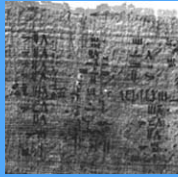


Mathematics in Ancient Egypt

- More about the presentation
- Egypt Overview
- Egyptian Math in a Nutshell
- How do we know what we know
- Scribes and papyri
- The Rosetta Stone
- The Rhind papyrus



1. Multiplication and division
2. Parts (fractions)
3. Method of false position
4. Areas and volumes
 - Area of the circle, π !
 - Volume of the truncated pyramid.
5. As I was going to St Ives...

Introduction

Welcome to History of Mathematics

MAT 336 - Spring 2024

Useful links

- **Course Schedule:** The course schedule of Lecture 1 and course schedule of Lecture 2 list what topics were (or will be) covered, as well as the *all* course work and deadlines.
- **Syllabus:** The description and policies of this course can be found (of course!) on the class syllabus.
- **Click on each of the names below for addresses, office hours, office location and zoom links.**
 - Moira Chas Instructor Lecture 1, TuTh 2:30-3:50pm.
 - Georgina Spence Instructor Lecture 2, TuTh 4-5:20pm.
 - Shuo Gao Grader of Lecture 1.
 - Daniel Brogan Grader of Lecture 2.
- The list topics, as well as a possible secondary source is [here](#).
- **Examples of abstract, outline, slides, paper and presentation** from past years. (Note that the required length varied. Also requirements for reference on one of the papers (paper2.pdf) were "lighter" than in our course, so the student mentions certain facts - which are not common knowledge - without citation. It is still a wonderful paper.)

Resources: Books, websites and databases to use during the semester

- **Links:** Readings, course materials and links organized by topic
- Stony Brook Library is a great source of materials, and librarians that can help you.
- There is a great deal of relevant material in the digital database JSTOR, which you can access with your Stony Brook Net ID.
- Another great math database is MathSciNet MSC primary classification is 01. For more details, see [here](#).
- MacTutor History of Mathematics Archive contains overviews and bibliography of most of the topics we will discuss.
- Wikimedia Commons is a good repository of images
- The Internet Archive has many books about math history. You can also find books at Project Gutenberg. Both websites are free because they post material whose copyright has expired. On one hand this is good, because books are easily available, on the other, some of the material will be outdated. Hence, make sure that you also look for more recent texts (for instance, in the library.)

<https://www.math.stonybrook.edu/~moira/courses/mat336-sp2024/links.html>

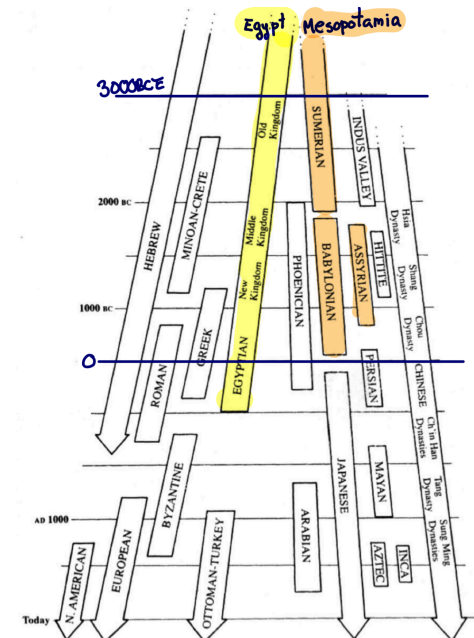


Illustration from *Pi in the Sky*
By Wendy Mass Little, Brown Books for Young Readers 2013

Make an educated guess about ancient Egyptian mathematics. Do you think it was developed for practical purposes or for its own sake? What branch of mathematics was more developed?



Ancient Egypt 101 | National Geographic

<https://youtu.be/hO1tzmi1V5g>

Write down something related to mathematics you heard in this clip.

Introduction

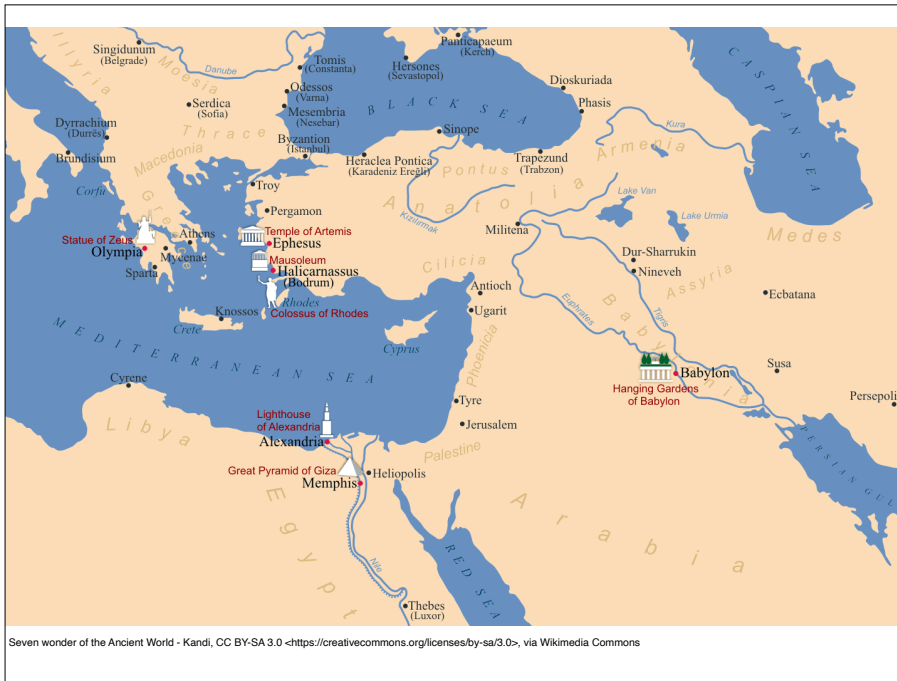
Ancient Egypt and Mesopotamia

- Interesting mathematical developments.
- Developed writing systems.
- Relatively warm climate, fertile lands.
- Flourished along major rivers.
- Strong centralized government.
- Strong religious life.

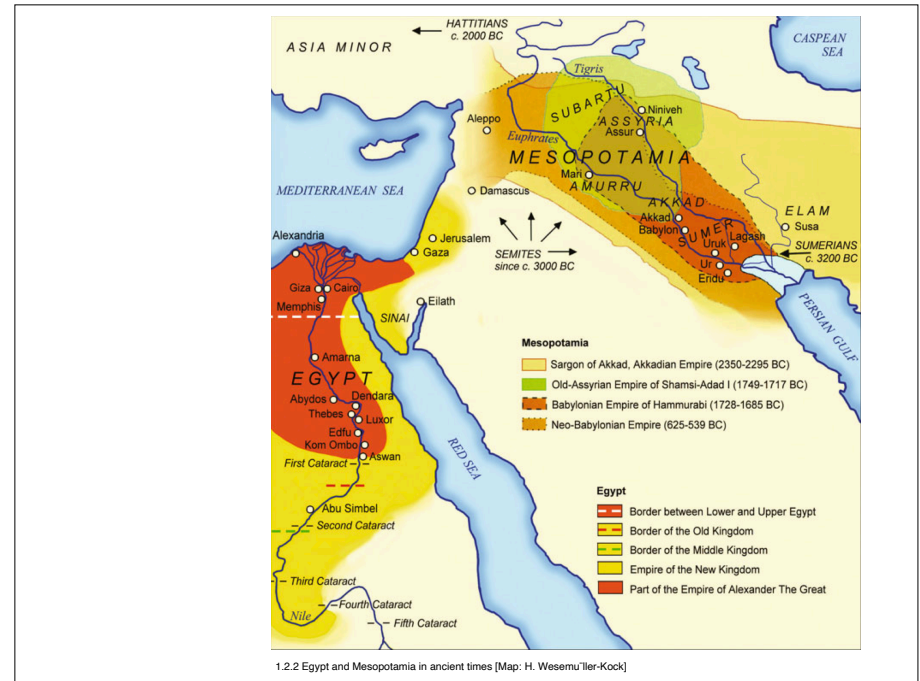
Practical science to facilitate

- computation of calendar
- organization of public works
- collection of taxes.
- Evidence suggests mathematics' development was motivated by administrative needs.
- Initial emphasis on arithmetic and mensuration (that is, determination of areas and volumes).
- Later became a more abstract discipline, studied some aspects of math for its own sake. (Although there is some debate between scholars about this point)

<https://www.math.tamu.edu/~don.allen/history/1000bc/1000bc.html>

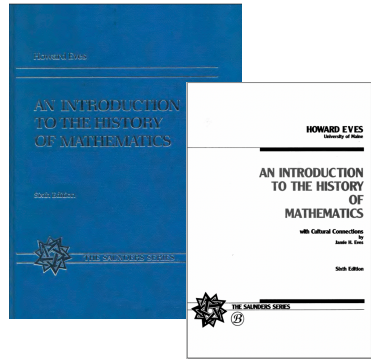


Seven wonder of the Ancient World - Kandi, CC BY-SA 3.0 <<https://creativecommons.org/licenses/by-sa/3.0/>>, via Wikimedia Commons



