Write down something about the beginnings of mathematics that we discussed last week

- All societies develop ideas of number.
- Counting as one-to-one correspondence
- Different ways of recording number (what do these ways depend on?)
- Formal definition of number is very hard (difficulty related to the barber paradox)
- **Primary sources**: A primary source is an original, firsthand, or direct piece of evidence or material that provides information about a particular topic or event.
- A **secondary source** is a document or material that is created based on information derived from primary sources. In academic research and historical analysis, secondary sources interpret, analyze, or comment on primary sources. They are **one or more steps** removed from the original events or materials and often involve synthesis, interpretation, or commentary by the author.
- The moment when the helper lowers their 10 fingers and the second helper lifts 1.

Image credit: https://creativekindergartenblog.com/one-to-one-correspondence-intervention-for-kindergarten/

It is crucial to use reliable sources of information.

From the Boston University Website https://www.bu.edu/africa/outreach/teachingresources/outreach-teachingresources-mathematics/

The Lebombo bone (top) is the oldest known mathematical artifact. It is a tally stick with 29 distinct notches that were deliberately cut into a baboon's fibula. It was discovered within the Border Cave in the Lebombo Mountains of Eswatini. The Lebombo bone (bottom) resembles a calendar stick still used in Namibia. See more about these artifacts under "Other Resources" below.

Mathematical Treasure: Ishango Bone
MAA website
https://www.maa.org/press/periodicals/convergence/mathematical-treasure-ishango-bone

- There is no need to be scare of paper or the presentation (memorize, master…)
- Beware of the use of AI. Some students who used it to write the paper, submitted a bad paper.
- About Wooclap and absences.
- HW0

Topics!

10101101

Number Systems

- The goal of this week is to give an overview of ideas related number representation, so we can later understand better how different societies represented numbers.
- We will briefly discuss number systems in Egypt, Mesopotamia, Greece, China and the Mayans.

Number systems

Number systems

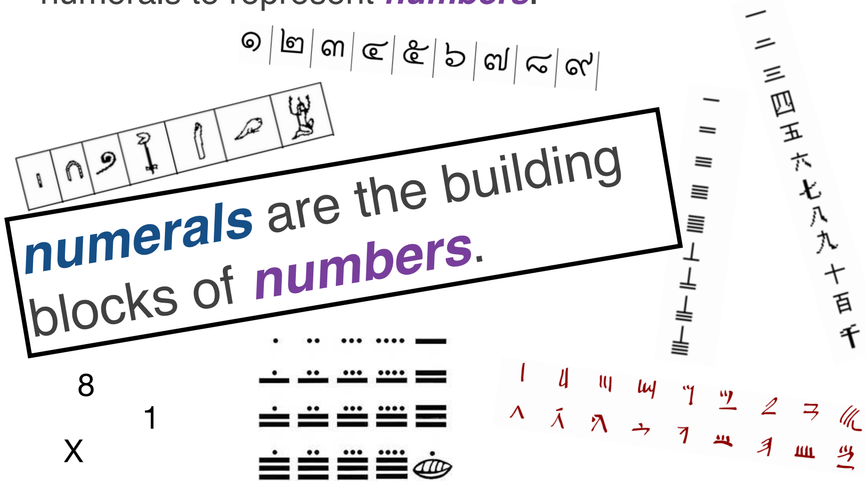
A ***number system*** consist on a set of symbols, called ***numerals***, and a set of ***rules*** for writing this numerals to represent ***numbers***.

Examples

number system	*numerals*	*numbers*	*rules*
Hindu-Arabic ("ours")			
Roman			
Binary			

Number systems

A ***number system*** consist on a set of symbols, called ***numerals***, and a set of ***rules*** for writing this numerals to represent ***numbers***.



Number systems

A ***number system*** consist on a set of symbols, called ***numerals***, and a set of ***rules*** for writing this numerals to represent ***numbers***.

We are going to consider four characteristics of number systems

- **Additive**
- **Ciphered or alphabetic**
- **Multiplicative**
- **Positional**

Note: characteristics ≠ classification:

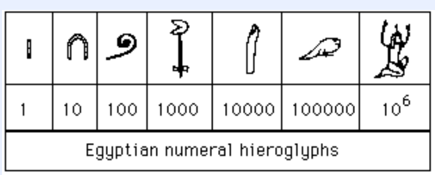
Additive number systems

Number systems

A ***number system*** consist on a set of symbols, called ***numerals***, and a set of ***rules*** for writing this numerals to represent ***numbers***.

