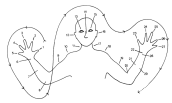
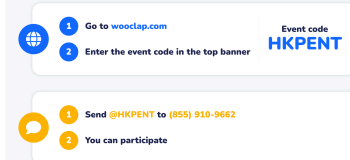


## The very beginning



- About this marvelous course
- What do we mean by mathematics in this course?
- Early math document: The Ishango bone. Interpretations.
- Reliability of sources
- Counting: history and meaning
- Different ways of counting
- History of counting hidden in words for numbers
- Timeline of mathematics
  
- And the syllabus.

## Important: log in to the platform with your SB email



Write down your first name and something you like about math.

## Who am I?



Instagram @moirart3

I did my PhD at the Autnoma University of Barcelona



<http://www.math.stonybrook.edu/~moira/>

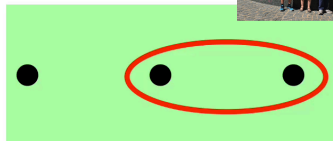
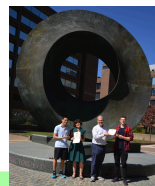


I was born in Buenos Aires, Argentina

I studied "licenciatura" in Mathematics at the University of Buenos Aires



I'm a professor at Stony Brook University, New York, US



## What is mathematics?

- Make groups of four students and go to a blackboard.
- Introduce yourself to the members of your group and exchange coordinates for future communication.
- Write down a definition of mathematics that you agree with (do not use sources other than your group to maximize your learning).
- You have 5 minutes.

## What is mathematics?

Write down in Wooclap your own definition.

# What is mathematics?

## What is mathematics?

## What do you mean by mathematics?

Fundamental type of question

All science requires mathematics. The knowledge of mathematical things is almost innate in us! This is the easiest of sciences, a fact which is obvious in that no ones brain's rejects it; for laymen and people who are utterly illiterate know how to count and reckon.



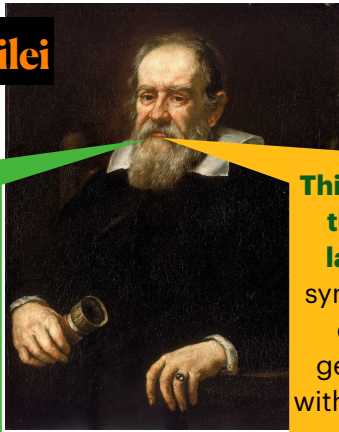
Roger Bacon. Stipple engraving, 1786.  
Wellcome Library no. 6961  
<https://wellcomecollection.org/works/nj73cnm?wellcomeimages/indexplus/image/00000285.html>

Roger Bacon (1214–1294), English philosopher and scientist

## Galileo Galilei

~1600

Philosophy is written in that **great book which ever lies before our eyes** — I mean the **universe** — but we cannot understand it if we do not first learn the language and grasp the symbols, in which it



Justus Sustermans - Portrait of Galileo Galilei, 1636  
Public Domain

**This book is written in the mathematical language**, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth.

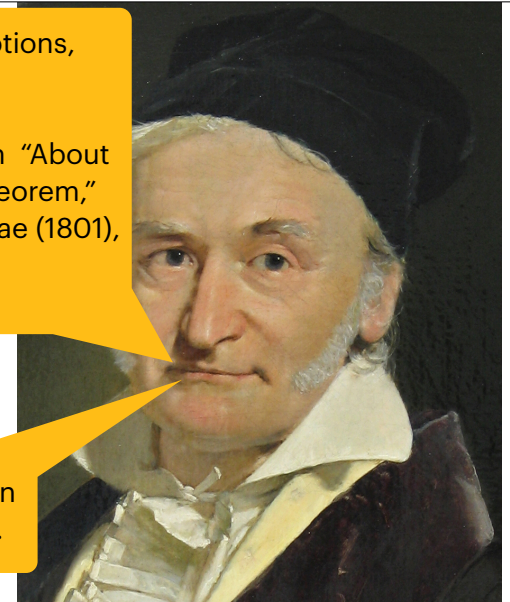
“What we need are notions, not notations.”