1.2.2 Egypt and Mesopotamia in ancient times [Map: H. Wesemu'iler-Kock]

MATHEMATICAL PERIODS with some major contributions and contributors (many dates are approximate)			
EGYPTIAN AND BABYLONIAN (3000 B.C. to A.D. 260)	GREEK (600 B.C. to A.D. 450)	CHINESE (1030 B.C. to A.D. 1644)	HINDU (200 B.C. to A.D. 1250)
Essentially empirical, or inductive, mathematics Introduction of early numeral systems (decimal and sexagesimal) Simple arithmetic, practical geometry Mathematical tables, collections of mathematical problems Chief primary sources: Moscow (1850 B.C.), Rhind (1650 B.C.), and other Egyptian papyri; Babylonian cuneiform tablets (2100 B.C. to 1600 B.C. and 600 B.C. to A.D. 300)	Significant introduction, then development, of deductive geometry (Thales, 600 B.C.; Pythagoras, 540 B.C.) Start of number theory (Pythagorean School, 540 B.C.) Discovery of incommensurable magnitudes (Pythagorean School, before 340 B.C.) Systematization of deductive logic (Aristotle, 380 B.C.) Axiomatic development of geometry (Euclid, 300 B.C.) Germs of the integral calculus (Archimedes, 225 B.C.) Geometry of conic sections (Apollonius, 225 B.C.) Practical geometry (Heron, A.D. 75?) Trigonometry (Hipparchus, 140 B.C.; Menelaus, A.D. 100; Ptolemy, A.D. 150) Number theory, syncopation of algebra (Diophantus, A.D. 250?)	Largely isolated from the mainstream of mathematical development Decimal numeral system, rod numerals magic squares (from earliest time) Chou-pei, oldest of Chinese mathematical classics (000 A.C.?) Arithmetic in Nine Sections (100 A.C.?) Horner's method (Ch'in Kiu-Shao, 1247) Pascal's arithmetic triangle, binomial theorem (Chu Shü-kai, 1303) Jesuit missionaries infiltrated China in early 1600s	Introduction of Hindu-Arabic numeral system (before A.D. 250) Negative numbers and invention of zero symbol (early centuries A.D.) Development of early computing algorithms (A.D. 900-1000) Syncopated algebra, indeterminate equations (Brahmagupta, A.D. 628; Bhaskara, A.D. 1150) ARABIAN (A.D. 650 to 1200) Preservers of Hindu arithmetic and Greek geometry (encouraged by caliph patrons of learning, such as Harun al-Rashid, A.D. 780) Influential treatise on algebra and a book on the Hindu numerals (Al-Khawarizmi, A.D. 820) Trigonometric tables (Abū Wafā, A.D. 980; Ulugh Beg, A.D. 1435) Geometric solution of cubic equations (Omar Khayyam, A.D. 1100)



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Naqada Tablets - 4th millennium BCE

The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook By Annette Imhausen, Eleanor Robson, Victor J. Katz, Victor J. Katz, Annette Imhausen · 2007

Ancient Egyptian Mathematics in a very small nutshell

- Very few extant sources (math was recorded in fragile papyri.)
- Problems and solutions to concrete algebraic and geometric problems
 - finding the area or volume of certain shapes,
 - fair division of loaves of bread.
 - feeding animals and storage of grain
 - solutions of linear equations with one unknown - false position.
- Some Problems with theoretical interest
 - Adding $7 + 7^2 + 7^3 \dots + 7^5$
- Examples (as opposed to rules); how (as opposed to why)
- Doubling and halving were the basic arithmetic operations.
- Two number systems: hieroglyphic and a ciphred (used for different purposes).
- Intriguingly good approximation to π
- Fractions were written as a sum of *parts* (fractions of form $1/n$).
- Development of calendar.



Looking at the table of hieratic numerals, what characteristic do you think this number system have? Why?

Credit tables: https://mathshistory.st-andrews.ac.uk/HistTopics/Egyptian_numerals/

Four **characteristics** of number systems

- **Additive:** The value of a number is the sum of the values of the numerals.
- **Ciphred or alphabetic:** Numerals design 1, 2,...9, and the powers of 10 (or, more generally, some base) but also to the multiples of this powers.
- **Multiplicative:** There are two sets of numerals, the elements of one set represent digits and the elements of the other set represent position. If necessary, a digit and a position symbols are used together, and the values of numerals are multiplied. Finally, all the products are added.
- **Positional:** The value of each numeral depends on its position. The system consists of a **base** (a natural number greater than one) and a **set of numerals** representing the numbers from zero to one less than the base. The numbers from zero to the base minus one are the digits in the system.

1	𐎁	10	𐎁𐎁	100	𐎁𐎁𐎁	1000	𐎁𐎁𐎁𐎁
2	𐎁𐎁	20	𐎁𐎁𐎁	200	𐎁𐎁𐎁𐎁	2000	𐎁𐎁𐎁𐎁𐎁
3	𐎁𐎁𐎁	30	𐎁𐎁𐎁𐎁	300	𐎁𐎁𐎁𐎁𐎁	3000	𐎁𐎁𐎁𐎁𐎁𐎁
4	𐎁𐎁𐎁𐎁	40	𐎁𐎁𐎁𐎁𐎁	400	𐎁𐎁𐎁𐎁𐎁𐎁	4000	𐎁𐎁𐎁𐎁𐎁𐎁𐎁
5	𐎁𐎁𐎁𐎁𐎁	50	𐎁𐎁𐎁𐎁𐎁𐎁	500	𐎁𐎁𐎁𐎁𐎁𐎁𐎁	5000	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁
6	𐎁𐎁𐎁𐎁𐎁𐎁	60	𐎁𐎁𐎁𐎁𐎁𐎁𐎁	600	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	6000	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁
7	𐎁𐎁𐎁𐎁𐎁𐎁𐎁	70	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	700	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	7000	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁
8	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	80	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	800	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	8000	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁
9	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	90	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	900	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	9000	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁

Hieratic numerals

Number systems in Ancient Egypt

Credit tables: https://mathshistory.st-andrews.ac.uk/HistTopics/Egyptian_numerals/

Hieroglyphic numerals

𐎁	𐎁𐎁	𐎁𐎁𐎁	𐎁𐎁𐎁𐎁	𐎁𐎁𐎁𐎁𐎁	𐎁𐎁𐎁𐎁𐎁𐎁	𐎁𐎁𐎁𐎁𐎁𐎁𐎁
1	10	100	1000	10000	100000	10^6

Egyptian numeral hieroglyphs



Example of a hieroglyphic number from a tomb inscription.

- Both hieroglyphs and hieratic symbols changed over time. Also, a third writing system emerged: demotic.
- The versions here of the hieratic numerals date from around 1800 BC.
- Both systems ran in parallel for around 2000 years with the hieratic symbols being used in writing on papyrus, while the hieroglyphs continued to be used when carved on stone.

1	𐎁	10	𐎁𐎁	100	𐎁𐎁𐎁	1000	𐎁𐎁𐎁𐎁
2	𐎁𐎁	20	𐎁𐎁𐎁	200	𐎁𐎁𐎁𐎁	2000	𐎁𐎁𐎁𐎁𐎁
3	𐎁𐎁𐎁	30	𐎁𐎁𐎁𐎁	300	𐎁𐎁𐎁𐎁𐎁	3000	𐎁𐎁𐎁𐎁𐎁𐎁
4	𐎁𐎁𐎁𐎁	40	𐎁𐎁𐎁𐎁𐎁	400	𐎁𐎁𐎁𐎁𐎁𐎁	4000	𐎁𐎁𐎁𐎁𐎁𐎁𐎁
5	𐎁𐎁𐎁𐎁𐎁	50	𐎁𐎁𐎁𐎁𐎁𐎁	500	𐎁𐎁𐎁𐎁𐎁𐎁𐎁	5000	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁
6	𐎁𐎁𐎁𐎁𐎁𐎁	60	𐎁𐎁𐎁𐎁𐎁𐎁𐎁	600	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	6000	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁
7	𐎁𐎁𐎁𐎁𐎁𐎁𐎁	70	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	700	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	7000	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁
8	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	80	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	800	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	8000	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁
9	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	90	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	900	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁	9000	𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁𐎁

Hieratic numerals



Rhind Papyrus Section
British Museum

Topics are posted
(with dates)
Next week: class, in
Wooclap, I will ask you
for a brief summary of
your topic.

ChatGPT 3.5 ▾

You

Is this correct: In the early 1880s, Felix Klein talked in one of his lectures of the surface that now carries his name.

ChatGPT

No, that statement is not accurate. Felix Klein did not give lectures in the early 1880s about a surface carrying his name. The surface you might be referring to is likely the "Klein bottle," a non-orientable surface with interesting mathematical properties. Felix Klein, a German mathematician, did make significant contributions to mathematics, but the Klein bottle was not explicitly mentioned by him.

The concept of the Klein bottle is attributed to another mathematician, Heinrich Scherk, who first described it in 1834. The name "Klein bottle" itself comes from Felix Klein, who discussed it in a more systematic way in the late 19th century, around 1882-1884. However, it's crucial to clarify that the Klein bottle was not explicitly named by Klein in his lectures during the early 1880s; rather, it was later associated with him due to his work on the broader field of topology.

Ancient Egypt

Uses and motivation of mathematics

- Writing systems.
- fertile lands, major river.
- Strong centralized government. TAXES!
- Strong religious life.

- computation of calendar
- Arithmetic and mensuration.
- Administrative needs
 - organization of public works
 - collection of taxes.
- Later mathematics became a more abstract discipline, studied some aspects of math for its own sake. (Although there is some debate between scholars about this point)

Very few primary sources in math

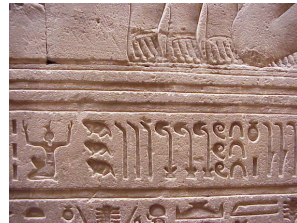
<https://www.math.tamu.edu/~don.allen/history/1000bc/1000bc.html>

Ancient Egypt Numbers

Hieratic

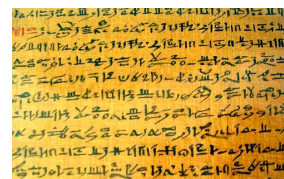


Some numbers in hieroglyphics



In their long history, Egyptians created more than one system of writing. Some of these systems were used simultaneously.