We are going to consider four characteristics of number systems

- **Additive:** The value of a number is the sum of the values of the numerals.
- **Ciphered or alphabetic**
- **Multiplicative**
- **Positional**

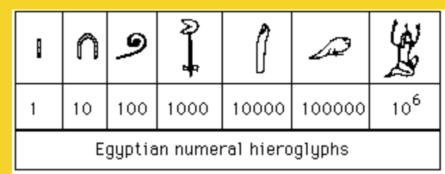


Images credits: https://mathshistory.st-andrews.ac.uk/HistTopics/Egyptian_numerals/

An additive number system: Egyptian Hieroglyphs

- based on a scale of 10
- used as far back as 3400 B.C.E.
- mostly for inscription in stones

numerals:



Write the number 752 in Egyptian hieroglyphics. In Wooclap, express number on the left in Hindu-Arabic numerals.

Images credits: https://mathshistory.st-andrews.ac.uk/HistTopics/Egyptian_numerals/

An additive number system: Egyptian Hieroglyphs

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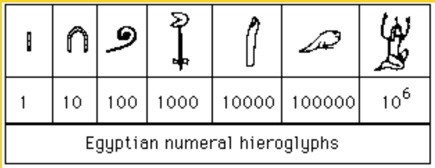
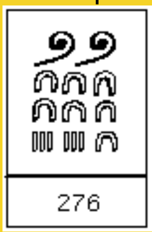
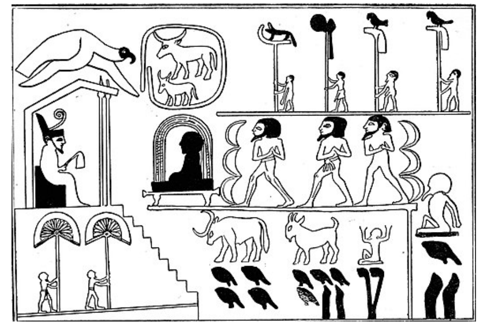


Example



Educated guess: what are the rules of the Egyptian hieroglyphic system?
Hint: One rule is related to the number of times a numeral can be repeated.

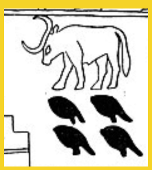
Ceremony in which captives and plunder are presented to Egyptian King Narmer (c. 31st century BCE)



Narmer Macehead (drawing). The design shows captives being presented to Pharaoh Narmer enthroned in a naos. Ashmolean Museum, Oxford.

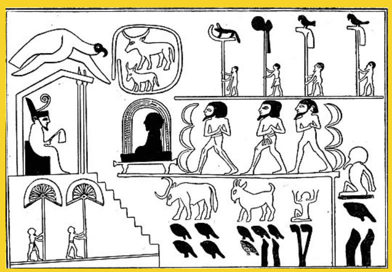
Decipher with your team 1, 2 and 3 (on the left). Each member of the team writes down their answer individually. You have 7 minutes.

2



1

3



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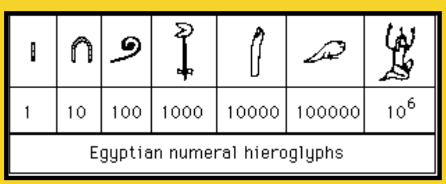
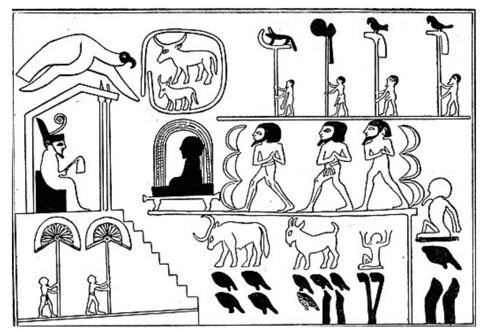


Ceremony in which captives and plunder are presented to Egyptian King Narmer (c. 31st century BCE)

Here is a good picture of this source
https://digi.ub.uni-heidelberg.de/diglit/quibell1900bd1/0038/image

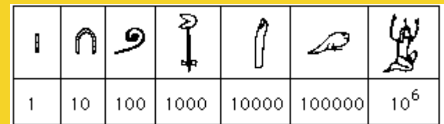


Narmer Macehead (drawing). The design shows captives being presented to Pharaoh Narmer enthroned in a naos. Ashmolean Museum, Oxford.