- I wrote that sentence in “About the proof of Wilson's theorem,” *Disquisitiones Arithmeticae* (1801), Article 76

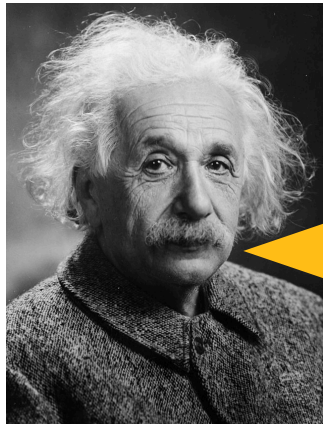
## Carl Friedrich Gauss

~1800

Mathematicians stand on each other's shoulders.



Portrait of the mathematician and philosopher Carl Friedrich Gauss (1777-1855) - Public domain



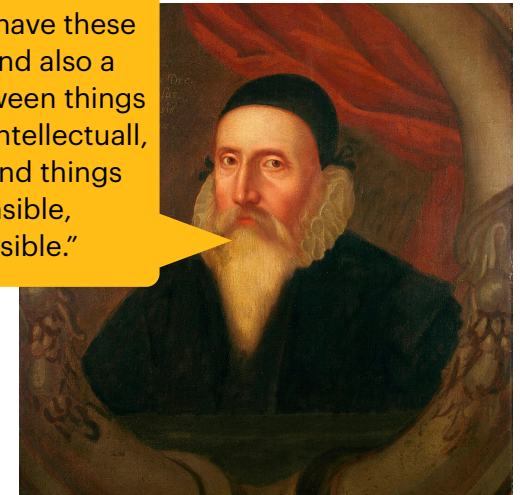
Photograph by Orren Jack Turner, Princeton, N.J. - Public domain

“**Pure mathematics is, in its way, the poetry of logical ideas.** One seeks the most general ideas of operation which will bring together in simple, logical and unified form the largest possible circle of formal relationships. In this effort toward logical beauty spiritual formulas are discovered necessary for the deeper penetration into the laws of nature.”

Albert Einstein, Obituary for Emmy

## Albert Einstein

“A marveilous newtrality have these things mathematicall, and also a strange participation between things supernaturall, immortall, intellectuall, simple and indivisible, and things naturall, mortall, sensible, compounded and divisible.”



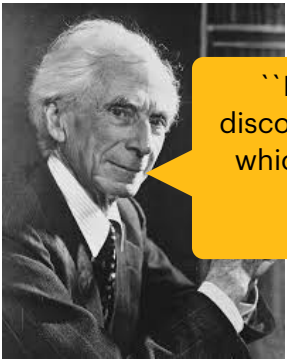
It seems that this portrait was painted when Dee was 67.

John Dee, 1527-1608. English mathematician and astrologer.  
Preface to his edition of Euclid's *Elements*, 1570

# What is mathematics?

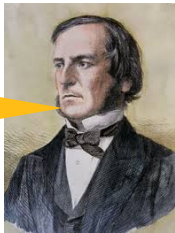
In the 1950s, Sawyer described mathematics as the "classification and study of all possible patterns". He explained that pattern was meant "to cover almost any kind of regularity that can be recognized by the mind".

W. W. Sawyer, Prelude to Mathematics, 1955



"Pure mathematics was discovered by Boole, in a work which he called The Laws of Thought (1854)".  
**1918**

I am George Boole



This file is made available under the Creative Commons CC0 1.0 Universal Public Domain Dedication

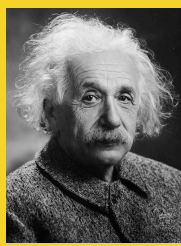
Bertrand Russell.

George Boole, mathematician, 1815-1864 - Public Domain

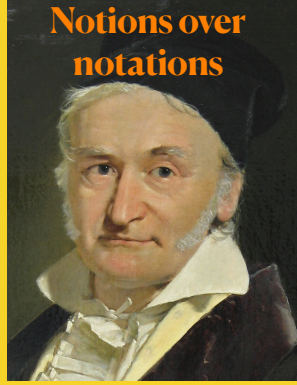
# What do you mean by mathematics?