	10^0	10^1	10^2	10^3
1	𐎠	𐎡	𐎢	𐎣
2	𐎠𐎠	𐎡𐎡	𐎢𐎢	𐎣𐎣
3	𐎠𐎠𐎠	𐎡𐎡𐎡	𐎢𐎢𐎢	𐎣𐎣𐎣
4	𐎠𐎠𐎠𐎠	𐎡𐎡𐎡𐎡	𐎢𐎢𐎢𐎢	𐎣𐎣𐎣𐎣
5	𐎠𐎠𐎠𐎠𐎠	𐎡𐎡𐎡𐎡𐎡	𐎢𐎢𐎢𐎢𐎢	𐎣𐎣𐎣𐎣𐎣
6	𐎠𐎠𐎠𐎠𐎠𐎠	𐎡𐎡𐎡𐎡𐎡𐎡	𐎢𐎢𐎢𐎢𐎢𐎢	𐎣𐎣𐎣𐎣𐎣𐎣
7	𐎠𐎠𐎠𐎠𐎠𐎠𐎠	𐎡𐎡𐎡𐎡𐎡𐎡𐎡	𐎢𐎢𐎢𐎢𐎢𐎢𐎢	𐎣𐎣𐎣𐎣𐎣𐎣𐎣
8	𐎠𐎠𐎠𐎠𐎠𐎠𐎠𐎠	𐎡𐎡𐎡𐎡𐎡𐎡𐎡𐎡	𐎢𐎢𐎢𐎢𐎢𐎢𐎢𐎢	𐎣𐎣𐎣𐎣𐎣𐎣𐎣𐎣
9	𐎠𐎠𐎠𐎠𐎠𐎠𐎠𐎠𐎠	𐎡𐎡𐎡𐎡𐎡𐎡𐎡𐎡𐎡	𐎢𐎢𐎢𐎢𐎢𐎢𐎢𐎢𐎢	𐎣𐎣𐎣𐎣𐎣𐎣𐎣𐎣𐎣

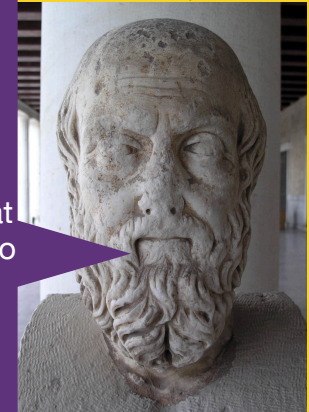


Demotic

“It was this king, moreover, who divided the country among all the Egyptians by giving each an equal square parcel of land, and made this his source of revenue, appointing the payment of a yearly tax. And any man who was robbed by the river of a part of his land would come to Sesostris and declare what had befallen him; then the king would send men to look into it and measure the space by which the land was diminished, so that thereafter it should pay in proportion to the tax originally imposed. **Perhaps this was the way in which the art of measuring land (geometry) was invented, and passed afterwards into Greece**”

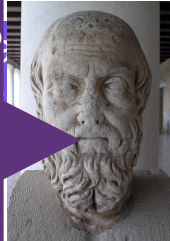
How was geometry invented, according to Herodotus?

Herodotus (~400BC) was an ancient Greek historian who was born in Halicarnassus in the Persian Empire (modern-day Bodrum, Turkey).

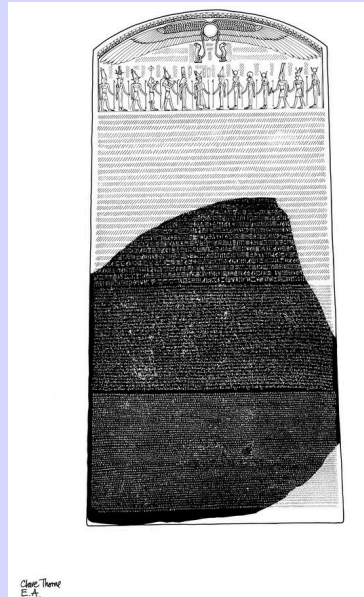


Bust of Herodotus. 2nd century AD. Roman copy after a Greek original. On display along the portico of the Stoa of Attalus, which houses the Ancient Agora Museum in Athens.

... Cheops became king over them and brought them to every kind of evil: (...) he then bade all the Egyptians work for him. So some were appointed to draw stones from the stone-quarries in the Arabian mountains to the Nile, and others he ordered to receive the stones after they had been carried over the river in boats, and to draw them to those which are called the Libyan mountains; and they worked by a hundred thousand men at a time, for each three months continually. Of this oppression there passed ten years (..) For this they said, the ten years were spent, and for the underground he caused to be made as sepulchral chambers for himself in an island, having conducted thither a channel from the Nile. For the making of the pyramid itself there passed a period of twenty years.

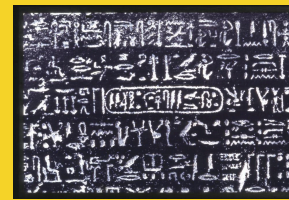


The Rosetta Stone

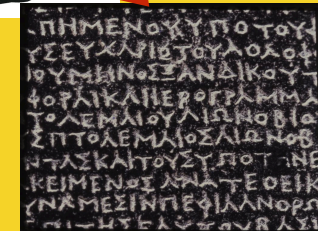
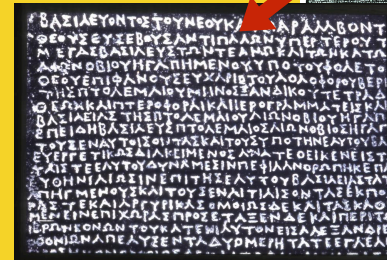
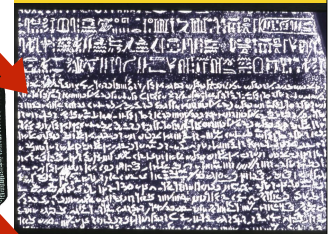
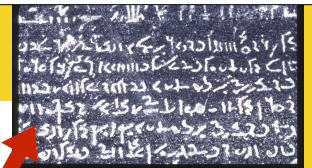
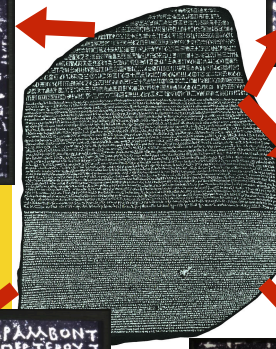


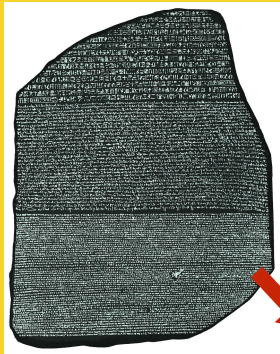
Cheops
E.A.

The Rosetta stone

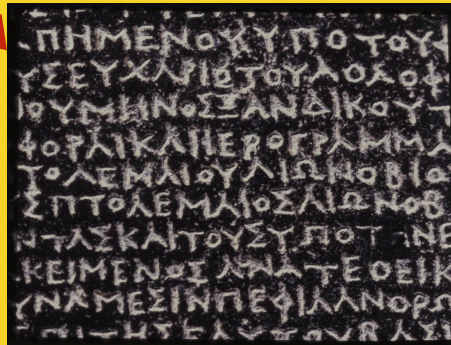


What languages are used in the Rosetta stone?

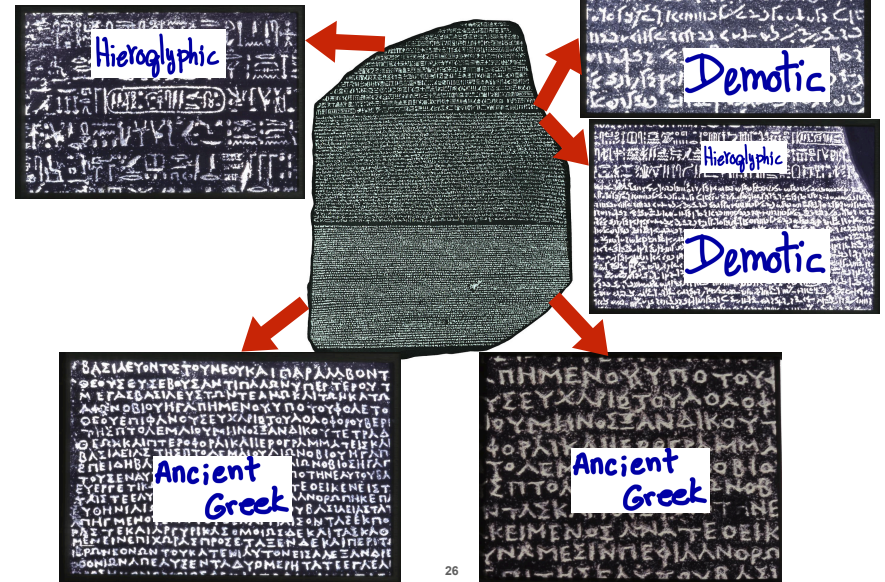




What language is used in the bottom part of the Rosetta stone?



