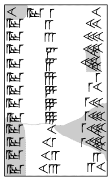
Mathematics in Mesopotamia



- Overview Mesopotamia.
- Cuneiform and the beginning of writing
- Review of numbers
- Areas
- Plimpton 322
- Number system**s**
- Square roots: algorithm and computations
- The “canonical“ triangle.
- Calculation of areas
- Solutions of Equations
- Reciprocals
- Multiplication tables

Write down a brief summary of your topic, and the date you are presenting.

Overview

Civilizations 3000 years’ span
- Sumerian
- Babylonian
- Assyrian
- Akkadian

Civilizations 3000 years' span

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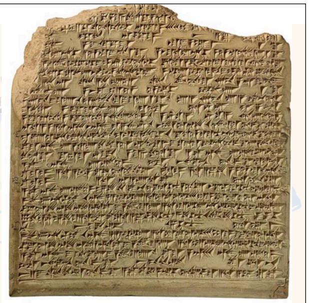


Tablet Inscribed in Babylonian with a Ritual for the Observances of Eclipses-The Morgan Library and Museum

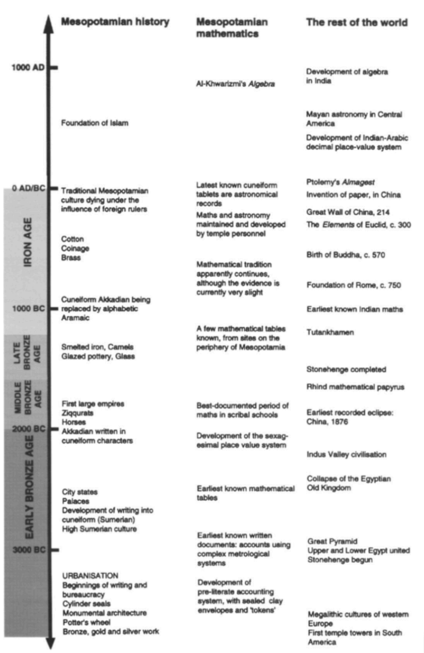
- Complicated, syllabic, wedge based or "cuneiform" script.
- Trained scribes wrote in tablets.
- Hundreds of thousand have arrived to us.
- Most of these tablets are administrative records (memos, receipts, wage records)
- Some of these tablets are
  - magical rituals,
  - law codes,
  - vivid descriptions of military campaigns and battles,
  - great myths such as the Epic of Gilgamesh

Robson, Eleanor. "Counting in Cuneiform." Mathematics in school 27.4 (1998): 2-9

Describe one or more mathematical developments of the Mesopotamian culture mentioned in this clip.

Ancient Mesopotamia 101- National Geographic

https://youtu.be/xVf5kZA0HtQ



Words:
Mesopotamia vs
Babylonia

Counting in Cuneiform
Author(s): Eleanor Robson
Source: *Mathematics in School*, Sep., 1998, Vol. 27, No. 4, History of Mathematics (Sep., 1998), pp. 2-9
Published by: The Mathematical Association

A praise poem of Šulgi

I am a king, offspring begotten by a king and borne by a queen.
I, Šulgi the noble, have been blessed with a favourable destiny right from the womb.
When I was small, I was at the academy, where I learned the scribal art from the tablets of Sumer and Akkad.
None of the nobles could write on clay as I could.
There where people regularly went for tutelage in the scribal art, **I qualified fully in subtraction, addition, reckoning and accounting.**
The fair Nanibgal, Nisaba, provided me amply with knowledge and comprehension.
I am an experienced scribe who does not neglect a thing.

Nanibgal, Nisaba, The patron goddess of the scribal art.

Cuneiform writing: origins, decipherment and texts.

Mesopotamian Accounting Tokens

https://www.maa.org/press/periodicals/convergence/mathematical-treasure-mesopotamian-accounting-tokens

- Each counter shape represented a specific quantity of a specific commodity.
- Each category of items was counted with special numerations or number words spe

The Mesopotamian accounting tokens were found in present Iraq, and date from about 4000 BCE. The **cone**, **spheres**, and flat disc were measures of cereals: smallest, larger, and largest. The **tetrahedron** designated a unit of work, perhaps one man-day, or the amount of work performed by one man in one day. (Image by Denise Schmandt-Besserat and the University of Pennsylvania Museum of Archaeology and Anthropology, University of Pennsylvania, Philadelphia.)

Envelope and contents from Susa, Iran, circa 3300 BCE. For instance, the large cone represented a very large measure of grain; the small **cones** designated small measures of grain. (Image Denise Schmandt-Besserat and Musée du Louvre, Département des Antiquités Orientales, Paris.)

Mesopotamian Writing

https://www.maa.org/press/periodicals/convergence/mathematical-treasure-mesopotamian-accounting-tokens

- According to Schmandt-Besserat, the transformation of three-dimensional tokens to two-dimensional signs to communicate information was **the beginning of writing in the Mesopotamia.**
- Eventually, the tokens were replaced by signs made by their impressions onto solid balls of clay, or tablets
- **The impressed signs evolved to become cuneiform writing.**

Impressed tablets from, Iran, circa 3200 BCE. Each circular impression stood for one large measure of grain; each wedge or conical impression designated a smaller measure of grain.(Image by by Denise Schmandt-Besserat and Musée du Louvre, Département des Antiquités Orientales, Paris.))

Writing seems to have been invented independently by *at least* by four societies: Mesopotamia, Egypt, China, Mesoamerica.

https://www.bl.uk/history-of-writing/articles/where-did-writing-begin#

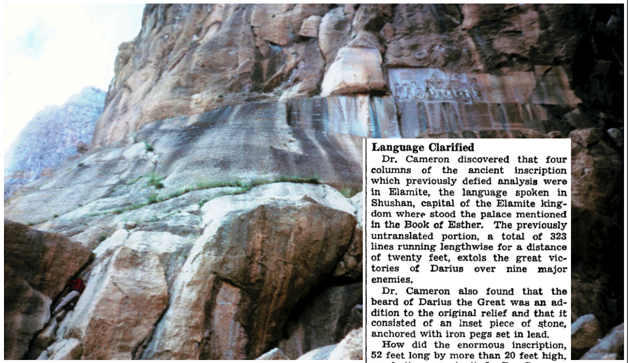
Behistun Inscription -the "cuneiform Rosetta stone-

Babylonian Rosetta

New Data Deciphered From the Monument of Darius the Great

On the face of precipitous Mount Behistun in Iran is the 2,500-year-old sculptured monument of Darius the Great, which has been likened to the famous Rosetta Stone of Egypt because it provides the key to the languages spoken in ancient Babylonia and Elam. The same story is told in three languages. One of these is ancient Persian, which was known. Hence it was possible to decipher the others.

The man who did the recent deciphering is Dr. George C. Cameron, who has been annual professor at the American School of Oriental Research at Baghdad. To decipher the inscription he had to make photographs and plastic impressions while he was suspended by steel cables from a mountain point 194 feet above the monument, which itself stands 225 feet above the ancient caravan route from Persia to Babylonia. There were times when the work had to be done in bitter cold, while snowstorms raged.



Language Clarified

Dr. Cameron discovered that four columns of the ancient inscription which previously defied analysis were in Elamite, the language spoken in Shushan, capital of the Elamite kingdom where stood the palace mentioned in the Book of Esther. The previously untranslated portion, a total of 323 lines running lengthwise for a distance of twenty feet, extols the great victories of Darius over nine major enemies.

Dr. Cameron also found that the beard of Darius the Great was an addition to the original relief and that it consisted of an inset piece of stone, anchored with iron pegs set in lead.

How did the enormous inscription, 52 feet long by more than 20 feet high, reach its present site? Dr. Cameron found the remains of an ancient staircase and evidence that sixty feet of it had been chiseled off the side of the mountain.

Darius wanted caravan voyagers to view the great work, but feared that some successor might destroy it. Evidently he knew that he was recording his deeds for posterity because a portion of the inscription says: "You who read my inscription even this, preserve it and do not conceal it. The god of Ahuramazda be thy friend, and thou shalt live long, and thou shalt have a big family."

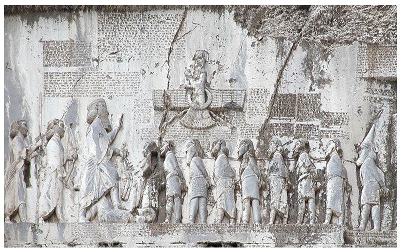
The New York Times
Published: March 27, 1949

See https://archive.org/details/sculpturesinscri00brituoft/page/n39/mode/2up for more information

Behistun Inscription
-the “cuneiform Rosetta stone-

written in three different cuneiform script languages: Old Persian, Elamite, and Babylonian (a variety of Akkadian).