In this course, by mathematics we mean study of ideas related to number, space and shapes, patterns and structure.

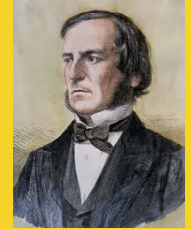
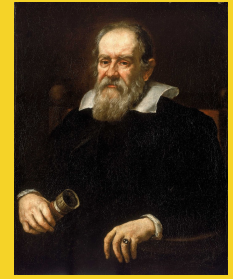
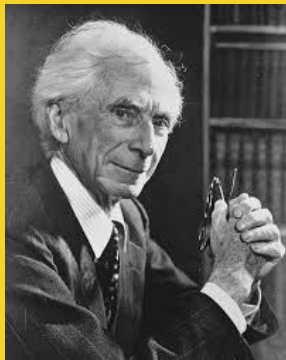
# Do you find a pattern?



Photograph by Orren Jack Turner, Princeton, N.J. - Public domain



# Notions over notations





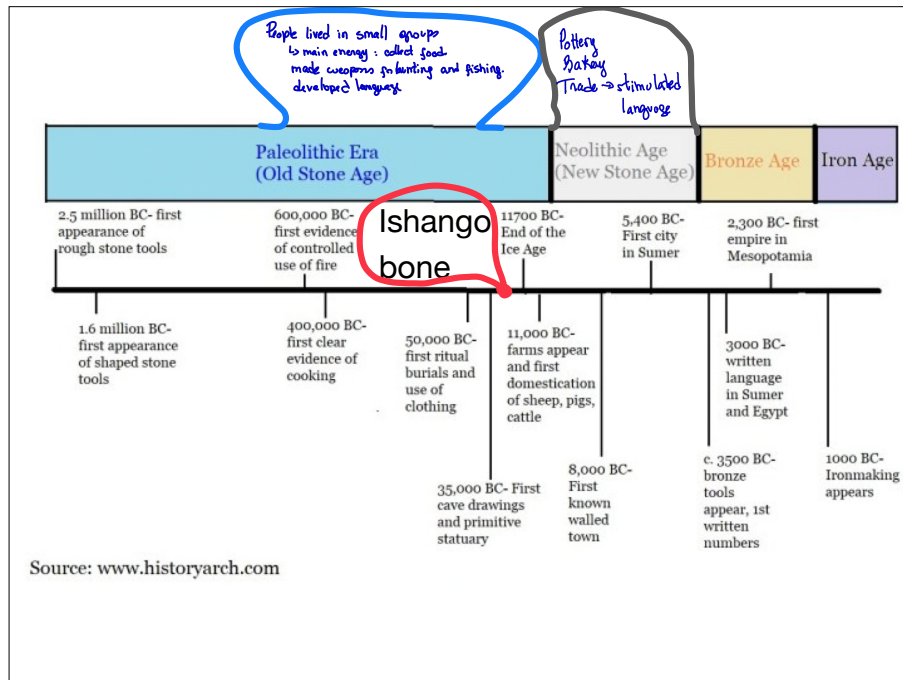
I have said that the love of study is the passion most necessary to our happiness. It is an unfailing resource against misfortunes, it is an unending source of pleasures,...

Madame du Châtelet  
~1700

Gabrielle Émilie Le Tonnelier de Breteuil, marquise de Châtelet (1706-1749), French mathematician and physicist - Maurice Quentin de La Tour

J'ai dit que l'amour de l'étude étoit la passion la plus nécessaire à notre bonheur : c'est une ressource sûre contre les malheurs ; c'est une ressource de plaisirs inépuisable,

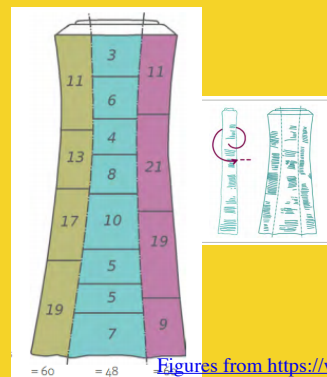
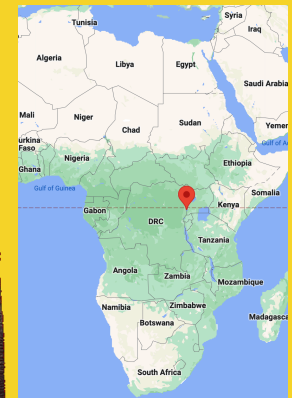
# The Ishango bone and its interpretations