The Rosetta stone



Rough History of the Rosetta Stone

- Made in 196 BC, on the first anniversary of the coronation of king Ptolemy V, by then a teenager.
- It's a decree issued by Egyptian priests, ostensibly to mark the coronation and to declare Ptolemy's new status as a living god - divinity went with the job of being a pharaoh
- It was the result of hard political negotiations with his extremely powerful Egyptian priests.
- Survived unread through two thousand years of further foreign occupations - Romans, Byzantines, Persians, Muslim Arabs and Ottoman Turks, all had stretches of rule in Egypt.
- A French invasion (which was not only military but intellectual) found it in the town of Rosetta (now el Rashid) in 1799.
- The French took it as cultural trophy of war. But Napoleon was defeated, and in 1801 the terms of the Treaty of Alexandria, signed by the French, British and Egyptian generals, included the handing over of antiquities - and the Rosetta Stone was one of them.
- on the broken side, you can see that in fact there are four. Because there, stencilled on in English, you can read: "CAPTURED BY THE BRITISH ARMY IN 1801; PRESENTED BY KING GEORGE III".

<https://www.bbc.co.uk/programmes/articles/3dtD0Nr8tUpG7pXvt302Ys/episode-transcript-episode-33-rosetta-stone>

Decipherment of the Rosetta Stone

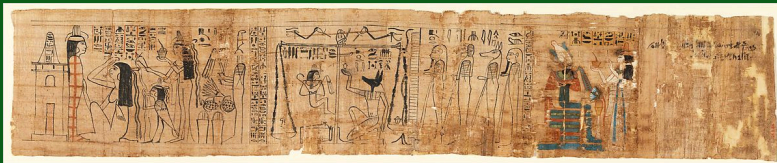
- It was hoped that the Egyptian text could be deciphered through its Greek translation.
- Phonetic glyphs in a **cartouche** containing the name of an Egyptian king of foreign origin, Ptolemy V.
- In the early 1820s Champollion compared Ptolemy's cartouche with others and realised the **hieroglyphic script was a mixture of phonetic and ideographic elements.**
- Young, meanwhile, largely **deciphered demotic** using the Rosetta Stone in combination with other Greek and demotic parallel texts.
- New progress was made in the second quarter of the 1800s.

Men Splitting Papyrus (public domain)
A.D. 1914–1916, original ca. 1479–1458 B.C.



This is a photo of Papyrus growing wild along the banks of the Nile River in Uganda. It was taken by Michael Shade in the fall of 2006.

Papyrus



Book of the Dead Papyrus of Tiyeca, 975–945 B.C. Discovered tucked inside her hollow wooden "Osiris figure" (now Cairo JE 49164) this papyrus was designed to assist Tye in her successful transition from death to eternal life. - (Public domain)

Papyri, reading, writing and math

- Papyri are fragile
- Papyri were expensive (labor intensive production)
- Only about 10 mathematical papyri have survived
- Mostly fragments, except for Rhind and Moscow papyri
- Scholars think that only about 1 or 2% of the population was able to read and write.

Aristotle writes (Metaphysics):
“Thus the mathematical sciences originated in the neighborhood of Egypt, because there the priestly class was allowed leisure.”



Scribes

The seated scribe, about 2500 BCE, Louvre Museum

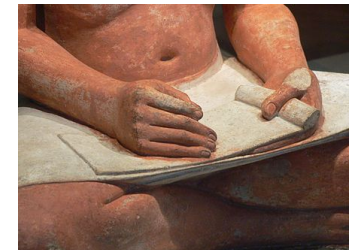
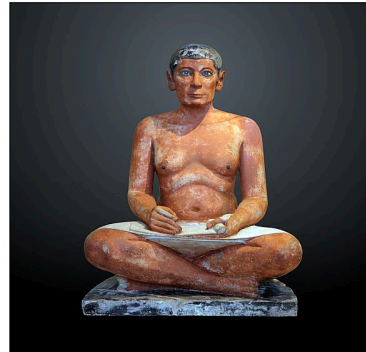


image <https://commons.wikimedia.org/>

Very often, in tombs of high officials, the tomb owner is shown as a **inspector in sciences of accounting cattle or product**, and sometimes several scribes are depicted working together as a group. Several models depict the filling of granaries, and a **scribe is always present to record the respective quantities.**

Scribes were, among other functions, accountants.

Katz, Victor J., ed. The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook. Princeton University Press, 2007.³²

A fragment from Papyrus Anastasi - A fictional letter, which forms part of a debate between two scribes

You are told: "Empty the magazine that has been loaded with sand under the monument for your lord—may he live, prosper, and be healthy—which has been brought from the Red Mountain. It makes 30 cubits stretched upon the ground with a width of 20 cubits, passing chambers filled with sand from the riverbank. The walls of its chambers have a breadth of 4 to 4 to 4 cubits. It has a height of 50 cubits in total. [...] You are commanded to find out what is before it. **How many men will it take to remove it in 6 hours if their minds are apt?** Their desire to remove it will be small if (a break at) noon does not come. You shall give the troops a break to receive their cakes, in order to establish the monument in its place. One wishes to see it beautiful.

The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook By Annette Imhausen, Eleanor Robson, Victor J. Katz, Victor J. Katz, Annette Imhausen · 2007, page 11

The Rhind or Ahmose Papyrus



The Rhind or Ahmes papyrus



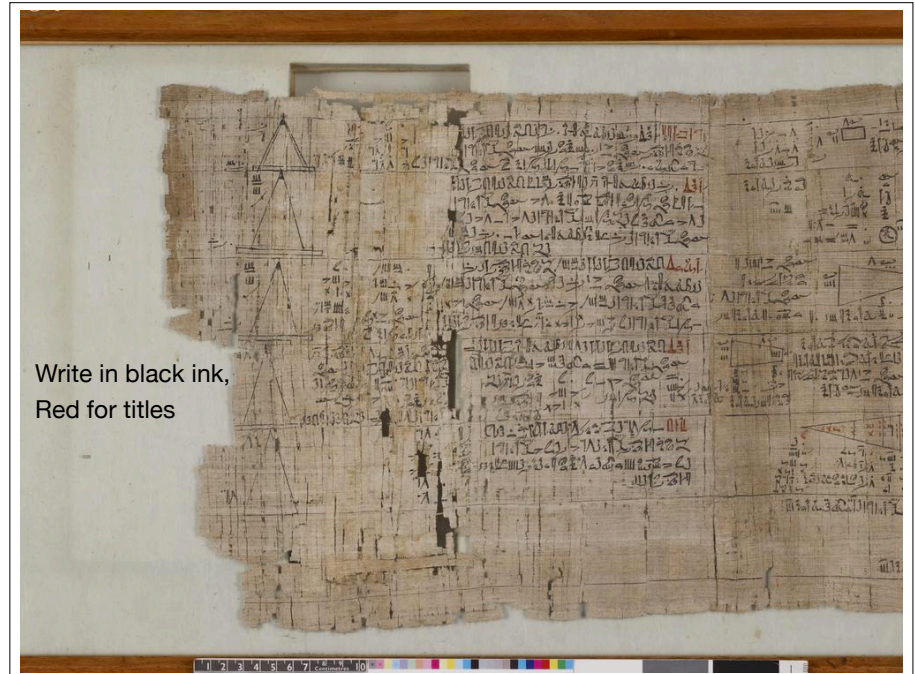
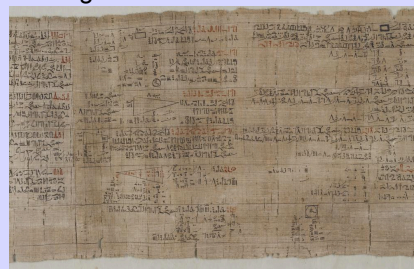
Image from the British Museum
<https://www.britishmuseum.org/collection/image/366145001>

Fragment

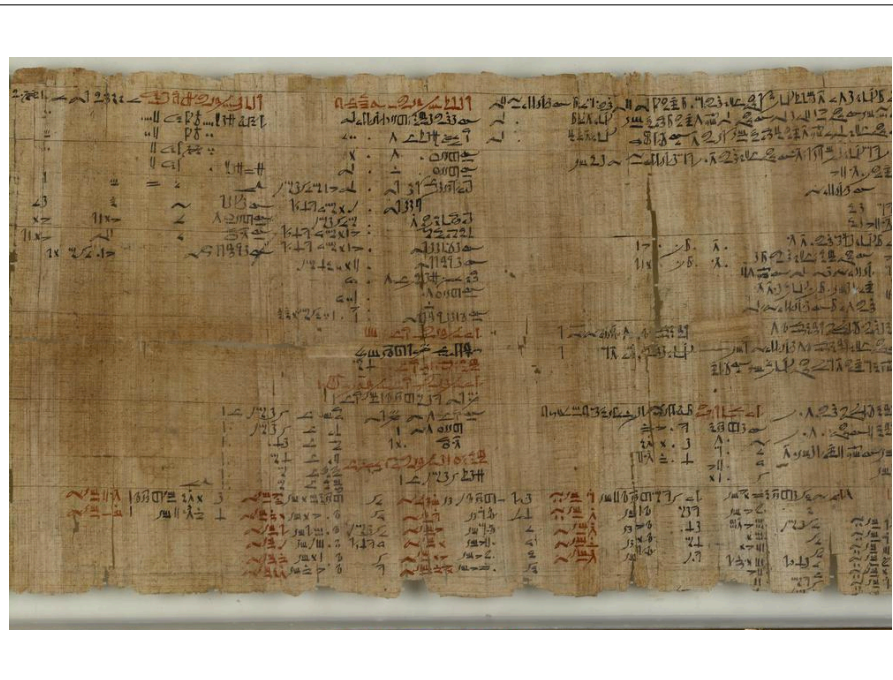
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985	986	987	988	989	990	991	992
993	994	995	996	997	998	999	1000