By Unknown author - http://titus.fkidg1.uni-frankfurt.de/didact/idg/iran/apers/DB1_1-15.GIF, Public Domain, https://commons.wikimedia.org/w/index.php?curid=124383

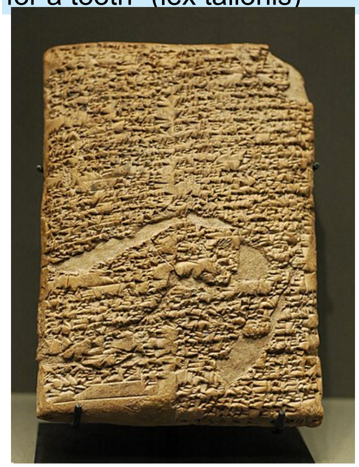


Punishment of captured impostors and conspirators: Gaumāta lies under the boot of Darius the Great.
Mount Behistun, Public domain, via Wikimedia Commons

Location of the Behistun complex

Code of Hammurabi about 1700 BC

"an eye for an eye, a tooth for a tooth" (lex talionis)



Epic of Gilgamesh

Ancient Assyrian statue currently in the Louvre, possibly representing Gilgamesh

Neo-Assyrian clay tablet. Epic of Gilgamesh, Tablet 11: Story of the Flood. Known as the "Flood Tablet" From the Library of Ashurbanipal, 7th century BC. British Museum

Mesopotamian Mathematics in a slide-nutshell

Overview of Mesopotamian Mathematics

Writing in Mesopotamia began about 3000 BCE (same as Egypt) with the needs of accountancy. Cuneiform (wedge shaped)

Cuneiform writing in clay tables. Many (many many) of them survived

Calendar

Mathematical tablets contain:

- Multiplication tables (by some numbers). The multiplication algorithm was likely to be similar to ours.
- Reciprocals tables
- Square roots tables
- **Pythagorean triples? Plimpton 322.**
- Rough work
- Problems - Verbal techniques

No tables of addition

- Scribes used **positional** number system with base 60 for computations(although they also used other number systems).
- No 0 - sometimes used a word to distinguish between, for instance, $(1)_{60}$ and $(1, 0)_{60}$.

Procedures to determine lengths, areas and volumes of many kinds of figures.

- The **defining component** of an equilateral triangle was a side and 7/8 was the **coefficient** for the height)
- The **defining component** of the circle was the circumference. The **coefficients** for
  - the diameter was $1/3=(0;20)_{60}$
  - the area $1/12= (0;5)_{60}$

- Algorithms to determine square roots.
- Solution of linear equations by false position (with one or two unknowns)
- Solution of certain quadratic equations (problems were often related to architecture and building)

positional! with no zero.

Educated guess: Answer one of the following: what do you think it means that

a. 7/8 is the coefficient for the height of an equilateral triangle?

b. 1/3 was the coefficient for the diameter of a circle?

c. 1/12 was the coefficient for the area of a circle?

Procedures to determine lengths, areas and volumes of many kinds of figures.

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Example of uses of mathematics

- It was an important task for the rulers of Mesopotamia to dig canals and to maintain them, because canals were not only necessary for irrigation but also useful for the transport of goods and armies.
- The rulers or high government officials must have ordered Babylonian mathematicians to calculate the number of workers and days necessary for the building of a canal, and to calculate the total expenses of wages of the workers.
- **There are several Old Babylonian mathematical texts in which various quantities concerning the digging of a canal are asked for.**

https://mathshistory.st-andrews.ac.uk/HistTopics/Babylonian_mathematics/

K Muroi, Small canal problems of Babylonian mathematics, Historia Sci. (2) 1 (3) (1992), 173-180.

Tables in Tablets

- One of the most amazing aspect of the Babylonian's calculating skills was their **construction of tables to aid calculation.** Two tablets which date from 2000 BC. give squares of the numbers up to 59 and cubes of the numbers up to 32.
- Babylonians used the formula

$$a.b = (1/2)((a+b)^2 - a^2 - b^2)$$

to make multiplication easier.

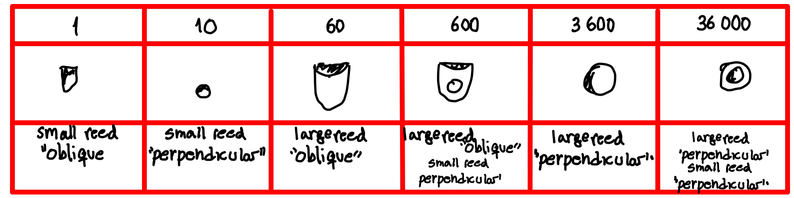
- The Babylonians did not have an algorithm for long division. Instead they based their method on the fact that

$$A \% B = A \times (1/B)$$

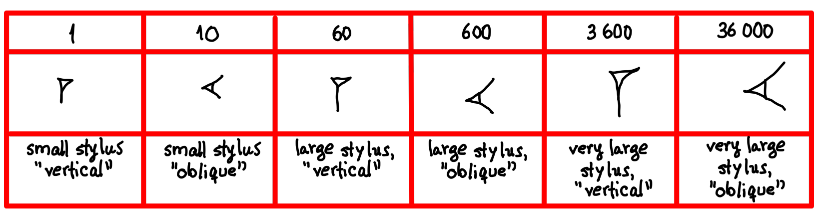
and used tables of reciprocals.

Numbers in Cuneiform

Early Sumerian Numerals



Later Sumerian Numerals



Base 60 was used in Mesopotamian mathematics because…

1. The approximate duration of the year is 60^2
2. 60 has many divisors (for instance, 2,3,4,5 and 6)
3. Of advantages for writing and calculating fractions

There are many theories trying to explain base 60

- Theon of Alexandria (~300CE): **60 is, among all the numbers the most convenient, because, being the smallest among all those which have the most divisors, it is the easiest to handle.**

  Can you imagine the Sumerians creating a committee to decide on their number base?

- Neugebauer (~1950CE): **a decimal counting system was modified to base 60 to allow for dividing weights and measures into thirds.**
- Several theories have been based on astronomical events.
  - **60 is the product of the number of months (moons) in the year with the number of planets** (Mercury, Venus, Mars, Jupiter, Saturn)
  - **The year was thought to have 360 days** was suggested as a reason for the number base of 60. *(However the Sumerians certainly knew that the year was longer than 360 days.)*
  - **The sun moves through its diameter 720 times during a day and, with 12 Sumerian hours in a day.** In that way, one can come up with 60.
- Some theories are based on geometry.
  - an equilateral triangle was considered the fundamental geometrical building block by the Sumerians. Now an angle of an equilateral triangle is 60° so if this were divided into 10, an angle of **6° would become the basic angular unit. Now there are sixty of these basic units in a circle** so again we have the proposed reason for choosing 60 as a base. *(it assumes 10 as the basic unit for division)*

https://mathshistory.st-andrews.ac.uk/HistTopics/Babylonian_numerals/

There are many theories trying to explain base 60

- **Sumerian civilization must have come about through the joining of two peoples, one of whom had base 12 for their counting and the other having base 5.** (5 was used as a number base among a few ancient civilizations). As the two peoples mixed and the two systems of counting were used by different members of the society trading with each other then base 60 would arise naturally as the system everyone understood.
- One of the nicest things about these theories is that it may be possible to find written evidence of the two mixing systems and thereby give what would essentially amount to a proof of the conjecture.

There are many theories trying to explain base 60

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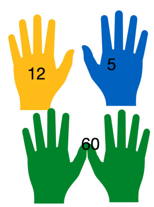
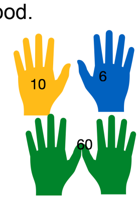


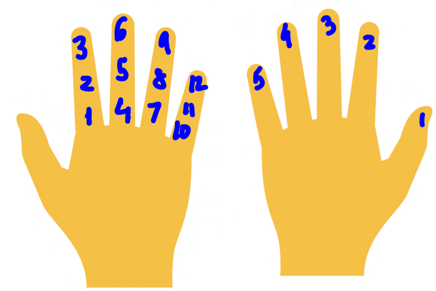


Two peoples who mixed to produce the Sumerians having 10 and 6 as their number bases. This version has the advantage that there is a natural unit for 10 in the Babylonian system which one could argue was a remnant of the earlier decimal system.

E. F. Robertson Suggested: **The reason has to involve the way that counting arose in the Sumerian civilization** (just as 10 became a base in other civilizations who began counting on their fingers, and twenty became a base for those who counted on both their fingers and toes.)

There are many theories trying to explain base 60

One can count up to 60 using your two hands. On your left hand there are three parts on each of four fingers (excluding the thumb). The parts are divided from each other by the joints in the fingers. Now **one can count up to 60 by pointing at one of the twelve parts of the fingers of the left hand with one of the five fingers of the right hand.** This gives a way of finger counting up to 60 rather than to 10.