The scene depicts a ceremony in which captives and plunder are presented to King Narmer, who is enthroned beneath a canopy on a stepped platform. He wears the Red Crown of Lower Egypt, holds a flail, and is wrapped in a long cloak. To the left, Narmer's name is written inside a representation of the palace facade (the *serekh*) surmounted by a falcon. At the bottom is a record of animal and human plunder; 400,000 cattle, 1,422,000 goats, and 120,000 captives

Answer as many questions as you can

1. **What is the maximum number of times a numeral can be repeated in a single number in the Egyptian hieroglyphic number system?**
2. **Suppose that L is the largest number that can be written in Egyptian hieroglyphics. What is L?**
3. **Suppose M is the number of numerals in L (L is as in the previous question). What is M?**



Egyptian numeral hieroglyphs

An additive system invented by me

value	1	5	25	125
numerals	a	b	c	d

1. Express abbcdd in Hindu-Arabic numerals.
2. Express 106 in this additive system

Rules:
- Numerals are written from left to right, from the numeral with smallest value to the numeral with largest value. (abbcdd)
- The number of numerals used must be the smallest possible (for instance, we should write “b” instead of “aaaaa”)

Ciphered or alphabetic number systems

Number systems

A ***number system*** consist on a set of symbols, called ***numerals***, and a set of ***rules*** for writing this numerals to represent ***numbers***.

A number system can be.

- **Additive**:
- **Ciphered or alphabetic**: Numerals design 1, 2,..9, and the powers of 10 (or, more generally, some base) but also to the multiples of this powers. Example: Greek Alphabetic
- **Multiplicative**
- **Positional**:

Letter	Value	Letter	Value	Letter	Value
α alpha	1	ι iota	10	ρ rho	100
β beta	2	κ kappa	20	σ sigma	200
γ gamma	3	λ lambda	30	τ tau	300
δ delta	4	μ mu	40	υ upsilon	400
ε epsilon	5	ν nu	50	φ phi	500
ς digamma	6	ξ xi	60	χ chi	600
ζ zeta	7	ο omicron	70	ψ psi	700
η eta	8	π pi	80	ω omega	800
θ theta	9	Q koppa	90	ϡ sampi	900

Table from https://online.math.uh.edu/Math2303-unpaid/ch1/s12/index.html

A ciphered number system: Greek Alphabetic

Rule: Numeral in ascending value, from right to left. Repetitions?

1. Write the number 752 in Greek numerals
2. Translate σπγ to Hindu-Arabic.

Numerals

Letter	Value	Letter	Value	Letter	Value
α alpha	1	ι iota	10	ρ rho	100
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Table from https://online.math.uh.edu/Math2303-unpaid/ch1/s12/index.html

Is the Greek alphabetic system additive? Why or why not?

Letter	Value	Letter	Value	Letter	Value
α alpha	1	ι iota	10	ρ rho	100
β beta	2	κ kappa	20	σ sigma	200
γ gamma	3	λ lambda	30	τ tau	300
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Greek alphabetic system

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Table from https://online.math.uh.edu/Math2303-unpaid/ch1/s12/index.html

Alphabetic Greek: For the numbers 1000 to 9000, they wrote: ‘α,’β, ‘ɣ…’Θ (For instance, ’β represents 2000)

10000 was written $\overset{\alpha}{M}$

There were rules for numbers up to 640,000,and even larger

Your questions

Are there any words or questions you don't like about the format and content of students' emails?
How did you choose your future job?
How you became interested in mathematics and your area of study. And how has your identity impacted your path as a mathematician.
I would like to know that if you have enjoyed math all along? Was there a time that you ever feel like math might not be the right path, or you have always loved it?
What initially sparked your interest in math?
What is your favorite mathematical object? Could be geometric, algebraic, etc.
What is your favorite topic to teach in this course?
Why did you choose to study math and which topic is your personal favorite? Besides math, what other subjects and hobbies do you enjoy?
What are your favorite topics regarding math?
What aspects of math are you most interested in and is that what we're going to be focusing on in class.
What mathematics did you mainly focus on studying when you got your PHD.

