count calendar counts how much time elapsed since a fixed event, the creation date.

Long count calendar

1. Initial Series Introductory Glyph: This symbol identifies this date as belonging to the Long Count system
2. Baktun: A number and the symbol of “baktun”
3. Katun: A number and the symbol for “katun”
4. Tun: A number and the symbol for “tun”
5. Uinal: A number and the symbol for “uinal”
6. K’in: A number and the symbol for “k‘in”

A long count calendar date it is written now as 2.3.4.5.6



Reading the Maya Calendar
https://maya.nmai.si.edu/calendar/maya-calendar-converter

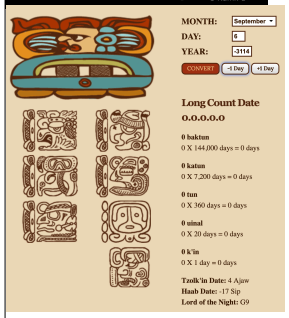
Long count calendar

Long Count calendar is a system that counts 5 cycles of time (kins, uinals, tuns, katuns, baktuns). The shortest period was the **kin** (day).

- 1 uinal = 20 kins
- 1 tun = 18 uinals
- 1 katun = 20 tuns
- 1 baktun = 20 katuns

- A Long Count date and a Calendar Round date were used by the to place mythical and historical events in chronological order.
- An era, or a full cycle, is comprised of 13 baktun cycles.
- Note that this is similar to the Gregorian calendar system that counts days, months, years, centuries and millennia.

How many years are there in a batkun? To facilitate computations, you can assume that all years are 365 days long. You can use calculators.

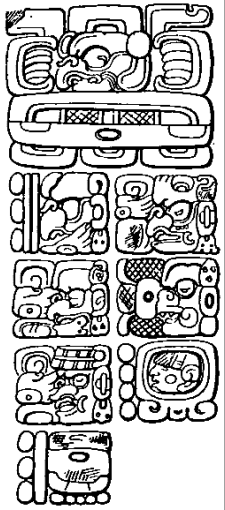


https://maya.nmai.si.edu/calendar/maya-calendar-converter

Maya era starts on Long count: 13.0.0.0.0, Tzolkin :4 Ajaw Haab: 8 Kumk’u,

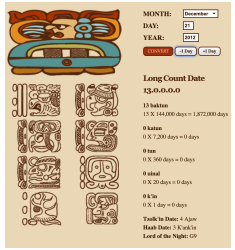
https://maya.nmai.si.edu/calendar/reading-calendar-glyphs

According to the Popol Vuh, the beginning date, or the day of the creation of the world, is the 13.0.0.0.0 in the Long Count Calendar.



East side of stela C, Quirigua with the mythical creation date of 13 baktuns, 0 katuns, 0 tuns, 0 uinals, 0 kins, 4 Ahau 8 Cumku – August 11, 3114 BCE in the proleptic Gregorian calendar.
Public Domain

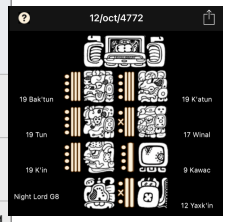
Beware of the confusion, added to the correlation with Western calendars



Maya era ends

Long Count unit	Long Count period	Days	Approximate Solar Years
1 K'in		1	
1 Winal	20 K'in	20	
1 Tun	18 Winal	360	1
1 K'atun	20 Tun	7,200	20
1 B'ak'tun	20 K'atun	144,000	394
1 Piktun	20 B'ak'tun	2,880,000	7,885
1 Kalabtun	20 Piktun	57,600,000	157,704
1 K'inchiltun	20 Kalabtun	1,152,000,000	3,154,071
1 Alautun	20 K'inchiltun	23,040,000,000	63,081,429

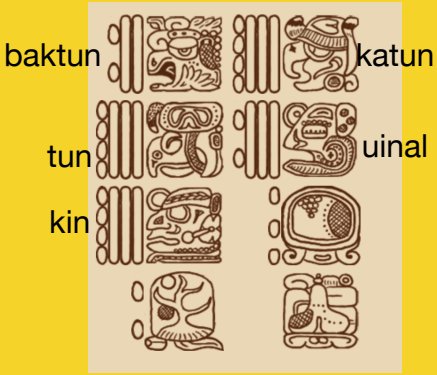
Piktun



Note that this is similar to the Gregorian calendar system that counts days, months, years, centuries and millennia.

Beware of the confusion

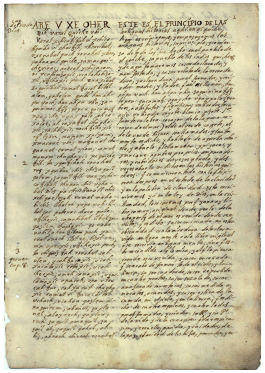
What is the day after the date below in the Long Count calendar? Write it as *baktun.katun.tun.uinal.kin*



Teams of 3

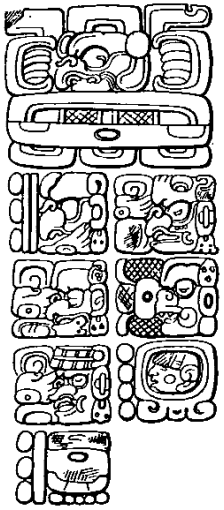
- 1 kin is one day.
- 1 uinal (month) = 20 kins
- 1 tun (year) = 18 uinals
- 1 katun = 20 tuns
- 1 baktun = 20 katuns

Long count calendar: 13.0.0.0.0



The oldest surviving written account of Popol Vuh (ms c. 1701 by Francisco Ximénez, O.P.) Transcription of the original K'iche' (Quiché), a language of the Mayans, side-by-side with a Spanish translation.