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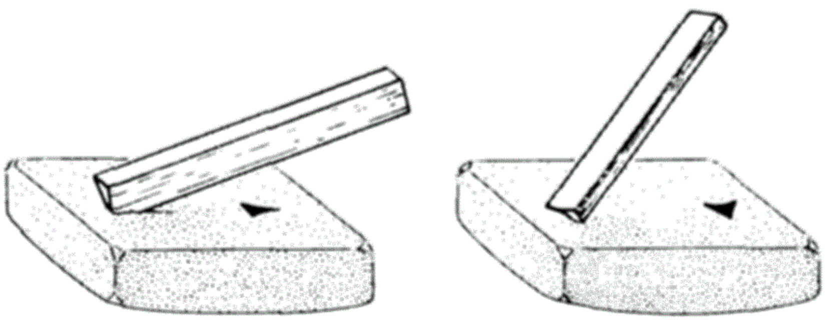
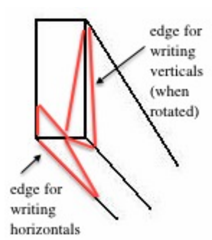
Babylonian Reciprocals Tablet 1900–1600 BCE
https://www.history-of-mathematics.org/artifacts/babylonian-reciprocals-tablet

Assume1in is 2.5cm (it is 2.54)
We could define a new unit
of measure a M336, of 0.5cm long. So
1in=5M336
1cm=2M336
A similar process could have
happened to the number system

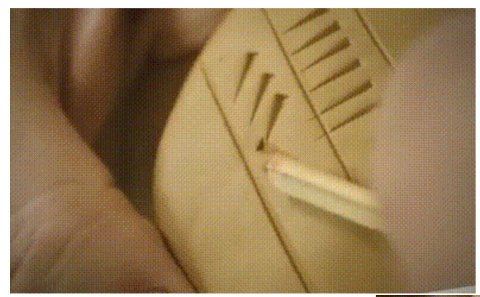
Writing cuneiform

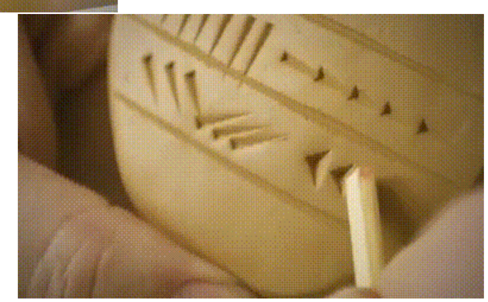


Impression of cuneiform symbols on clay tablets. (Redrawn from Neugebauer[3]



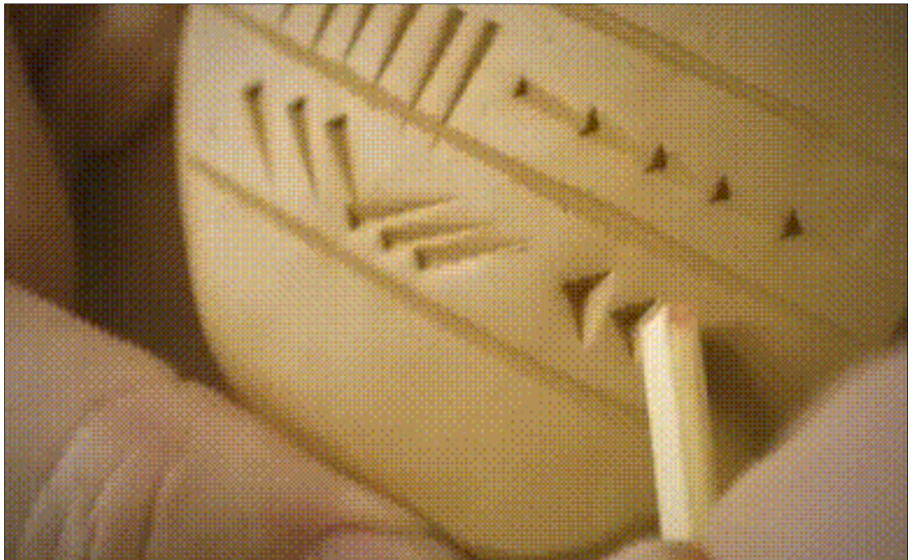
http://writingcuneiform.blogspot.com/
2012/10/5-making-basic-wedges.html





Cuneiform writing

Clips from https://cuneiform.neocities.org/CWT/Figures/2_keilschrift_hwc.gif



Cuneiform writing: How it was done

https://cuneiform.neocities.org/CWT/Figures/2_keilschrift_hwc.gif

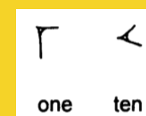
- Annotated bibliography due on 2/27.
- https://www.math.stonybrook.edu/~moira/courses/mat336-sp2024/bib.html
- Abstract and outline of the paper. Advise: write an outline for your presentation.

Fractions in base 60 - Review

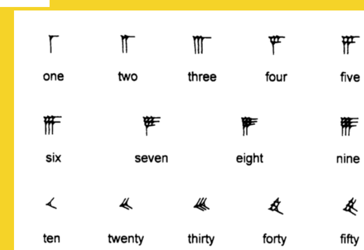
Express numbers a. and b. in Hindu-Arabic numerals

a.

b.



Recall that the symbol for one and the symbol for ten were combined to make the symbols for all numerals from one to fifty-nine.



Some sexagesimal numerals

Figures from Robson, Eleanor. "Counting in Cuneiform." Mathematics in school 27.4 (1998): 2-9

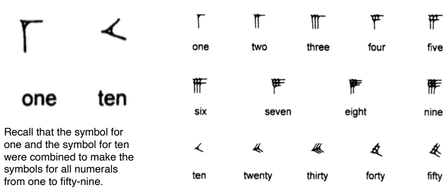
What are the following numbers?

45/60 = 3/4 or 0;45
45
45x60 = 2700 or 45 00

5/60 + 40/3600 0.944 or 0;05 40
5 + 40/60 = 5 2/3 or 5;40
5x60 + 40 = 340 or 5 40
5x3600 + 40x60 = 20,400 or 5 40 00

First and second numbers

Ways of making the medial zero across time



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Some sexagesimal numerals

Figures from Robson, Eleanor. "Counting in Cuneiform." Mathematics in school 27.4 (1998): 2-9

Express 1/20, 21/80 or 1/7 in cuneiform (answer in Wooclap in base 60)

$\frac{21}{80}$ Recall: We need $a_1, a_2, a_3 \ldots$ such that 1) a_i's are integers 2) $0 \le a_i < 60$, 3) $\frac{21}{80} = \frac{a_1}{60} + \frac{a_2}{60^2} + \frac{a_3}{60^3} + \ldots$

1) Note $0 < \frac{21}{80} < 1$.

2) $\frac{21}{80} \cdot 60 = \frac{63}{4} = 15 + \frac{3}{4}$

integer

This is a number larger or equal to 0 and smaller than 1.

3) Also $\frac{21}{80} \cdot 60 = a_1 + \frac{a_2}{60} + \frac{a_3}{60^2} + \ldots$

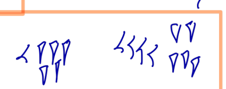
Then $a_1 = 15$

$\frac{3}{4} = \frac{a_2}{60} + \frac{a_3}{60^2} + \ldots$

4) $\frac{3}{4} \cdot 60 = 45$. Then $a_2 = 45$

$\frac{21}{80} = (0; 15; 45)_{60}$

→ In cuneiform



(Remember: There was no decimal period in Ancient Mesopotamia

Consider a number a between 0 and 1. We want to write a in base 60.
That is, we want to find numbers $c_1, c_2, c_3 \ldots$ such that

1) Each c_i is an integer
2) $0 \le c_i < 60$
3) $a = \frac{c_1}{60} + \frac{c_2}{60^2} + \frac{c_3}{60^3} + \cdots$

$c_1 = \lfloor 60 \cdot a \rfloor$ where $\lfloor x \rfloor$ denotes the largest integer smaller or equal than x.