About 17 x 1 feet

https://www.britishmuseum.org/collection/object/Y_EA10057



Write in black ink,
 Red for titles



Three types of problem

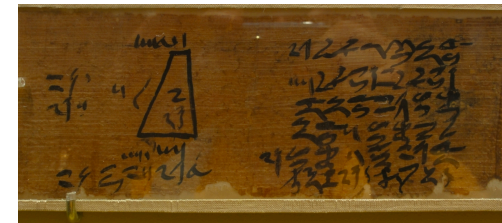
- pure mathematical problems teaching basic techniques
- practical problems, which contain an additional layer of knowledge from their respective practical setting
- non-utilitarian problems, which are phrased with a pseudo-daily life setting without having a practical application (only very few examples extant)
 - No symbols (like + or -)
 - No variables (like x)
 - Algorithmic: a list of concrete instructions to solve them

The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook By Annette Imhausen, Eleanor Robson, Victor J. Katz, Victor J. Katz, Annette Imhausen · 2007

- Table $2/n$ as a sum of parts, $n=3$ to 103
- divisions of a certain number of loaves of bread among 10 men
- addition of fractions, summing up to 1.
- solution of linear equations (but not as we understand them)
- unequal distribution of goods and other problems
- find the volume of cylindrical and rectangular granaries.
- show how to compute an assortment of areas
- slopes (of pyramids.)
- multiplications of fractions.
- Value, fair exchange and feeding

The Rhind Papyrus (in present language)

The Moscow Papyrus



Moscow Mathematical Papyrus
Fotography by Charles Dorca
<https://thematheaticaltourist.wordpress.com/2012/10/19/moscow-mathematical-papyrus/>

14th problem of the Moscow Mathematical Papyrus



Dimensions of the Moscow Papyrus
Length: 5.5 metres (18 ft)
Width: 3.8 to 7.6 cm (1.5 to 3 in)

Multiplication in Ancient Egypt

What does it mean to multiply a positive integer A by another positive integer B ? (If you prefer, you can explain it with an example, say $A=20$ and $B=45$)

Ancient Egyptian multiplication

Find the product of A and B

- Set up two columns. Write 1 in the left column and B in the right column.
- Keep doubling the number on both columns. Stop when the number on the 1-column will be greater than A .
- Mark all the numbers in the 1-column that add up to A . Mark the corresponding numbers on the B-column.
- Add the marked numbers of the A-column. The sum of these numbers is the product of $A \cdot B$.

A.B		EXAMPLE: $A=45,$ $B=20$
1	B	
2	2B	
4	4B	
...	...	
2^n	$2^n B$	
n is the largest integer such that $2^n < A$		

47

Ancient Egyptian multiplication

Find the product of A and B

- Set up two columns. Write 1 in the left column and B in the right column.
- Keep doubling the number on both columns. Stop when the number on the 1-column will be greater than A .
- Mark all the numbers in the 1-column that add up to B . Mark the corresponding numbers on the B-column.
- Add the marked numbers of the B-column the sum is the product of $A \cdot B$.

A.B		EXAMPLE: $A=45,$ $B=20$
1	B	1 20
2	2B	2 40
4	4B	4 80
...	...	8 160
2^n	$2^n B$	16 320
n is the largest integer such that $2^n < A$		32 640

STOP

$$20 \cdot 45 = 640 + 160 + 80 + 20 = 900$$

Mark all the numbers in the 1-column that add up to B ??

48

Exercise: Multiply 20 by 45 as we just did, but exchanging the roles of A and B.

Multiply 25 by 26

EXAMPLE:

1 45

Blank boxes for the example calculation.

ANOTHER EXAMPLE:

Blank boxes for another example calculation.

Recall that to multiply two positive integers A and B in the Egyptian algorithm, one needs to write one of those numbers, say B, as a sum of powers of 2, in such a way that each power appears at most once. Is that possible for B? Why or why not?

Recall that a number and the representation of the number in a number systems are two different concepts. There are many ways to represent a given number, but each number is "unique".

In a similar way, multiplication and how multiplication is performed are different concepts. Again, there are many algorithms, that is, many ways to multiply two numbers. But the meaning of multiplication is only one.

what ≠ how

However,

Rhind Mathematical Papyrus, problem 69

.	80
\ 10	800
2	160
\ 4	320
Total	1120

Division in Ancient Egypt

Ancient Egyptian division

- **Example:** Let's find the quotient of 130 divided by 10 in the Egyptian way.
- This computation will be stated as "multiply 10 so to get 130". (We can think about this as "solve $10 \cdot x = 130$ ")
- Again, we set up two columns, one for 1, the other one for 10, and we double as much as possible but in such a way that the B-column does not surpass 130
- Mark all the numbers in the 1-column that add up to 130. Mark the corresponding numbers on the 1-column.
- Add the marked numbers of the 1-column. The sum of these number is quotient of 130 divided by 10.



54

Ancient Egyptian division

Find the quotient of 130 divided by 10 in the ancient Egyptian way.

- This computation will be stated as "multiply 10 so to get 130". (We can think about this as "solve $10 \cdot x = 130$ ")
- Again, we set up two columns, one for 1, the other one for 10, and we double as much as possible but in such a way that the B-column does not surpass 130
- Mark all the numbers in the 1-column that add up to 130. Mark the corresponding numbers on the 1-column.
- Add the marked numbers of the 1-column. The sum of these number is quotient of 130 divided by 10.

1	10
2	20
4	40
8	80

$$80 < 130 \text{ but } 2 \cdot 80 = 160 > 130$$

$$130 = 80 + 40 + 10$$

$$130/10 = 8 + 4 + 1$$

55

Ancient Egyptian division

Find the quotient of A divided by B

- This computation will be stated as "multiply B so to get A". (We can think about this as "solve $B \cdot x = A$ ")
- Again, we set up two columns, one for 1, the other one for B, and we double as much as possible but in such a way that the B-column does not surpass A.
- Mark all the numbers in the 1-column that add up to A. Mark the corresponding numbers on the 1-column.
- Add the marked numbers of the 1-column. The sum of these numbers is quotient of A divided by B.

1	B
2	2B
4	4B
...	...
2^n	$2^n B$

n is the largest integer such that $2^n B < A$

56

Exercise: Find the quotient of 420 divided by 35

Find the quotient of 561 divided by 17.

EXAMPLE:

Blank lines for the example division of 420 by 35.

ANOTHER EXAMPLE:

Blank lines for the example division of 561 by 17.

The fundamental operations of Egyptian arithmetic are adding and doubling.

A multiplicative system Traditional Chinese numerals

Write the numbers below (in traditional Chinese numerals) in Hindu-Arabic numerals.

(a) 八十三 (b) 四百一十 (c) 二萬九千五百二十四 (d) 五千六百

1	一	10	十
2	二	100	百
3	三	1000	千
4	四		
5	五		
8	八	100,000	億
9	九		

100,000 should be “十萬”

Areas and Volumes

1. How do you determine the length of a segment?
2. What does "measure the length of a segment" mean?
3. How do you find the area of a plane figure?
4. What does "measurement of the area of a plane figure" mean?
5. How do you find the volume of a solid?
6. What does it mean to "measure the volume of a solid"?

The same amount unit squares can be rearranged to occupy both planar shapes

that they are the same size but can be different shapes

What does it mean for two planar shapes to have the same area?

That means we can put the same amount of water or some kind of unit into two planar shapes

They must be congruent.

identical

The measurement of area agrees

Measuring a segment means comparing its length with that of a chosen unit, and finding how many times the unit "fits" into the segment. (Of course, the unit may not "fit" into the segment an integer or fractional number of times.)

Similarly, measuring a planar figure means finding how many times a given unit of area (and/or fractions of that unit) "fits" into the figure.

Finally, measuring a solid can be defined in a similar way.

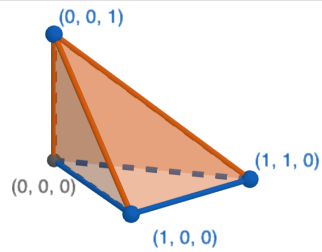
What do you think the scissor congruence app shows?



<https://dmsm.github.io/scissors-congruence/>

Two planar polygonal shape have the same area if and only if they are scissor congruent




Max Dehn showed that the analogous result does not hold in \mathbb{R}^3



Max Dehn

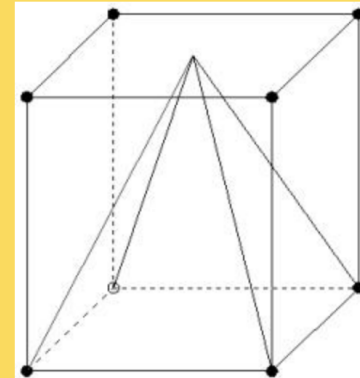
Idea: $\sum_{\text{all edges}} \text{length of edge} \otimes \text{dihedral angle} \pmod{Q\pi}$

Measurement in Egypt: length, area, volume

- Volumes of
 - Cylindrical containers. 
 - Rectangular parallelepipedal containers. 
 - Truncated pyramid 
- Areas of
 - Rectangles
 - Circles
 - Triangles
 - Trapezoids
- Division of given area of land into equal-sized fields.
- A quantity related to slope

Volume of the truncated pyramid

Denote by V the volume a pyramid with square base A and height A . Denote by C the volume of a cube of side length A (that is, $C=A^3$) Then:



- 1 $C=V/2$
- 2 C is larger than $V/2$ and smaller than V
- 3 $C=V$
- 4 C is larger than V and smaller than $3V$
- 5 $C=3V$
- 6 C is larger than $3V$ and smaller than $4V$
- 7 $C=4V$
- 8 C is larger than $4V$
- 9 None of the above

Denote by P the volume a pyramid with square base A and height A . Denote by C the volume of a cube of side length A (that is, $C=A^3$) Then:

① $C=P/2$

⑤ $C=3P$

② $P/2 < C < P$

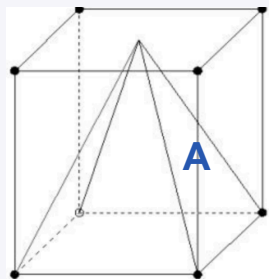
⑥ $3P < C < 4P$

③ $C=P$

⑦ $C=4P$

④ $P < C < 3P$

⑧ None of the above



What is the volume of a pyramid of square base?