Multiplicative number systems

Number systems

A ***number system*** consist on a set of symbols, called ***numerals***, and a set of ***rules*** for writing this numerals to represent ***numbers***.

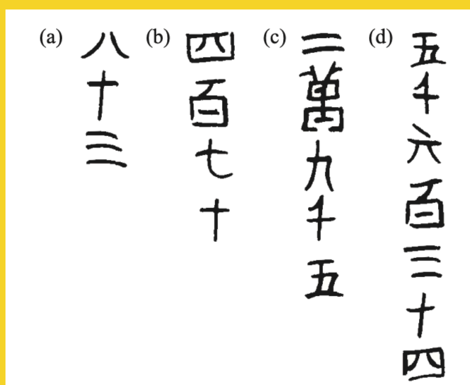
A number system can be.

- **Additive**:
- **Ciphered or alphabetic**:
- **Multiplicative**: There are two sets of numerals, the elements of one set represent digits and the elements of the other set represent position. If necessary, a digit and a position symbols are used together, and the values of numerals are multiplied. Finally, all the products are added.
- **Positional**

Number	Symbol
0	零
1	一
2	二
3	三
4	四
5	五
6	六
7	七
8	八
9	九
10	十
100	百
1000	千

A multiplicative system
Traditional Chinese numerals

Write the numbers below (in traditional Chinese numerals) in Hindu-Arabic numerals.



Burton, David M. "The history of mathematics: An introduction." Group 3.3 (1985)

1	一
2	二
3	三
4	四
5	五
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7	七
8	八
9	九

10	十
100	百
1000	千
10,000	萬
100,000	億

Burton, David M. "The history of mathematics: An introduction." *Group* 3.3 (1985)

Multiplicative system 2

	1	5	25	125	625	3125	15625
numerals representing position	a	b	c	d	e	f	g

digits (numerals)	0	1	2	3	4

1. Write 1060 down the numbers in this system
2. Translate d2 b3 a4 to the Hindu-Arabic number system

Multiplicative number system

- There are two sets of numerals, the elements of one set represent digits and the elements of the other set represent position. If necessary, a digit and a position symbols are used together, and the values of numerals are multiplied. Finally, all the products are added.

Invented multiplicative system 1

	1	10	100	1000
numerals representing position	a	b	c	d

Digits (numerals)	0	1	2	3	4	5	6	7	8	9

1. Translate d2c7b3a8 from the multiplicative system 1 to the Hindu-Arabic number system.
2. Write 1065 down the numbers in the multiplicative system 1

Multiplicative number system

- There are two sets of numerals, the elements of one set represent digits and the elements of the other set represent position. If necessary, a digit and a position symbols are used together, and the values of numerals are multiplied. Finally, all the products are added.

Number systems

A ***number system*** consist on a set of symbols, called ***numerals***, and a set of ***rules*** for writing this numerals to represent ***numbers***.

Four characteristics of number systems

- **Additive:**
- **Ciphered or alphabetic**
- **Multiplicative**
- **Positional:** The value of each numeral depends on its position. The system consists of a **base** (a natural number greater than one) and a **set of numerals** representing the numbers from zero to one less than the base. The numbers from zero to the base minus one are the digits in the system.

Example
$345 = 3.\ 10^2 + 4.10 + 5$
$5 = (101)_2$

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A ***number system*** consist on a set of symbols, called ***numerals***, and a set of ***rules*** for writing this numerals to represent ***numbers***.

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- **Additive**
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- **Positional**

Note:
characteristics ≠ classification:

Positional number systems

Examples of a Positional Systems Around the World

- Binary
- Hindu-Arabic ("ours")
- Mayan
- Babilonian (Mesopotamian)
- Chinese Rod Number System (different from the Traditional Chinese number system we discussed before)

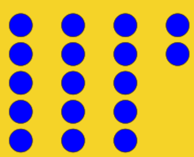
- **Positional**: The value of each numeral depends on its position. The system consists of a base (a natural number greater than one) and a set of numerals representing the numbers from zero to one less than the base. The numbers from zero to the base minus one are the digits in the system.

Important statement for Positional Number Systems

Integer division:

Given two integers **a** and **b**, with b > 0, there exist unique integers **q** and **r** such that **a = b.q + r** and 0 ≤ **r** < **b**

In this figure, a=17. What are the values of b, q, and r?