$a_1 = 60a - c_1$

$c_2 = \lfloor 60 a_1 \rfloor$

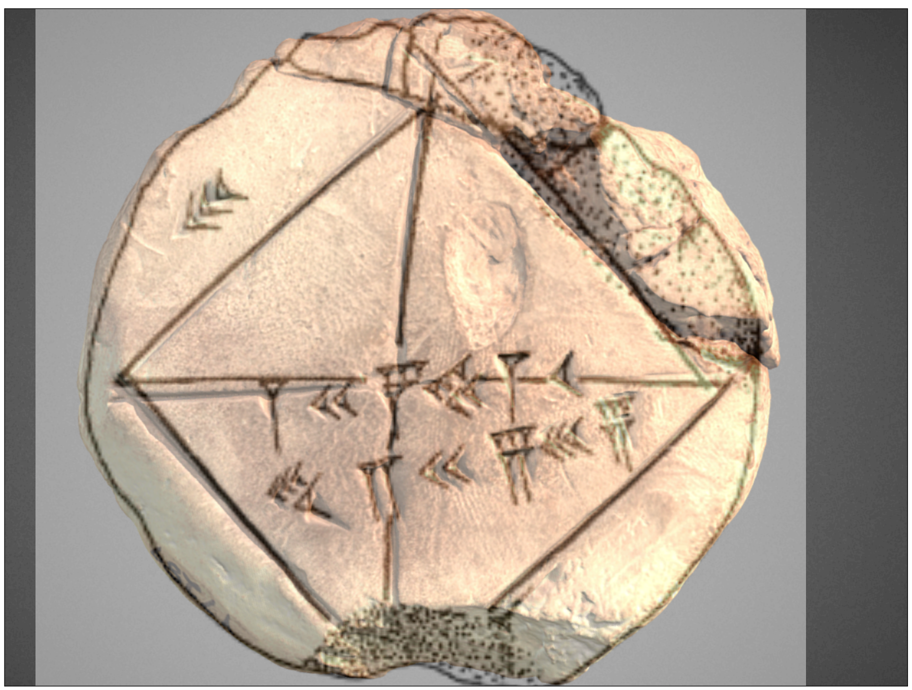
$a_2 = 60 a_1 - c_2$

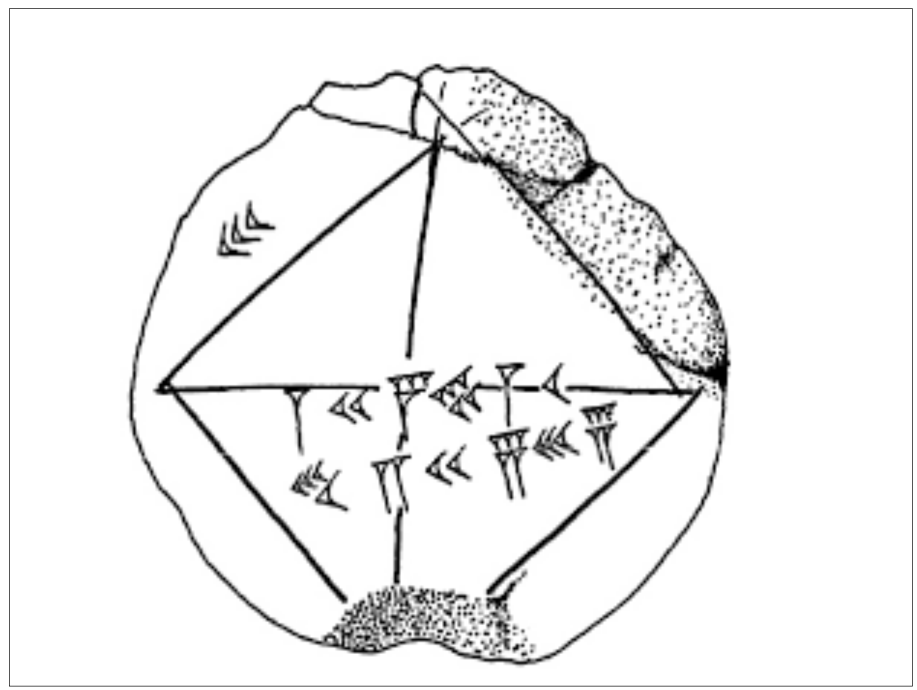
And so on $\left(\begin{matrix} c_n = \lfloor 60 a_{n-1} \rfloor \\ a_n = 60 a_{n-1} - c_n \end{matrix} \right)$

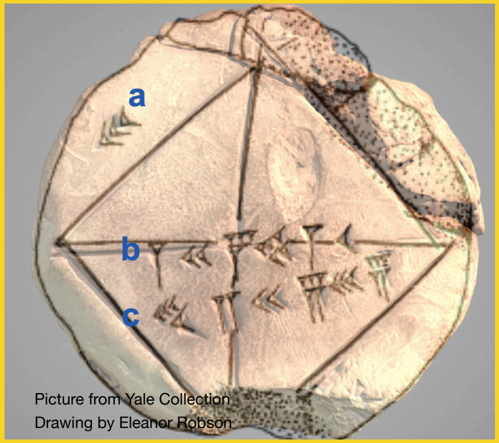
If we reach i such that $a_i = 0$ the process ends.

Review

A school exercise, most likely.







Picture from Yale Collection
Drawing by Eleanor Robson

1. What are the values of a, b and c?
2. What is do you think the purpose of this tablet?

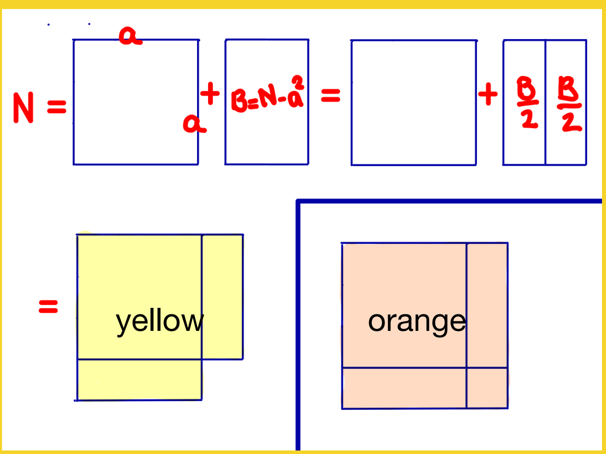
√2 tablet

- YBC 7289 is an Old Babylonian clay tablet (circa **1800–1600 BCE**) from the Yale Babylonian Collection.
- Appears to be a **practice school exercise** undertaken by a novice scribe.
- Contains not only a constructed illustration of a geometric square with intersecting diagonals, but also, in its text, a numerical estimate of √2 correct to three sexagesimal or six decimal places.
- The value demonstrates one of the **greatest known computational accuracy** obtained anywhere in the ancient world.
- It is believed that the tablet's **author copied** some of the results from an existing table of values and did not compute them himself.


Picture from Yale Collection
Drawing by Eleanor Robson

A digital replica to hold in your digital hands

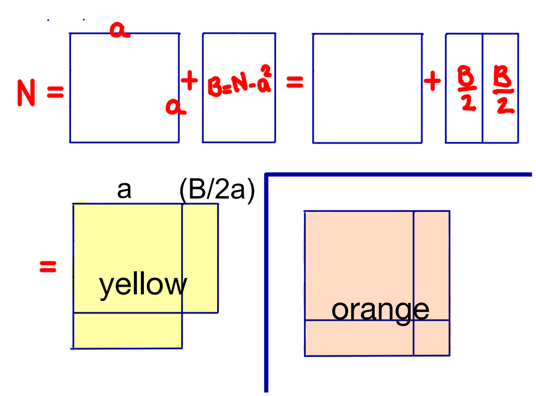
Babylonian approximation of the square root of a number N, starting from a number a such that $a^2<N$



What is the area of the yellow figure? What is the area of the orange figure? (Express both areas in terms of N, a, and B)

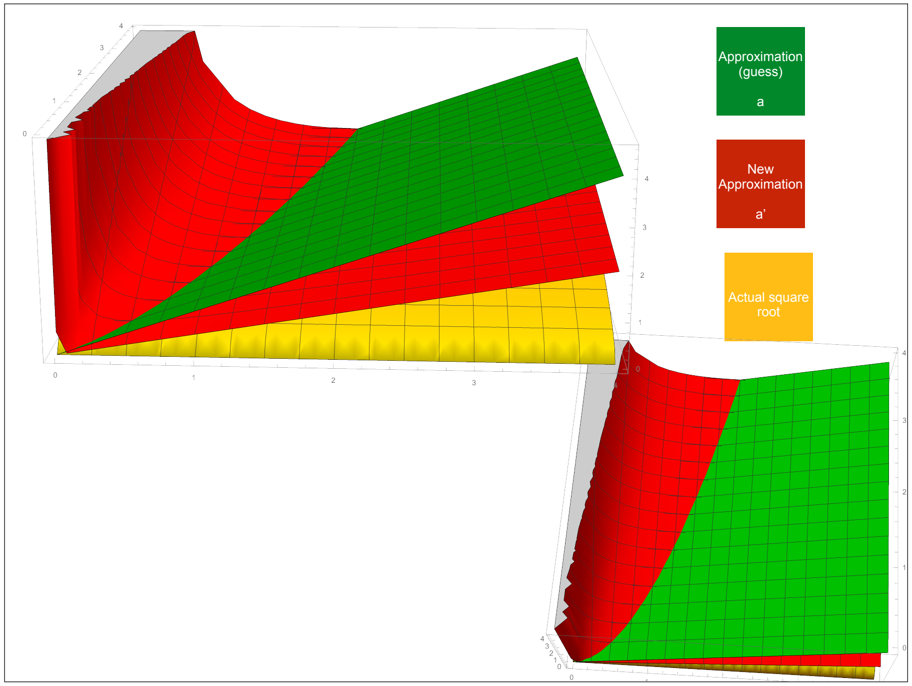
One starts with a rectangle of area N, with a side of length a.

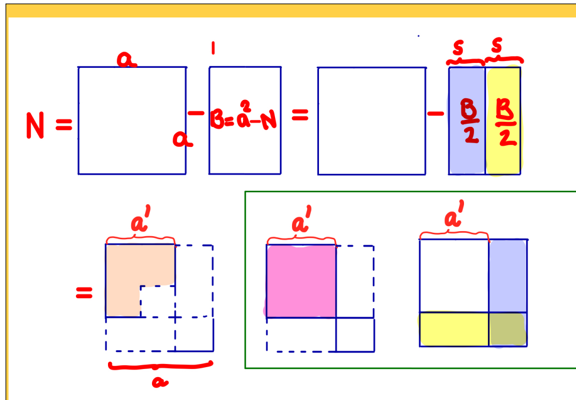
Babylonian approximation of the square root of a number N, starting from a number a such that $a^2<N$. One starts with a rectangle of area N, with a side of length a.