Image credit: wikimedia, by Emöke Dénes

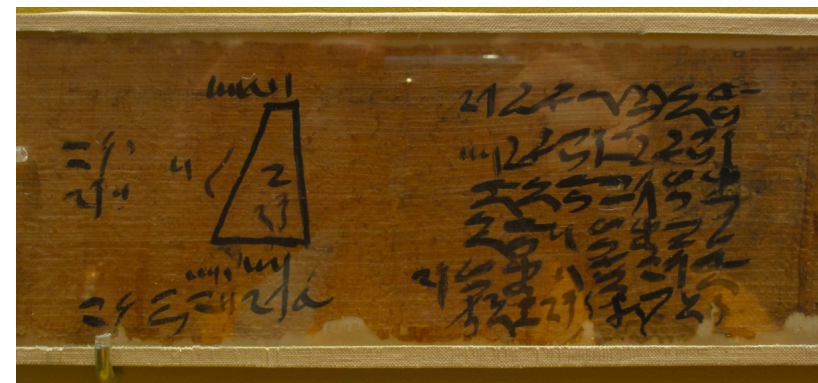
Demo

The Great Pyramid of Giza

We did a demonstration in which we saw that a cube can be filled exactly with three squared pyramids (of square base of same side length and height as the cube).

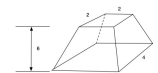
Does the demonstration constitute a proof that the volume of the cube is three times the volume of the pyramid?

Problem 14 of the Moscow Mathematical Papyrus



Charles Dorce photographed problem 14 of the Moscow Mathematical Papyrus

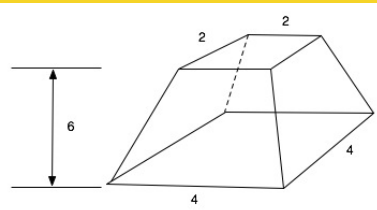
If someone says to you: a truncated pyramid of 6 for the height and by 4 on the base by 2 on the top. You are to square this 4; the result is 16. You are to double 4; the result is 8. You are to square this 2; the result is 4. You are to add the 16 and the 8 and the 4; the result is 28. You have to take 1/3 of the 6 the result is 2. You have to take 28 two times; the result is 56. Behold, the volume is 56. You will find that this is correct.



Problem 14 of the Moscow Mathematical Papyrus

Consider a truncated pyramid of square base of side length 4, square top base of side length 2 and height 6. Complete the "translation" of the explanations below and give a formula of the volume without performing the computations (Hint: The first 6 lines translate to $4^2+2\cdot4+2^2$)

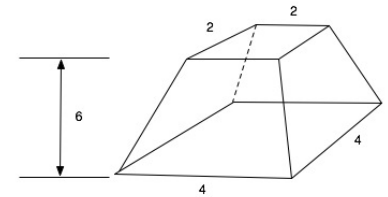
- A. If someone says to you:
- B. a truncated pyramid of 6 for the height and by 4 on the base by 2 on the top.
- C. You are to square this 4; the result is 16.
- D. You are to double (multiply by 2) 4; the result is 8.
- E. You are to square this 2; the result is 4.
- F. You are to add the 16 and the 8 and the 4; the result is 28.
- G. You have to take 1/3 of the 6 the result is 2.
- H. You have to take 28 two times; the result is 56. Behold, the volume is 56.
- I. You will find that this is correct.



73

Problem 14 of the Moscow Mathematical Papyrus

- A. If someone says to you:
- B. a truncated pyramid of H for the height and by A on the base by B on the top.
- C. You are to square this A; the result is A^2 .
- D. You are to multiply B by A; the result is $A\cdot B$.
- E. You are to square this B; the result is B^2 .
- F. You are to add the A^2 and the $A\cdot B$ and the B^2 ; the result is $A^2+A\cdot B+B^2$.
- G. You have to take 1/3 of the H the result is $H/3$.
- H. You have to multiply $(A^2+A\cdot B+B^2)$ by $H/3$; the result is $(A^2+A\cdot B+B^2)H/3$. Behold, the volume is $(A^2+A\cdot B+B^2)H/3$.
- I. You will find that this is correct.



Problem 14 of the Moscow Mathematical Papyrus

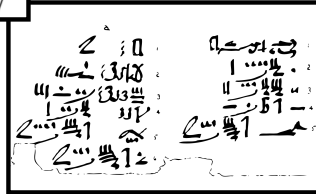
Conjecture: to find the formula the truncated pyramid was broken into pieces.

Note: This 3D "cut and paste" is a fundamental property of volume.

Problem 79

A house inventory:		houses	7
1	2,801	cats	49
2	5,602	mice	343
4	11,204	spelt	2,301
		hekat	16,807
Total	19,607	Total	19,607

Rhind or Ahmose Papyrus Problem 79



Possible interpretation

In each of the seven houses there are seven cats: each cat kills seven mice; each mouse would have eaten seven sheaves of wheat; and each sheaf of wheat was capable of yielding seven hekat measures of grain. How much grain was thereby saved?



<https://youtu.be/7vtszdW8MTs?t=11>

As I was going to Saint Ives,
I met a man with seven wives.
Every wife had seven sacks,
Every sack had seven cats,
Every cat had seven kits;
Kits, cats, sacks and wives,
How many were there going to
Saint Ives?

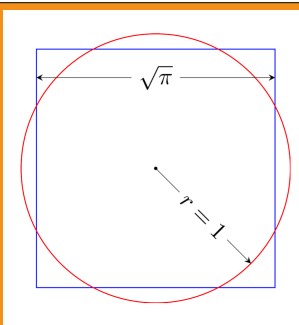


Mother Goose



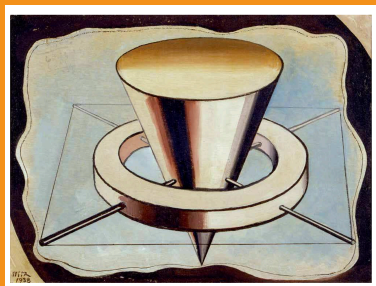
Image credit: <https://www.cornwalls.co.uk/st-ives/as-i-was-going-to-st-ives>

There are seven old women on the road to Rome.
Each woman has seven mules;
each mule carries seven sacks;
each sack contains seven loaves;
with each loaf are seven knives;
and each knife is in seven sheaths.
Women, mules, sacks, loaves, knives, and sheaths,
how many are there in all on the road to Rome?
(Translation from Fibonacci's Liber Abacci)



Squaring the circle

By 蔡望 - Own work, CC BY-SA 4.0, <https://commons.wikimedia.org/w/index.php?curid=132030577>



La Quadrature, 1938. Oil on panel Man Ray

Problem 50

Area of the circle

Approximation of π

Problem 50 of Rhind or Ahmes papyrus

Example of a round field of diameter 9 khet. What is its area?

Take away $\frac{1}{9}$ of the diameter, namely 1; the remainder is 8. Multiply 8 times 8; it makes 64. Therefore it contains 64 *setat* of land.

Do it thus:

1	9
$\frac{1}{9}$	1;
this taken away leaves 8	
1	8
2	16
4	32
$\setminus 8$	• 64.

Its area is 64 *setat*.

A *khet* is a length of about 50 meters.

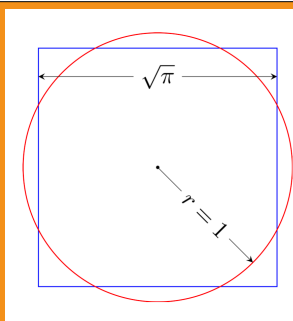
A *setat* is an area (one khet squared)

Problem 50 of Rhind or Ahmes papyrus

Find the formula of the area of the circle that the scribe would have obtained by starting with a circle of diameter d , instead of a circle of diameter 9. (Hint: Start by taking away $\frac{1}{9}$ of the diameter, that is $d/9$.)