"Some scholars describe this as a moment of creation. When a similar cycle comes to an end, as indeed happened on 24 December AD 2012, it was thought to anticipate another cycle of destruction and creation." Michael D. Coe. "The Maya ."



East side of stela C, Quirigua with the mythical creation date of 13 baktuns, 0 katuns, 0 tuns, 0 uinals, 0 kins, 4 Ahau 8 Cumku – August 11, 3114 BCE in the proleptic Gregorian calendar. Public Domain

All calendars again

All calendars

Round calendar

- Haab 365 days
- Tzolkin 260 (= 13 x 20) days

- Long count:
  - Counts days since a certain date.
  - Expressed as a.b.c.d.e= 144,000a+7200b+360c+20d+e
  - Some scholars believe it is cyclic, and goes back to zero after 13.0.0.0.0=1,872,000 days (roughly 5,125 years). Or maybe after 20.0.0.0.0 days….

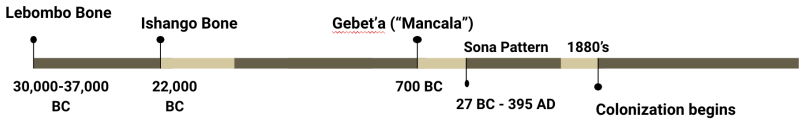
Mayan Calendar

- Two problems
  - Given a date (as a triple) and a number of days later, determine the new date.
  - Given two dates determine the least number days between them

Mayan Calendar Converter

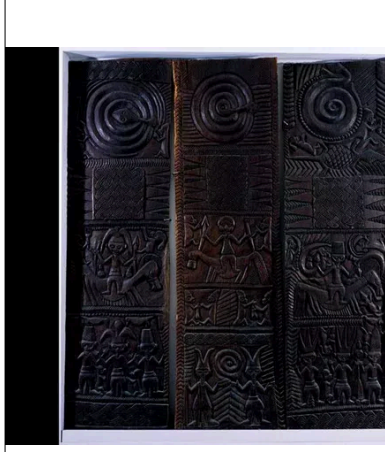
https://maya.nmai.si.edu/calendar/maya-calendar-converter

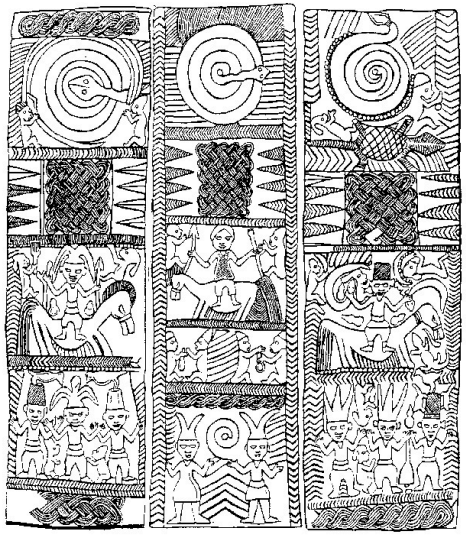
Mathematics of the Yoruba people



Yoruba number system

- The Yoruba were the dominant cultural force some regions of Africa during the 11 century and possibly before.
- Count using cowries shells may explain their remarkable number system.





Yoruba Panels, late 19th Century #2
A detailed drawing of wood panel doors, carved by the Yoruba people in the late 19th century. (African, Nigeria.) Medium: Rosewood; each panel approx. 70" X 20." Source: Speed Art Museum, Kentucky, USA. Gift of Mrs. William Belknap, 76.12.1-.3.

Yoruba number system

- Complex
- Base 20
- Strong reliance of subtraction
- Numbers between 200 and 2000 (except 400) are treated as multiples of 200.
- Terms for
  - 1 to 20
  - 20, 30,
  - 200, 400.

Numerals	
1	10
2	20
3	30
4	200
5	400
6	
7	
8	
9	
10	

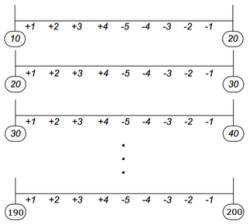


Figure 2:
Yorùbá number scale

Computational modelling
of Yorùbá numerals
in a number-to-text conversion system

Olúgbénga O. Akínadé and Odétúnjí A. Odéjobí
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Department of Computer Science and Engineering
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Ilé-Ifè, Nigeria

11	10+1
12	10+2
13	10+3
14	10+4
15	20-5
16	20-4
17	20-3
18	20-2
19	20-1
30	30
40	20x2
45	(20x3) -10 - 5
50	20x3 - 10
54	20x3 - 10+4
60	20x3
106	(20X6) - 10 - 4
108	(20x6) - 10 - 2
300	20 x (20 - 5)
318	400 - (20 x 4) - 2
525	(200 x 3) - (20x4)+5

Yoruba number system

Example: Compute 19 x 17

- Start with 20 piles of 20 shells each.
- Remove one shell from each pile (-20)
- Remove three of the piles (with 19 shells each)
- Take two shells from one of the removed piles, add one to each of the other two removed piles. (Get one pile with 17 shells and two with 20 shells each)
- 400 - 20 - (20x2) - (20 - 3) = 323

11	10+1
12	10+2
13	10+3
14	10+4
15	20-5
16	20-4
17	20-3
18	20-2
19	20-1
30	30
40	20x2
45	(20x3) -10 - 5
50	20x3 - 10
54	20x3 - 10+4
60	20x3
106	(20X6) - 10 - 4
108	(20x6) - 10 - 2
300	20 x (20 - 5)
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