- The goal is to find a square of area as close to N as possible, so the side of this square will be close to √N.
- The orange square is what we found. Its area is larger than that of N.
- If $(B/a)< a$ then $B = N-a^2 > (B/(2a))^2$

The new approximation is
$a' = a+B/(2a) = a+(N-a^2)/(2a)$

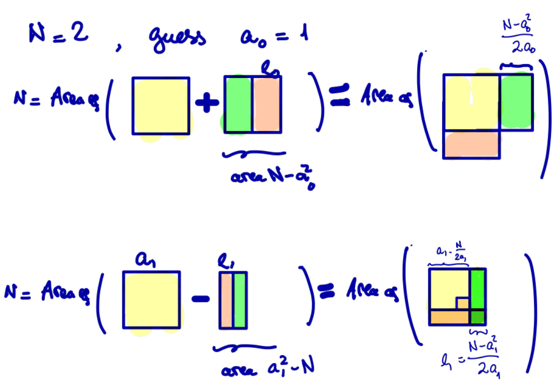




Babylonian method to find square roots

Given **N** and **a** such that:
- N>0, a>0
- $a^2>N$
- a^2 is close to N (thus a is close to $\sqrt{N}$)
- Hint: Write **s** in terms of **N** and **a**

Following the figures, find a' such that (in terms of a and N) such that $(a')^2$ is even closer N than a^2 (so a' is even closer than a to $\sqrt{N}$)



$a_i = a_{i-1}+l_{i-1}$	$\lvert N-a_i^2\rvert$	$l_i = \frac{\lvert N-a_i^2\rvert}{2a_i}$
1	1	1/2
3/2	1/4	1/12
17/12	1/144	1/408
577/408	-1/166464	

$$\frac{3}{2} = 2 - \frac{1}{2}$$

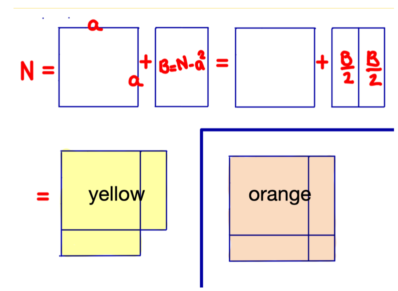
$$\frac{17}{12} = \frac{3}{2} - \frac{1/4}{2\cdot\frac{3}{2}} = \frac{3}{2} - \frac{1}{12}$$

$$\frac{577}{408} = \frac{17}{12} - \frac{1/144}{2\cdot\frac{17}{12}}$$

$$2 - \left(\frac{577}{408}\right)^2 = -\frac{1}{166464} \approx 6\cdot 10^{-6}$$

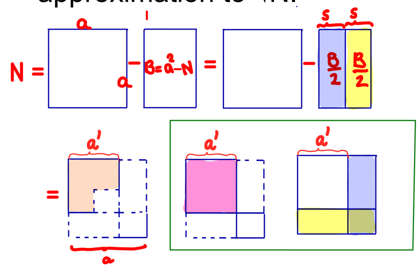
Babylonian approximation of the square root of a number N when $a^2 < N$

1. Find a such that a^2 is close to N and $a^2 < N$
2. $N=a^2 + (N - a^2)= a^2 + B$
3. $N=a^2 + B$ is area of the "yellow"shape.
4. Since a^2 is close to N, the area of the yellow shape is close to the area of the orange square.
5. Hence, the square root of N is close to the square root of the area of the orange square.
6. The square root of the area of the orange square is equal to the length of its side.
7. The length of the side of the orange square is a+B/(2a). Thus, the a+B/(2a) is the new approximation to $\sqrt{N}$.



Babylonian approximation of the square root of a number N when $a^2 > N$

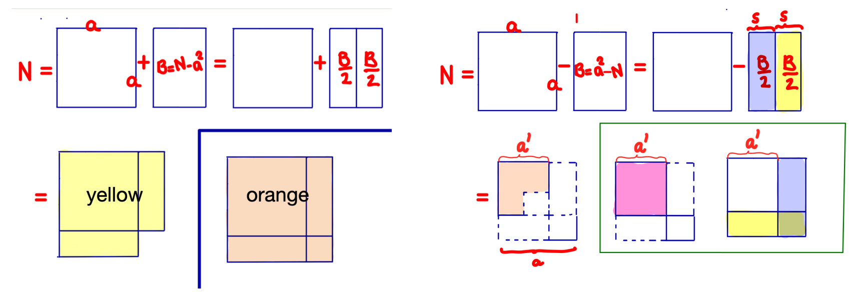
1. Find a such that a^2 is close to N and $a^2 > N$
2. $N = a^2 - (a^2 - N) = a^2 - B$
3. $N = a^2 - B$ is area of the "orange" shape.
4. The area of the orange shape is close to the area of the pink square (the difference is a square smaller than B)
5. Hence, the square root of N is close to the square root of the area of the orange square.
6. The square root of the area of the pink square is equal to the length of its side, a'.
7. The length of the side of the orange square is a-B/(2a). Thus, the a-B/(2a) is the new approximation to √N.



The new approximation is
$a' = a-(a^2-N)/(2a) = a+(N-a^2)/(2a)$

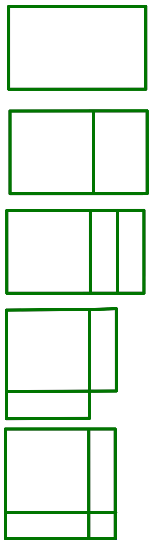
Babylonian approximation of the square root of a number N

Given a guess a, one finds a new guess $a' = a + (N-a^2)/(2a)$.
In both cases (whether we start with $a^2>N$ or $a^2<N$.



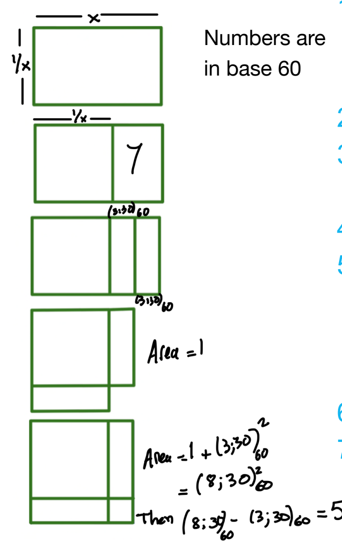
A number and its reciprocal differ in 7. What is the number?

A reciprocal exceeds its reciprocal by 7. What are the reciprocal and its reciprocal?



1. You: break in half the 7 by which the reciprocal exceeds its reciprocal, and 3;30 (will come up).
2. Multiply 3;30 by 3;30 and 12;15 (will come up).
3. Append [1 00, the area,] to the 12;15 which came up for you and 1 12;15 (will come up).
4. What is [the square-side of 1] 12;15? 8;30.
5. Put down [t8;30 and] 8;30, its equivalent, and subtract 3;30, the takiltum-square, from one (of them); append (3;30) to one (of them).
6. One is 12, the other is 5.
7. The reciprocal is 12, its reciprocal 5.

A reciprocal exceeds its reciprocal by 7. What are the reciprocal and its reciprocal?



Numbers are in base 60

1. You: break in half the 7 by which the reciprocal exceeds its reciprocal, and 3;30 (will come up).
2. Multiply 3;30 by 3;30 and get 12;15.
3. Append [1 00, the area,] to the 12;15 which came up for you and (1 12;15 (will come up).
4. What is [the square-side of 1] 12;15? 8;30.
5. Put down [8;30 and] 8;30, its equivalent, and subtract 3;30, the takiltum-square, from one (of them); append (3;30) to one (of them).
6. One is 12, the other is 5.
7. The reciprocal is 12, its reciprocal 5.

$u=(3;30)=(1/2)(x-1/x)$ $v^2=1+u^2$

$v=(8;30)=(1/2)(x+1/x)$ Answer $v+1$, $v-1$

Note:

Suppose x and y are reciprocal pairs (that is, x.y=1) and set

$$a=1$$

$$b=1+((x-y)/2)^2$$

$$c=1+((x+y)/2)^2$$

Then a, b and c are a Pythagorean triple, that is

$$a^2+b^2=c^2.$$

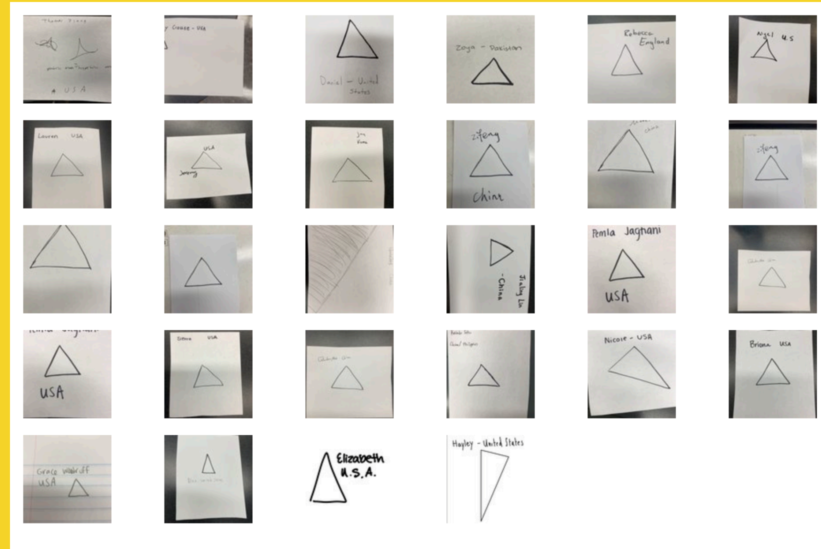
Draw a triangle

•Draw a triangle and your first name, and take a photo.