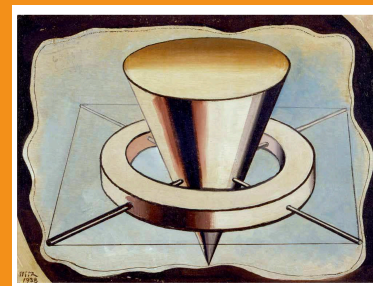
Problem 50 of Rhind or Ahmes papyrus

The area of a disk is a constant (π) times the radius of the circle squared. What is the value of π that Egyptians assumed in their computation of the area of the disk? (in the problem we are discussing)



By 蔡强 - Own work, CC BY-SA 4.0, <https://commons.wikimedia.org/w/index.php?curid=132030577>

Squaring the circle



La Quadrature, 1938. Oil on panel Man Ray

2/n table from the Rhind Mathematical Papyrus

2/3 = 1/2 + 1/6	2/5 = 1/3 + 1/15	2/7 = 1/4 + 1/28
2/9 = 1/6 + 1/18	2/11 = 1/6 + 1/66	2/13 = 1/8 + 1/52 + 1/104
2/15 = 1/10 + 1/30	2/17 = 1/12 + 1/51 + 1/68	2/19 = 1/12 + 1/76 + 1/114
2/21 = 1/14 + 1/42	2/23 = 1/12 + 1/276	2/25 = 1/15 + 1/75
2/27 = 1/18 + 1/54	2/29 = 1/24 + 1/58 + 1/174 + 1/232	2/31 = 1/20 + 1/124 + 1/155
2/33 = 1/22 + 1/66	2/35 = 1/30 + 1/42	2/37 = 1/24 + 1/111 + 1/296
2/39 = 1/26 + 1/78	2/41 = 1/24 + 1/246 + 1/328	2/43 = 1/42 + 1/86 + 1/129 + 1/301
2/45 = 1/30 + 1/90	2/47 = 1/30 + 1/141 + 1/470	2/49 = 1/28 + 1/196
2/51 = 1/34 + 1/102	2/53 = 1/30 + 1/318 + 1/795	2/55 = 1/30 + 1/330
2/57 = 1/38 + 1/114	2/59 = 1/36 + 1/236 + 1/531	2/61 = 1/40 + 1/244 + 1/488 + 1/610
2/63 = 1/42 + 1/126	2/65 = 1/39 + 1/195	2/67 = 1/40 + 1/335 + 1/536
2/69 = 1/46 + 1/138	2/71 = 1/40 + 1/568 + 1/710	2/73 = 1/60 + 1/219 + 1/292 + 1/365
2/75 = 1/50 + 1/150	2/77 = 1/44 + 1/308	2/79 = 1/60 + 1/237 + 1/316 + 1/790
2/81 = 1/54 + 1/162	2/83 = 1/60 + 1/332 + 1/415 + 1/498	2/85 = 1/51 + 1/255
2/87 = 1/58 + 1/174	2/89 = 1/60 + 1/356 + 1/534 + 1/890	2/91 = 1/70 + 1/130
2/93 = 1/62 + 1/186	2/95 = 1/60 + 1/380 + 1/570	2/97 = 1/56 + 1/679 + 1/776
2/99 = 1/66 + 1/198	2/101 = 1/101 + 1/202 + 1/303 + 1/606	

It is the year 1500 BCE. You are a scribe in Egypt

- Multiply 1/6 by 17.
- Multiply 1/5 by 17.

1	1/6
2	1/3
4	2/3=1/2+1/6
8	1+1/3
16	2+1/2+1/6
17(1/6)=	2+1/2+1/6+1/6
17(1/6)=	2+1/2+1/3

1	1/5
2	2/5=1/3+1/15
4	1/2+1/6+1/10+1/30
8	1+1/3+1/5+1/15
16	2+1/2+1/6+1/3+1/15+1/10+1/30
17(1/5)=	2+1/2+1/3+1/5+1/6+1/10+1/5+1/30

2/n table from the Rhind Mathematical Papyrus

2/3 = 1/2 + 1/6	2/5 = 1/3 + 1/15	2/7 = 1/4 + 1/28
2/9 = 1/6 + 1/18	2/11 = 1/6 + 1/66	2/13 = 1/8 + 1/52 + 1/104
2/15 = 1/10 + 1/30	2/17 = 1/12 + 1/51 + 1/68	2/19 = 1/12 + 1/76 + 1/114
2/21 = 1/14 + 1/42	2/23 = 1/12 + 1/276	2/25 = 1/15 + 1/75
2/27 = 1/18 + 1/54	2/29 = 1/24 + 1/58 + 1/174 + 1/232	2/31 = 1/20 + 1/124 + 1/155
2/33 = 1/22 + 1/66	2/35 = 1/30 + 1/42	2/37 = 1/24 + 1/111 + 1/296
2/39 = 1/26 + 1/78	2/41 = 1/24 + 1/246 + 1/328	2/43 = 1/42 + 1/86 + 1/129 + 1/301
2/45 = 1/30 + 1/90	2/47 = 1/30 + 1/141 + 1/470	2/49 = 1/28 + 1/196
2/51 = 1/34 + 1/102	2/53 = 1/30 + 1/318 + 1/795	2/55 = 1/30 + 1/330
2/57 = 1/38 + 1/114	2/59 = 1/36 + 1/236 + 1/531	2/61 = 1/40 + 1/244 + 1/488 + 1/610
2/63 = 1/42 + 1/126	2/65 = 1/39 + 1/195	2/67 = 1/40 + 1/335 + 1/536
2/69 = 1/46 + 1/138	2/71 = 1/40 + 1/568 + 1/710	2/73 = 1/60 + 1/219 + 1/292 + 1/365
2/75 = 1/50 + 1/150	2/77 = 1/44 + 1/308	2/79 = 1/60 + 1/237 + 1/316 + 1/790
2/81 = 1/54 + 1/162	2/83 = 1/60 + 1/332 + 1/415 + 1/498	2/85 = 1/51 + 1/255
2/87 = 1/58 + 1/174	2/89 = 1/60 + 1/356 + 1/534 + 1/890	2/91 = 1/70 + 1/130
2/93 = 1/62 + 1/186	2/95 = 1/60 + 1/380 + 1/570	2/97 = 1/56 + 1/679 + 1/776
2/99 = 1/66 + 1/198	2/101 = 1/101 + 1/202 + 1/303 + 1/606	

90

Problem 3 of the Rhind Mathematical Papyrus

Divide 6 loaves of bread among 10 men.

Each man receives

1/2+1/10.

Do it thus

1	1/2 + 1/10
2	
4	
8	

2/n table from the Rhind Mathematical Papyrus

2/3 = 1/2 + 1/6	2/5 = 1/3 + 1/15	2/7 = 1/4 + 1/28
2/9 = 1/6 + 1/18	2/11 = 1/6 + 1/66	2/13 = 1/8 + 1/52 + 1/104
2/15 = 1/10 + 1/30	2/17 = 1/12 + 1/51 + 1/68	2/19 = 1/12 + 1/76 + 1/114
2/21 = 1/14 + 1/42	2/23 = 1/12 + 1/276	2/25 = 1/15 + 1/75
2/27 = 1/18 + 1/54	2/29 = 1/24 + 1/58 + 1/174 + 1/232	2/31 = 1/20 + 1/124 + 1/155
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2/99 = 1/66 + 1/198	2/101 = 1/101 + 1/202 + 1/303 + 1/606	

The answer is given: each man gets 1/2 + 1/10 loaves. The scribe checks this answer is correct by multiplying the answer by 10.

Problem 3 of the Rhind Mathematical Papyrus

Problem 3

ir-[t]¹ t¹-w² 6 n s ir-hr · k wih-tp m i0 sp 10
 The making of loaves 6 for man [10]. Make thou the multiplication : [½] 1/10 times 10.

ir-t my
 The doing as [it occurs]:

[1] 2] i0
 [2] 1] 5³
 [4] 2] 3 is
 [8] 4 [3 i0] 30⁴
 dmd 6 my[t-t] pw
 Total 6; the same, this is.

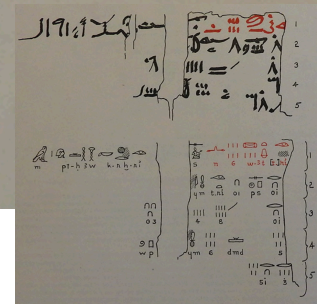


Image Credit: The Rhind Mathematical Papyrus translation by Arnolds Buffum Chace

**Problem 3 of the Rhind Mathematical Papyrus:
Divide 6 loaves of bread among 10 men.**

2/3 = 1/2 + 1/6	2/5 = 1/3 + 1/15	2/7 = 1/4 + 1/28
2/9 = 1/6 + 1/18	2/11 = 1/6 + 1/66	2/13 = 1/8 + 1/52 + 1/104
2/15 = 1/10 + 1/30	2/17 = 1/12 + 1/51 + 1/68	2/19 = 1/12 + 1/76 + 1/114
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2/75 = 1/50 + 1/150	2/77 = 1/44 + 1/308	2/79 = 1/60 + 1/237 + 1/316 + 1/790
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2/87 = 1/58 + 1/174	2/89 = 1/60 + 1/356 + 1/534 + 1/890	2/91 = 1/70 + 1/130
2/93 = 1/62 + 1/186	2/95 = 1/60 + 1/380 + 1/570	2/97 = 1/56 + 1/679 + 1/776
2/99 = 1/66 + 1/198	2/101 = 1/101 + 1/202 + 1/303 + 1/606	

NOTE: The symbol + will come MUCH LATER

Explanation from (in [] and () are comments for us to understand it better

Problems 1 to 6 of the Rhind Mathematical Papyrus
Author(s): R. J. Gillings
Source: *The Mathematics Teacher*, Vol. 55, No. 1 (JANUARY 1962), pp. 61-69
Published by: National Council of Teachers of Mathematics
Stable URL: <https://www.jstor.org/stable/27956523>
Accessed: 13-02-2020 00:53 UTC

Problem 3

Divide 6 loaves among 10 men.

Each man receives $\frac{1}{2} \frac{1}{10}$.

Proof. Multiply $\frac{1}{2} \frac{1}{10}$ by 10.

Do it thus:

1	$\frac{1}{2} \frac{1}{10}$
2	$1 \frac{1}{5}$
4	$2 \frac{2}{5} \frac{1}{5}$
8	$4 \frac{4}{5} \frac{1}{10} \frac{1}{50}$

Total 6 loaves, which is correct.