- a is called the *dividend*,
- b is called the *divisor*,
- q is called the *quotient,*
- r is called the *remainder.*

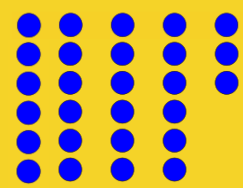
This statement answers the question: *What is the maximum number of times* **b** *"enters" into* **a**, *and what is remaining after this maximum number of* **b** *is subtracted from* **a**?

Note: The result works for a, b integers, b ≠ 0, but we will only work with positive numbers.

Important statement for Positional Number Systems

<u>Integer division</u>: Given two integers **a** and **b**, with b > 0, there exist unique integers **q** and **r** such that **a = b.q + r** and 0 ≤ **r** < **b**,

In this figure, a=27 (the total umber of blue dots). What are the values of b, q, and r?



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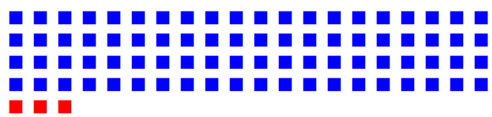
Note: The result works for a, b integers, b ≠ 0, but we will only work with positive numbers.

Division Algorithm

Theorem (Integer or Euclidean Division) For each pair a and b of integers, a positive there exists unique integers q and r such that

- a = q.b+r
- $0 \leq r < b$.

Example: If a=83, b=20, then q=4 and r=3



$83 = 4 \times 20 + 3$

From base 10 to base b≠10

We are given N (in base 10).
Suppose that we know that $N=(u,v)_b$,
then N=u.b +v, with $0 \leq u, v < b$.
In this case, to write N in base b need to find u and v.

if N=100 and b=11, find u and v.

Recall: Given two integers **a** and **b**, with b > 0, there exist unique integers **q** and **r** such that **a = b.q + r** and $0 \leq$ **r** < **b**

From a base b≠10 to base 10.

If $N=(18,6)_b$ then N=18.b +6
For instance, if the base b is 20, then
$N=(18,6)_{20}$ =18.20 +6 =376

Analogously, if $N=(15, 0, 10)_b$
then N=15. b^2 +0.b +10.

(N without parenthesis is assumed to be in base 10)

A positional system in base b (from base b to base 10)

Examples
$345 = 3.\ 10^2 + 4.10+5$
$5 = (101)_2$

Consider a positive integer $b \geq 2$.
In a positional number system on base b the numerals are
0, 1, 2,…, b-1.
A number in base **b** is denoted by $N=(a_n, a_{n-1}, \ldots, a_2, a_1, a_0)_b$
where each a_i is a base **b** numeral. Hence, to find N in base 10 we compute $a_n b^n + a_{n-1} b^{n-1} + \ldots + a_1 b + a_0$.

Write $(2, 10, 5)_{11}$ in base 10.

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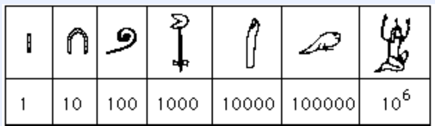

Egyptian numeral hieroglyphs

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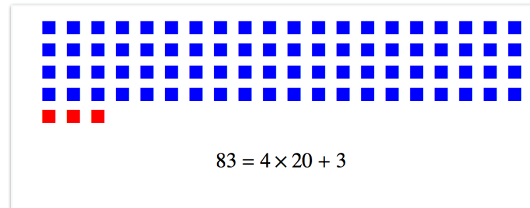
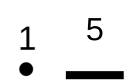


From base 10 to base b≠10: integer division

From base b≠10 to base 10: replace

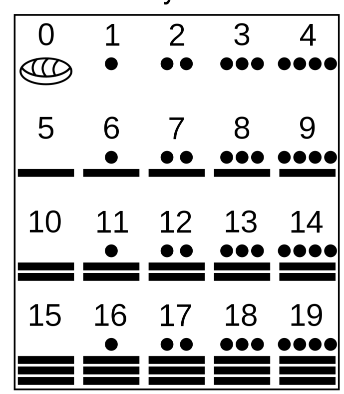
A positional system in 2 Mayan (in Mesoamerica)

Two special numerals



Most likely, these two numerals are from an older additive number system.