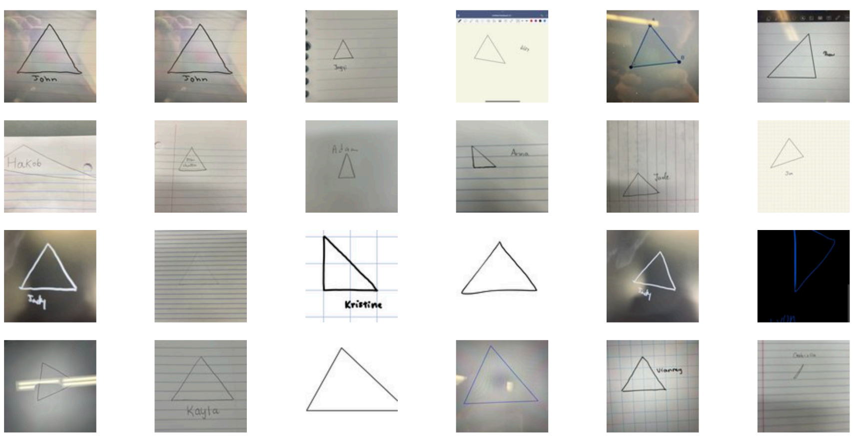
•Drag the photo to https://www.yogile.com/mat336/upload (the QR code is for that website) or mat336@yogile.com

•Note: You do not need to sign in to Yogile.

List features common to most drawings (besides the obvious fact that they are triangles)

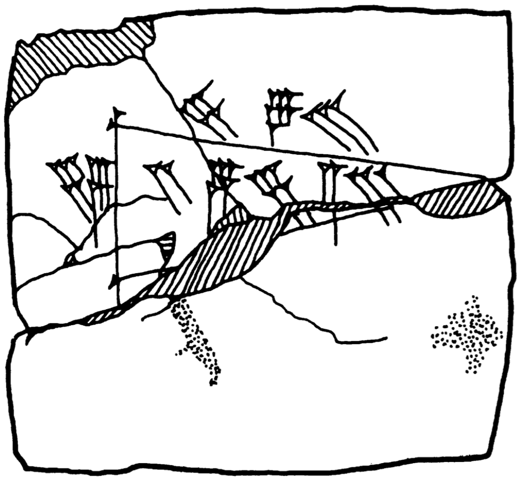


Fall 2023



List features common to most drawings (besides the obvious fact that they are triangles)

A Babylonian typical triangle



M 29-15-709 (obverse). Drawing by the Eleanor Robson - Words and Pictures: New Light on Plimpton 322 - The American Mathematical Monthly , Feb., 2002, Vol. 109, No. 2 (Feb., 2002), pp. 105-120

Plimpton 322

https://www.nytimes.com/2010/11/23/science/23babylon.html?smid=url-share

Plimpton 322

An Exhibition That Gets to the (Square) Root of Sumerian Math - NYTimes - Nov 22, 2010

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- Surviving correspondence shows that he bought the tablet for $10 from a dealer called Edgar J. Banks.
- Banks told him it came from an archaeological site called Senkereh in southern Iraq, whose ancient name was Larsa
- Approximate date of the tablet: 1800 BCE.

ta-k]i *ša* 1 *in*]	- *il- ti ṣi - li - ip* - *tim* -*na-as-sà-hu-ú-ma* sag *i-il-lu-ú*	íb-si8 sag	íb-si8 *ṣi-li-ip-tim*	mu-bi-im
1 59	15	1 59	2 49	ki 1
1 56 56	58 14 *56* 15	56 7	*3* *12* *1*	ki 2
1 55 7	41 15 33 45	1 16 41	1 50 49	ki 3
1 53 10	29 32 52 16	3 31 49	5 9 1	ki 4
1 48 54	1 40	1 5	1 37	ki 5
1 47 6	41 40	5 19	8 1	ki 6
1 43 11	56 28 26 40	38 11	59 1	ki 7
1 41 33	*59* 3 45	13 19	20 49	ki 8
1 38 33	36 36	*9* 1	12 49	ki 9
1 35 10	2 28 27 24 26 40	1 22 41	2 16 1	ki 10
1 33 45		45	1 15	ki 11
1 29 21	54 2 15	27 59	48 49	ki 12
1 27	3 45	*7* *12* *1*	4 49	ki 13
1 25 48	51 35 6 40	29 31	53 49	ki 14
1 23 13	46 40	*56*	53	ki 15

Plimpton 322: a review and a different perspective
Author(s): John P. Britton, Christine Proust and Steve Shnider
Source: Archive for History of Exact Sciences , September 2011, Vol. 65, No. 5 (September 2011), pp. 519-566

Errors are italicized and underlined
In dark gray, unreadable numbers

Plimpton 322 in “our” number system

(d/l)^2 or (s/l)^2	Short side s	Diagonal d	Row
(1).9834028	119	169	1
(1).9491586	3367	4825	2
(1).9188021	4601	6649	3
(1).8862479	12709	18541	4
(1).8150077	65	97	5
(1).7851929	319	481	6
(1).7199837	2291	3541	7
(1).6927094	799	1249	8
(1).6426694	481	769	9
(1).5861226	4961	8161	10
(1).5625	45	75	11
(1).4894168	1679	2929	12
(1).4500174	161	289	13
(1).4302388	1771	3229	14
(1).3871605	56	106	15

Plimpton 322 in “our” number system. Do you see pattern in the numbers of the last column?

$(d/l)^2$ or $(s/l)^2$	Short side s	Diagonal d	Row	d^2-s^2
(1).9834028	119	169	1	14400
(1).9491586	3367	4825	2	11943936
(1).9188021	4601	6649	3	23040000
(1).8862479	12709	18541	4	182250000
(1).8150077	65	97	5	5184
(1).7851929	319	481	6	129600
(1).7199837	2291	3541	7	7290000
(1).6927094	799	1249	8	921600
(1).6426694	481	769	9	360000
(1).5861226	4961	8161	10	41990400
(1).5625	45	75	11	3600
(1).4894168	1679	2929	12	5760000
(1).4500174	161	289	13	57600
(1).4302388	1771	3229	14	7290000
(1).3871605	56	106	15	8100

Plimpton 322 in “our” number system

$(d/l)^2$ or $(s/l)^2$	Short side s	Diagonal d	Row	d^2-s^2	$(d^2-s^2)^{1/2}$
(1).9834028	119	169	1	14400	120
(1).9491586	3367	4825	2	11943936	3456
(1).9188021	4601	6649	3	23040000	4800
(1).8862479	12709	18541	4	182250000	13500
(1).8150077	65	97	5	5184	72
(1).7851929	319	481	6	129600	360
(1).7199837	2291	3541	7	7290000	2700
(1).6927094	799	1249	8	921600	960
(1).6426694	481	769	9	360000	600
(1).5861226	4961	8161	10	41990400	6480
(1).5625	45	75	11	3600	60
(1).4894168	1679	2929	12	5760000	2400
(1).4500174	161	289	13	57600	240
(1).4302388	1771	3229	14	7290000	2700
(1).3871605	56	106	15	8100	90

Plimpton 322 in “our” number system

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(1).9834028	119	169	1	14400	120
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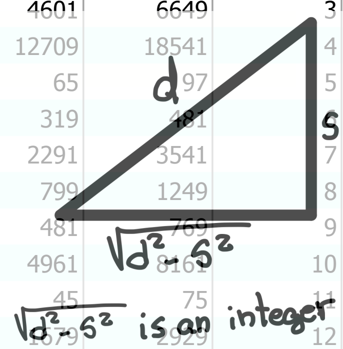

What do you think Plimpton 322 is about and why?

Three interpretations by scholars.

Trigonometric table: if Columns L and D contain the Legs and Diagonals of right-triangles, then the values in the first column are $\tan^2$ or $1/\cos^2$
The acute angles of the triangles decrease by approximately 1°

Pythagorean triples (that is integer numbers A, L, D such that $A^2+L^2=D^2$)
In this case, entries are generated by pairs (p, q), with no common divisor, not both odd and such that p>q.
L = 2pq,
$D=p^2+q^2$
The remaining leg is p^2-q^2

List of possible problems for students (using reciprocals of regular numbers)

Trigonometric table and pythagorean triples interpretations

Pythagorean triples (that is integer numbers A, L, D such that $A^2+L^2=D^2$)
In this case, entries are generated by pairs (p, q), with no common divisor, not both odd and such that p>q.
L = 2pq,
$D=p^2+q^2$
The remaining leg is p^2-q^2

line	α	*p*	*q*
1	44.76°	12	5
2	44.25°	1 04	27
3	43.79°	1 15	32
4	43.27°	2 05	54
5	42.08°	9	4
6	41.54°	20	9
7	40.32°	54	25
8	39.77°	32	15
9	38.72°	25	12
10	37.44°	1 21	40
11	36.87°	2	1
12	34.98°	48	25
13	33.86°	15	8
14	33.26°	50	27
15	31.89°	9	5

Eleanor Robson, Words and Pictures: New Light on Plimpton 322, The American Mathematical Monthly , Feb., 2002, Vol. 109, No. 2 (Feb., 2002), pp. 105-120

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List of possible problems for students (using reciprocals of regular numbers)