Problem 3

Division of 6 loaves among 10 men.
[Each man receives $\frac{1}{2} + \frac{1}{10}$.] (From his reference table.)

For proof multiply $\frac{1}{2} + \frac{1}{10}$ by 10.

Do it thus:

[If] 1 [part is] $\frac{1}{2} + \frac{1}{10}$
[then] $\sqrt{2}$ [parts are] $1 + \frac{1}{5}$
" 4 " $2 + \frac{1}{3} + \frac{1}{15}$
(From the Recto, $2 \div 5 = \frac{1}{3} + \frac{1}{15}$.)
" $\sqrt{8}$ " $4 + \frac{2}{3} + \frac{1}{10} + \frac{1}{30}$
(From the Recto, $2 \div 15 = \frac{1}{10} + \frac{1}{30}$.)
[Add the fractions on the lines with check marks (since $2 + 8 = 10$): $1 + \frac{1}{5} + 4 + \frac{2}{3} + \frac{1}{10} + \frac{1}{30} = 6$.]
93 Total 6 loaves which is correct.

Problem 5 of the Rhind Mathematical Papyrus:

Divide 8 loaves of bread among 10 men.

Each man receives $\frac{2}{3} + \frac{1}{10} + \frac{1}{30}$.

Do it thus

1	$\frac{2}{3} + \frac{1}{10} + \frac{1}{30}$
2	
4	
8	

2/3 = 1/2 + 1/6	2/5 = 1/3 + 1/15	2/7 = 1/4 + 1/28
2/9 = 1/6 + 1/18	2/11 = 1/6 + 1/66	2/13 = 1/8 + 1/52 + 1/104
2/15 = 1/10 + 1/30	2/17 = 1/12 + 1/51 + 1/68	2/19 = 1/12 + 1/76 + 1/114
2/21 = 1/14 + 1/42	2/23 = 1/12 + 1/276	2/25 = 1/15 + 1/75
2/27 = 1/18 + 1/54	2/29 = 1/24 + 1/58 + 1/174 + 1/232	2/31 = 1/20 + 1/124 + 1/155
2/33 = 1/22 + 1/66	2/35 = 1/30 + 1/42	2/37 = 1/24 + 1/111 + 1/296
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2/63 = 1/42 + 1/126	2/65 = 1/39 + 1/195	2/67 = 1/40 + 1/335 + 1/536
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2/93 = 1/62 + 1/186	2/95 = 1/60 + 1/380 + 1/570	2/97 = 1/56 + 1/679 + 1/776
2/99 = 1/66 + 1/198	2/101 = 1/101 + 1/202 + 1/303 + 1/606	

Problem 5 of the Rhind Mathematical Papyrus:

Problem 5

Divide 8 loaves among 10 men.

Each man receives $\frac{2}{3} \frac{1}{10} \frac{1}{30}$.

Proof. Multiply $\frac{2}{3} \frac{1}{10} \frac{1}{30}$ by 10; the result is 8.

Explanation from (in [] and () are comments for us to understand it better

Translation the papyrus

Problem 5

Division of 8 loaves among 10 men.
[Each man receives $\frac{2}{3} + \frac{1}{10} + \frac{1}{30}$.] (From his reference table.)

For proof multiply $\frac{2}{3} + \frac{1}{10} + \frac{1}{30}$ by 10.

Do it thus:

[If] 1 [part is] $\frac{2}{3} + \frac{1}{10} + \frac{1}{30}$
[then] $\sqrt{2}$ [parts are] $1 + \frac{1}{2} + \frac{1}{10}$ (As in Problem 4 [6].)
" 4 " $3 + \frac{1}{5}$
" $\sqrt{8}$ " $6 + \frac{1}{3} + \frac{1}{15}$
(From the Recto, $2 \div 5 = \frac{1}{3} + \frac{1}{15}$.)
[Add the fractions on the lines with check marks.]
Total 8 loaves which is correct.

2/n table from the Rhind Mathematical Papyrus

2/3 = 1/2 + 1/6	2/5 = 1/3 + 1/15	2/7 = 1/4 + 1/28
2/9 = 1/6 + 1/18	2/11 = 1/6 + 1/66	2/13 = 1/8 + 1/52 + 1/104
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2/87 = 1/58 + 1/174	2/89 = 1/60 + 1/356 + 1/534 + 1/890	2/91 = 1/70 + 1/130
2/93 = 1/62 + 1/186	2/95 = 1/60 + 1/380 + 1/570	2/97 = 1/56 + 1/679 + 1/776
2/99 = 1/66 + 1/198	2/101 = 1/101 + 1/202 + 1/303 + 1/606	

Method of false position

Exercise: Find the quotient of 19 divided by 8

Exercise: Find the quotient of 19 divided by 8

EXAMPLE:

1	8
2	16
1/2	4
1/4	2
1/8	1

Then the answer is $2 + 1/4 + 1/8$

Problem 24 of the Rhind Mathematical Papyrus:

A quantity and its $1/7$ added together become 19.
What is the quantity?

Method of false position

Problem 24 of the Rhind Mathematical Papyrus:

A quantity and its $1/7$ added together become 19. What is the quantity?

Assume 7	$\backslash 1/7$	1
Total		8
	1	8
	$\backslash 2$	16
	1/2	4
	$\backslash 1/4$	2
	$\backslash 1/8$	1
Total		$2 \frac{1}{4} \frac{1}{8}$
	$\backslash 1$	$2 \frac{1}{4} \frac{1}{8}$
	$\backslash 2$	$4 \frac{1}{2} \frac{1}{4}$
	$\backslash 4$	$9 \frac{1}{2}$

Do it thus:

1	$16 \frac{1}{2} \frac{1}{8}$
$1/7$	$2 \frac{1}{4} \frac{1}{8}$
Total	19.

As many times as 8 must be multiplied to give 19, so many times 7 must be multiplied to give the required number.

In other words, find x , so the ratio $7/8 = x/19$ holds.

Method of false position

Problem 24 of the Rhind Mathematical Papyrus:

A quantity and its $1/7$ added together become 19. What is the quantity?

Assume 7.

$\setminus 1$	7
$\setminus \frac{1}{7}$	1
Total	8.

First, let's solve the problem in the "spirit of ancient Egypt" but using modern notation and language.

To obtain 8, the quantity is 7. (We choose 7 because it is easy to compute and then we obtain 8)

To obtain 19, what is the quantity?

We have the ratio,

$19/8 = x/7$ where x is the number we are trying to find.

1	8
$\setminus 2$	16
$\frac{1}{2}$	4
$\setminus \frac{1}{4}$	2
$\setminus \frac{1}{8}$	1
Total $2 \frac{1}{4} \frac{1}{8}$.	

By proportionally, we know that to obtain 19, x - the quantity we are looking for- is 7 multiplied by $19/8$.

Before we found that $19/8 = 2 + 1/4 + 1/8$

$\setminus 1$	$2 \frac{1}{4} \frac{1}{8}$
$\setminus 2$	$4 \frac{1}{2} \frac{1}{4}$
$\setminus 4$	$9 \frac{1}{2}$

Do it thus: The quantity is	$16 \frac{1}{2} \frac{1}{8}$,
$\frac{1}{7}$	$2 \frac{1}{4} \frac{1}{8}$,
Total	19.

Hence, the answer is $7(2 + 1/4 + 1/8) = 16 + 1/2 + 1/8$

Method of false position

Problem 25 of the Rhind Mathematical Papyrus:

A quantity and its $1/2$ added together become 16.

What is the quantity?

Method of false position

Problem 25

A quantity and its $1/2$ added together become 16. What is the quantity?

Assume 2.

$\setminus 1$	2
$\setminus \frac{1}{2}$	1
Total	3.

As many times as 3 must be multiplied to give 16, so many times 2 must be multiplied to give the required number.

$\setminus 1$	3
2	6
$\setminus 4$	12
$\frac{2}{3}$	2
$\setminus \frac{1}{3}$	1
Total $5 \frac{1}{3}$.	

1	$5 \frac{1}{3}$
$\setminus 2$	$10 \frac{2}{3}$
Do it thus: The quantity is	$10 \frac{2}{3}$
$\frac{1}{2}$	$5 \frac{1}{3}$
Total	16.

A careful study of the Rhind Papyrus convinced me several years ago that this work is not a mere selection of practical problems especially useful to determine land values, and that the Egyptians were not a nation of shopkeepers, interested only in that which they could use. Rather I believe that they studied mathematics and other subjects for their own sakes.

The Rhind Mathematical Papyrus, Arnold Buffum Chace

After working on the Rhind Papyrus, with which of the two paragraphs you agree more and why?

The Rhind and Moscow papyri are handbooks for the scribe, giving model examples of how to do things which were a part of his everyday tasks . . . The sheer difficulties of calculation with such a crude numeral system and primitive methods effectively prevented any advance or interest in developing the science for its own sake. It served the needs of everyday life . . . and that was enough.

Mathematics and Astronomy, in The Legacy of Egypt, J. R. Harris (ed.), 27-54, Oxford University Press. - Gerald J. Toomer

**Problem 50 of Ahmes
Papyrus: Conjectures
about how the area of
the circle was found.**

If D is the diameter of each of the small black disks below then the length of the side of the square of the figure is:



Suppose that A is the area of each of the small black disks below. Which of the expressions approximates below best the area of the square?



The diameter of the large circle below is approximately $n \cdot D$ where D is the diameter of each of the small black disks. The value of n is...



Say the area of the small disk is A . Which of the expressions below approximates best the area of the large circle?