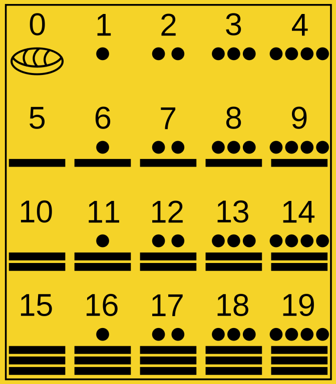
All the Mayan numerals



Express the number 752 Mayan number system.

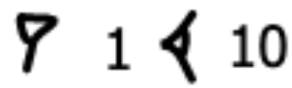
In Wooclap, express 752 in base 20.

For instance, 445 can be expressed as (1,2,5)_20



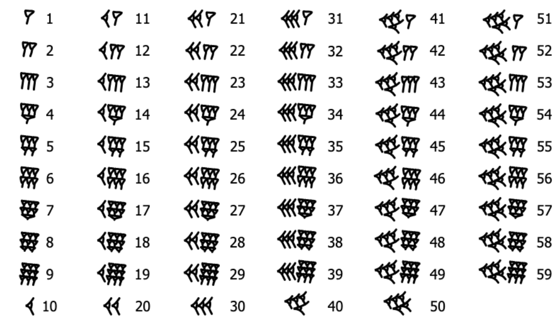
A positional system in base 60 Mesopotamian

Two special numerals



Most likely, these two numerals are from an older additive number system.

All the Mesopotamian numerals



Express the number 752 cuneoiform number system.

In Slido, express 752 in base 60.

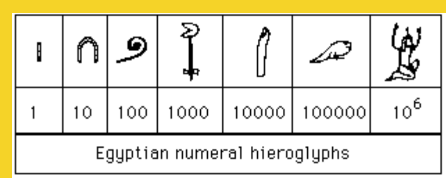
For instance, 70 can be expressed as (1,10)_60

Numerals

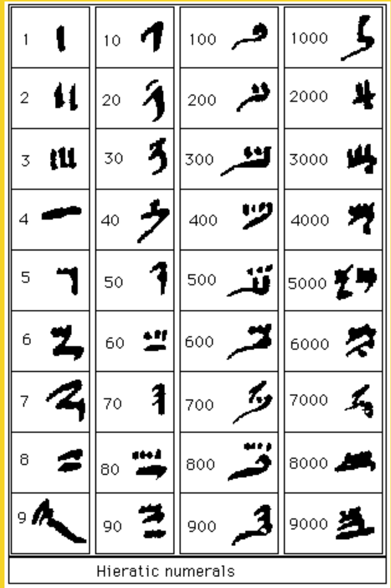
Answer as many questions as you can

1. **What is the maximum number of times a numeral can be repeated in a single number in the Egyptian hieroglyphic number system?**
2. **Suppose that L is the largest number that can be written in Egyptian hieroglyphics. What is L?**
3. **Suppose M is the number of numerals in L (L is as in the previous question). What is M?**

Reminder: The Egyptian hieroglyphic system is **additive**, the value of a number is the sum of the values of the numerals.



Hieratic script is the cursive form of hieroglyphic. It was used for administrative and literary purposes. The hieratic numerals below suggest that the hieratic number system is additive, multiplicative, ciphered or positional? Why?



https://mathshistory.st-andrews.ac.uk/HistTopics/Egyptian_numerals/

Number systems

A ***number system*** consist on a set of symbols, called ***numerals***, and a set of ***rules*** for writing this numerals to represent ***numbers***.

We are going to consider four characteristics of number systems

- **Additive**
- **Ciphered or alphabetic**
- **Multiplicative**
- **Positional**

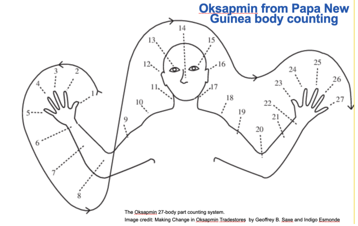
Note:
characteristics ≠ classification:

Note:
There are two different concepts, number and representation of number (in symbols or words).

Numbers and numerals are also different concepts.

	Means *two* of the following
Mex	trees, sticks, pencils and some other long things
Met	leaves or pieces of textiles or other flat items.
Mik	berries, balls and other round things.

Oksapmin from Papa New Guinea body counting



To count, say, a pile of coconuts, (let's say) she collected a heap of sticks.