TABLE 6
From Reciprocal Pairs to Plimpton 322 Entries

x	$1/x$	$(x-1/x)/2$	$(x+1/x)/2$	$\{(x+1/x)/2\}^2$	Short side	Diagonal	Long side	Line
2;24	0;25	0;59 30	1;24 30	1;59 00 15	1 59	2 49	2 00	1
2;22 13 20	0;25 18 45	0;58 27 17 30	1;23 46 02 30	1;56 56 58 14 50 06 15	56 07	1 20 25	57 36	2
2;20 37 30	0;25 36	0;57 30 45	1;23 06 45	1;55 07 41 15 33 45	1 16 41	1 50 49	1 20 00	3
2;18 53 20	0;25 55 12	0;56 29 04	1;22 24 16	1;53 10 29 32 52 16	3 31 49	5 09 01	3 45 00	4
2;15	0;26 40	0;54 10	1;20 50	1;48 54 01 40	1 05	1 37	1 12	5
2;13 20	0;27	0;53 10	1;20 10	1;47 06 41 40	5 19	8 01	6 00	6
2;09 36	0;27 46 40	0;50 54 40	1;18 41 20	1;43 11 56 28 26 40	38 11	59 01	45 00	7
2;08	0;28 07 30	0;49 56 15	1;18 03 45	1;41 33 45 14 03 45	13 19	20 49	16 00	8
2;05	0;28 48	0;48 06	1;16 54	1;38 33 36 36	8 01	12 49	10 00	9
2;01 30	0;29 37 46 40	0;45 56 06 40	1;15 33 53 20	1;35 10 02 28 27 24 26 40	1 22 41	2 16 01	1 48 00	10
2	0;30	0;45	1;15	1;33 45	45	1 15	1 00	11
1;55 12	0;31 15	0;41 58 30	1;13 13 30	1;29 21 54 02 15	27 59	48 49	40 00	12
1;52 30	0;32	0;40 15	1;12 15	1;27 00 03 45	2 41	4 49	4 00	13
1;51 06 40	0;32 24	0;39 21 20	1;11 45 20	1;25 48 51 35 06 40	29 31	53 49	45 00	14
1;48	0;33 20	0;33 20	1;10 40	1;23 13 46 40	28	53	45	15

ELEANOR ROBSON

Eleanor Robson, New light on Plimpton

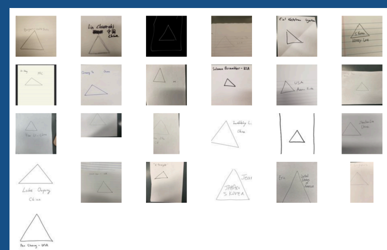
On balance, then, Plimpton 322 was probably (but not certainly!) a good copy of a teachers' list, with two or three columns, now missing, containing starting parameters for a set of problems, one or two columns with intermediate results (Column I and perhaps a missing column to its left), and two columns with final results (II–III). All that remains is for us to decide what the problem type might have been. Eleanor Robson

Robson's reasons for stating that the numbers in Plimpton 322 were not thought as Pythagorean triples

- Use of reciprocals
- Use of regular numbers
- Study of mistakes in the table
- Place of the table
- No similar table.
- No notion of angle

*Ancient mathematical texts and artefacts, if we are to understand them fully, **must be viewed in the light of their mathematico-historical context**, and not treated as artificial, self-contained creations in the style of detective stories.*

Eleanor Robson



https://www.nytimes.com/2010/11/23/science/23babylon.html?smid=url-share

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(1).6927094	799	1249	8	921600	960
(1).6426694	481	769	9	360000	600
(1).5861226	4961	8161	10	41990400	6480
(1).5625	45	75	11	3600	60
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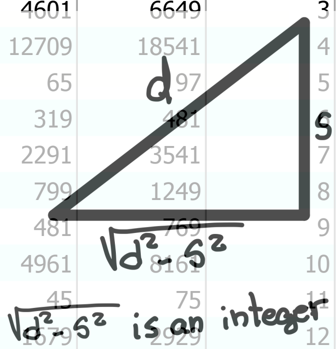
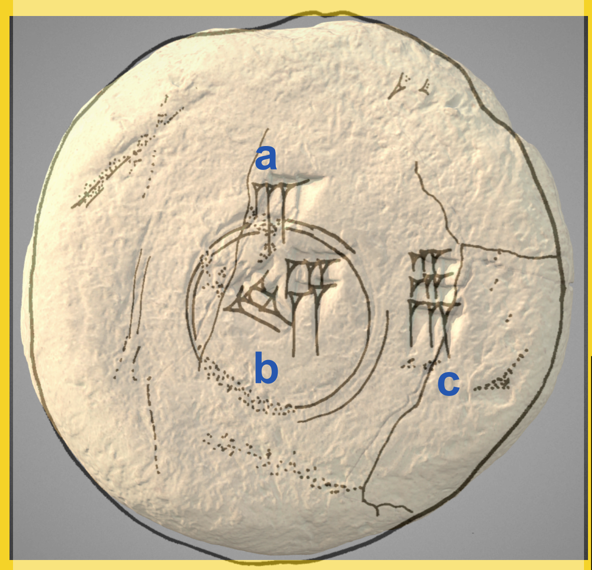


Calculation of Areas



Picture from Yale Collection

Drawing by Eleanor Robson The American Mathematical Monthly , Feb., 2002, Vol. 109, No. 2 (Feb., 2002), pp. 105-120)

What are the values of a, b and c? What do these numbers represent? What is the purpose of this tablet?

Hint: To find the purpose, consider the arrangement of the numbers with respect to the circle.

Recall

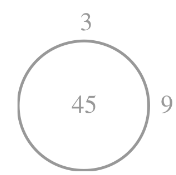
The formula for the area A of a circle in terms of the radius r is $\pi . r^2$.

The formula for the circumference c of a circle in terms of the radius r is $2.\pi.r$.

Find the formula of the area of the circle in terms of the circumference and π.

Picture from Yale Collection

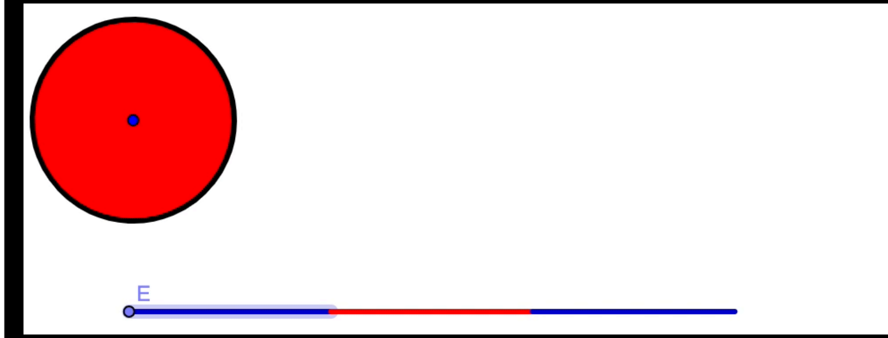
Drawing by Eleanor Robson



Procedures to determine lengths, areas and volumes of many kinds of figures.

- The **defining component** of an equilateral triangle was a side and 7/8 was the **coefficient** for the height)
- The **defining component** of the circle was the circumference. The **coefficients** for
  - the diameter was $1/3=(0;20)_{60}$
  - the area $1/12= (0;5)_{60}$

How to find (or approximate) π
in a desert island.

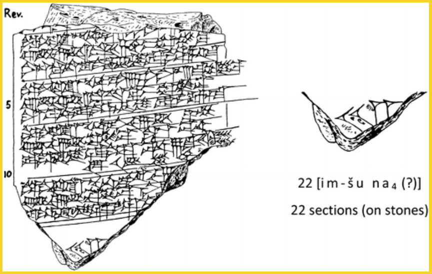


A trapezoid

Solutions of equations

Write an equation whose solution will be the answer to the problem below

Do you think this is an actual practical problem? Have you seen aa problem like this before? Can you suggest what the tablet might have been for?



Colophon of Tablet C4: end of the reverse (Neugebauer/Sachs 1945, Plate 13)

- I found a stone, (but) did not weigh it;
- (after) I subtracted one-seventh,
- added one-eleventh,
- (and) subtracted one-thir[teenth],
- I weighed (it): 1 ma-na.
- What was the origin(al weight) of the stone?

The origin(al weight)] of the stone was 1 ma–na, 9½ gin, (and) 2½ se.

- 60 gin = 1 ma-na
- 180 se = 1 gin

Educated guess: This problem is giving the instructions to solve a certain kind of equations. Which kind? (Linear, quadratic cubic… one unknown, two three…)

Numbers are in base 60

1. I summed the area and my square-side so that it was 0;45.
2. You put down 1, the projection.
3. You break off half of 1.
4. You combine 0;30 and 0;30.
5. You add 0;15 to 0;45.
6. 1 squares 1.
7. You take away 0;30 which you combined from inside 1 so that the square-side is 0;30.

Translation by Eleanor Robson

How was this formula found?
Conjecture: By completing the square

1. I summed the area and my square-side so that it was 0;45.
2. You put down 1, the projection.
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6. 1 squares 1.
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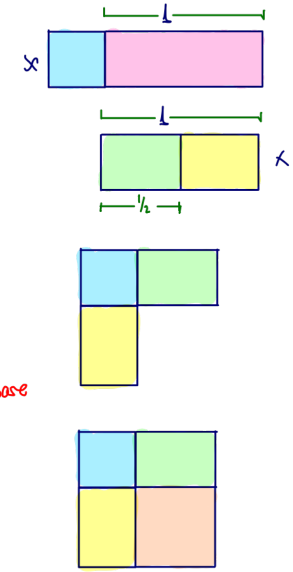
Set x = length of the side of the square

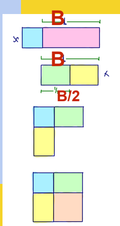


Problem from a tablet

Write down u, v and w in terms of B and C

1. I summed B^2 and my **B times** square-side so that it was **C.**
2. You put down **B**, the projection.
3. You break off half of **B**.
4. You combine (**B/2**) and (**B/2**).
5. You add $(B/2)^2$ to **C.**
6. $\sqrt{(B/2)^2+C}$ squares $(B/2)^2+C$.
7. You take away **…u…** which you combined from inside **… v…** so that the square-side is **…w…**

Translation by Eleanor Robson



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Translation by Eleanor Robson

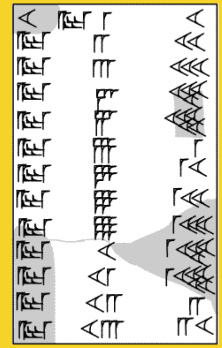
Tablets in Cuneiform Deciphering and finding meaning

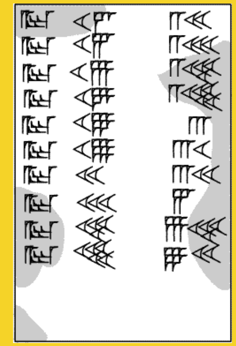
Photographs by Bill Casselman, Peter Damerow, Jöran Friberg.

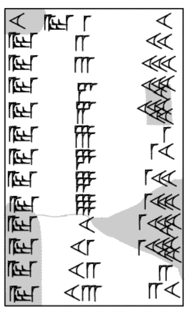
Deciphering and finding meaning. What s this table about?

Ask for help if you need it

Please do not use any other tools (except your mind and writing instruments)







Decipher tablet

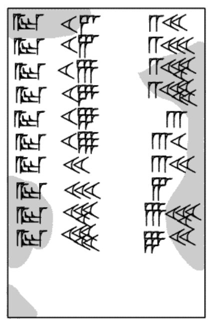
The first line (partially illegible here) reads

10 a-rà(times) 1 10.

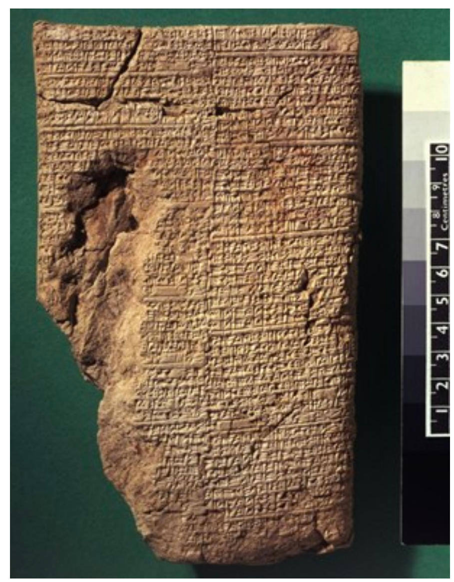
From line 2 on, in this format, the multiplier is left out:

a-rà 2 20

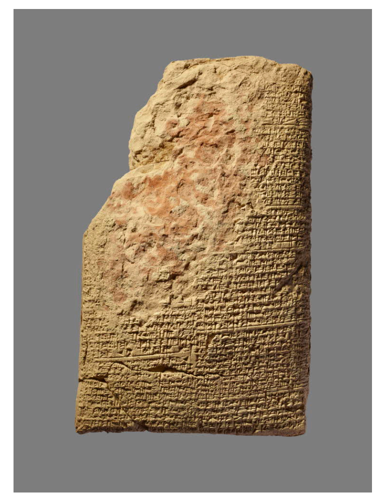
a-rà 3 30…



http://www.ams.org/publicoutreach/feature-column/fc-2012-05



Tablet 13901
British Museum
complete the square problems



A Babylonian Table of ??

n	1/n
2	0.5
3	0.333333333333333
4	0.25
5	0.2
6	0.166666666666667
7	0.142857142857143
8	0.125
9	0.111111111111111
10	0.1
11	0.0909090909090909
12	0.0833333333333333
13	0.0769230769230769
14	0.0714285714285714
15	0.0666666666666667
16	0.0625
17	0.0588235294117647
18	0.0555555555555556
19	0.0526315789473684
20	0.05
21	0.0476190476190476

Describe the numbers n such the 1/n has a finite decimal development (here you have some approximations that might help.)

n	1/n
22	0.0454545454545455
23	0.0434782608695652
24	0.0416666666666667
25	0.04
26	0.0384615384615385
27	0.037037037037037
28	0.0357142857142857
29	0.0344827586206897
30	0.0333333333333333
31	0.032258064516129
32	0.03125
33	0.0303030303030303
34	0.0294117647058824
35	0.0285714285714286
36	0.0277777777777778
37	0.027027027027027
38	0.0263157894736842
39	0.0256410256410256
40	0.025
41	0.024390243902439

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37	0.027027027027027
38	0.0263157894736842
39	0.0256410256410256
40	0.025
41	0.024390243902439

Consider the numbers n such the 1/n has a finite decimal development.

n	1/n	C= 1/n with integer fraction separator removed	C.n
2	0.5	5	
4	0.25	25	
5	0.2	2	
8	0.125	125	
10	0.1	1	
16	0.0625	625	
20	0.05	5	
25	0.04	4	
32	0.03125	3125	
40	0.025	25	
50	0.02	2	

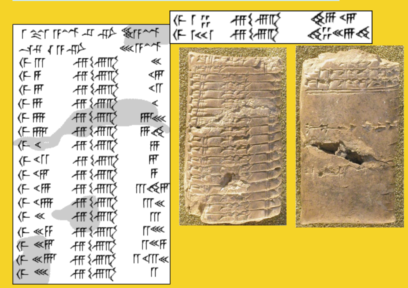
Integer fraction separator is the decimal point

2	30
3	20
4	15
5	12
6	10
8	7,30
9	6,40
10	6
12	5
15	4
16	3,45
18	3,20
20	3
24	2,30
25	2,24

27	2,13,20
30	2
32	1,52,30
36	1,40
40	1,30
45	1,20
48	1,15
50	1,12
54	1,6,40
1	1
1,4	56,15
1,12	50
1,15	48
1,20	45
1,21	44,26,40

What do you think this table is about? Can you find a pattern?

Notation:
- the value within each sexagesimal place is represented in Hindu-Arabic (our) numerals,
- places are separated by commas



Please do not use any other tools (except your mind, writing instruments and a calculator if you *really* need it) https://docs.google.com/spreadsheets/d/1i5OZZAiXEUIHJgR098cY5ItHyl_dpidTlx1IPt9eXTg/edit?usp=sharing

n	B=1/n in sexagesimal with integer-fraction separator removed	n	B decimal	n.B
2	30	2	30	60
3	20	3	20	60
4	15	4	15	60
5	12	5	12	60
6	10	6	10	60
8	7,30	8	450	3600
9	6,40	9	400	3600
10	6	10	6	60
12	5	12	5	60
15	4	15	4	60
16	3,45	16	225	3600
18	3,20	18	200	3600
20	3	20	3	60
24	2,30	24	150	3600
25	2,24	25	144	3600
27	2,13,20	27	8000	216000
30	2	30	2	60
32	1,52,30	32	6750	216000
36	1,40	36	100	3600

n	60^n
0	1
1	60
2	3600
3	216000
4	12960000

n	1/n	C(n)	n*C(n)
2	0.5	5	10
4	0.25	25	100
5	0.2	2	10
8	0.125	125	1000
10	0.1	1	10
16	0.0625	625	10000
20	0.05	5	100
25	0.04	4	100

Table 1

A	B	A in Dec.	B in Dec	A.B
2	30	2	30	60
3	20	3	20	60
4	15	4	15	60
5	12	5	12	60
6	10	6	10	60
8	7,30	8	450	3600
9	6,40	9	400	3600
10	6	10	6	60
12	5	12	5	60
15	4	15	4	60
16	3,45	16	225	3600
18	3,20	18	200	3600
20	3	20	3	60
24	2,30	24	150	3600
25	2,24	25	144	3600
27	2,13,20	27	8000	216000
30	2	30	2	60
32	1,52,30	32	6750	216000
36	1,40	36	100	3600
40	1,30	40	90	3600
45	1,20	45	80	3600
48	1,15	48	75	3600

n	60^n
1	60
2	3600
3	216000
4	12960000

Calendar

Fragment of a circular clay tablet with depictions of constellations (planisphere). Neo-Assyrian. - British Museum

Concrete impact:
360 degrees angle
60 minutes in an hour
60 seconds in a minute

Impact

Every culture has mathematics, but some have more than others. The cuneiform cultures of the pre-Islamic Middle East left a particularly rich mathematical heritage, some of which profoundly influenced late Classical and medieval Arabic traditions, but which was for the most part lost in antiquity and has begun to be recovered only in the last century or so. Eleanor Robson -The Uses of Mathematics in Ancient Iraq, 6000–600 BC

Concrete impact:
360 degrees angle
60 minutes in an hour
60 seconds in a minute