- For each coconut in the pile, she took a stick.
- Each time she took a stick, she said "another one"
- When finished, she pointed out the pile of sticks she took and said "That many".

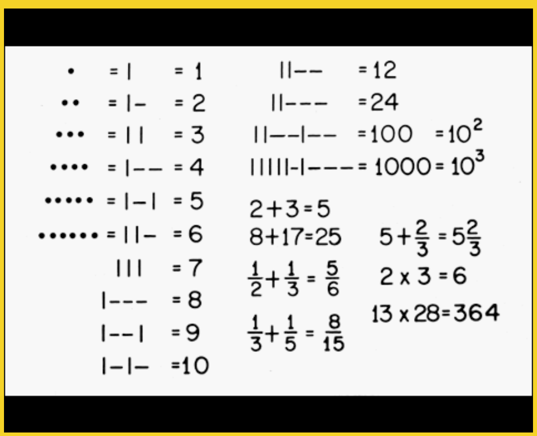
What happens with these counting systems when there is a need of using large numbers?

Express 20 in base 20.

Is the Roman number system positional? Why or why not?

One of the images from the Golden Record launched in 1977is shown below.
Can you relate it to the topic we are studying, number systems?

Images on the Golden Record



Source: https://voyager.jpl.nasa.gov/galleries/images-on-the-golden-record/
Image credit: Francis Drake

For more info: https://voyager.jpl.nasa.gov/golden-record/

Traditional Chinese

1	一
2	二
3	三
4	四
5	五
6	六
7	七
8	八
9	九

10	十
100	百
1000	千
10,000	萬
100,000	億

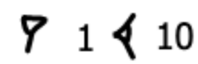
Burton, David M. "The history of mathematics: An introduction." *Group* 3.3 (1985)

Moira's multiplicative system

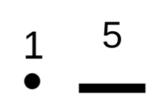
	1	5	25	125	625	3125	15625
numerals representing position	a	b	c	d	e	f	g

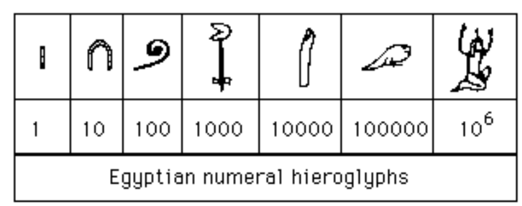
digits (numerals)	0	1	2	3	4

Mesopotamia (base 60)



Maya (base 20)





Complete the table

Mayan	*Hindu-Arabic ("ours") and Rod numerals*	*Roman*	Egyptian hieroglyphics	Babylonian Cuneiform	Traditional Chinese	Greek alphabetic	Moira's Multiplicative system
	25						
		DCCXIX					
	45625						

Base 20	*Hindu-Arabic ("ours") and Rod numerals*	*Roman*	*Binary*	Base 60	Moira's Multiplicative system
$(1,5)_{20}$	25	XXV	$(1,1,0,0,1)_2$	$(25)_{60}$	$(1,0,0)_5$=1c
$(8, 15, 19)_{20}$	3519	MMMDXIX	$(1, 1, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1)_2$	$(58, 39)_{60}$	$(1, 0, 3, 0, 3, 4)_5$=
$(5, 14, 1, 5)_{20}$	45625	~~XLV~~DCXXV	$(1, 0, 1, 1, 0, 0, 1, 0, 0, 0, 1, 1, 1, 0, 0, 1)_2$	$(12, 40, 25)_{60}$	$(2, 4, 3, 0, 0, 0, 0)_5$=
$(16, 11, 9)_{20}$	6629	~~V~~MDCXXIX	$(1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 1, 0, 1)_2$	$(1, 50, 29)_{60}$	$(2, 0, 3, 0, 0, 4)_5$ = 2g 3e 4a

Completed table

Mayan	*Hindu-Arabic ("ours")*	*Roman*	Egyptian hieroglyphics	Babylonian Cuneiform	Traditional Chinese	Greek alphabetic
	25	XXV			二十五	κε
304 = $(15,4)_{20}$	304	MMMDXIX		304 = $(5,4)_{60}$	三百四	τδ
	45625	~~XLV~~ DCXXV			The numerals for 10000 was not given.	Numerals larger than 900 were not given

Fractions in base 60

Express 1/16 and 1/11 in base 60