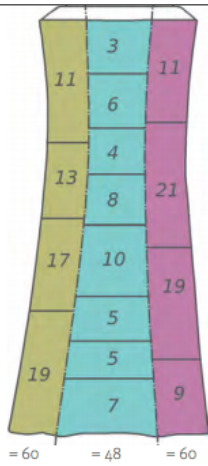
## Ishango Bone (About 20,000 BC)

Educated guess: The purpose of the Ishango bone is:

- counting animals
- serve as a calendar
- unknown



Figures from <https://www.naturalsciences.be/sites/default/files/Discover%20Ishango.pdf>



## Interpretations: Ishango Bone

Jean de Heinzelin (who found it in 1960) proposed  
 - arithmetical game,  
 - counting system based on 10  
 - a knowledge of multiplication by 2 and of prime numbers.

Alexander Marshack (1972)  
 - six-month lunar calendar.

Claudia Zaslavsky (1991) suggested  
 - lunar calendar made by women to keep track of their monthly cycles.

Olivier Keller (2010)  
 - there is a great deal of mathematical fiction in the study of the Ishango bone and nothing definite can be said.

“We do not see things as they are, we see them as we are.” Anais Nin, probably quoting from the Talmud.



Are other cultures so different that they are incomprehensible to us?

## Let's distinguish conjectures from facts!

“All history is contemporary history.”  
 Benedetto Croce

“As the South African archeologist David Lewis-Williams says of prehistoric art, ‘Meaning is always culturally bound’”  
 R. Leaky - The origin of humankind

“We do not see things as they are, we see them as we are.” Anais Nin, probably quoting from the Talmud.



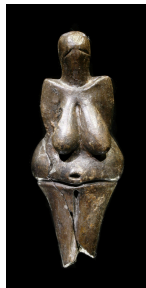
Are other cultures so different that they are incomprehensible to us?

## Dolní Věstonice (in Czech Republic) archeological site

- From (roughly) 27,000 to 20,000 B.C
- Many archeological artifacts.
- Representations of men, women, and animals, along with personal ornaments, human burials and engravings.

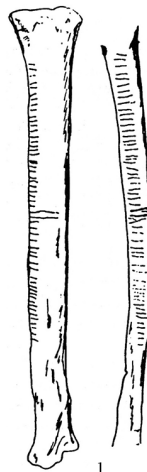


Carved female head from Dolní Věstonice, Krahuletz-Museum (replica)  
 Source: Wikipedia



Venus of Dolní Věstonice, the earliest discovered use of ceramics (29,000 BCE – 25,000 BCE) by Petr Novák, Wikipedia

Tally Sticks of the Stone Age  
 Andrej Kapcar  
[https://www.academia.edu/18709816/Tally\\_Sticks\\_of\\_the\\_Stone\\_Age](https://www.academia.edu/18709816/Tally_Sticks_of_the_Stone_Age)



The Wolf Bone of Dolní Věstonice

## Did Neanderthals count?

Hyena Bone, about 60,000 years old, found in France



Prehistoric accounting? Markings made on a hyena bone by a Neanderthal might have recorded numerical information.

Barras, C. (2021). How did Neanderthals and other ancient humans learn to count?. *Nature*, 594(7861), 22-25.  
<https://media.nature.com/original/magazine-assets/d41586-021-01429-6/d41586-021-01429-6.pdf>

Photo Credit Seawright, Caroline. (2014). ARC3PAL Essay: Ochre and the African middle stone age record. - 70,000 years old - found in Blombos Cave in South Africa



Recall: Ishango bone, 20,000BCE

Can you see any math in these photos?

Reproductions one of Lascaux artworks by Jack Versloot (about 17,000 years old)



## Lebombo bone about 44,000 years old



The oldest mathematical artefact  
The Mathematical Gazette , Volume 71 , Issue 458 , December 1987 , pp. 294  
DOI: <https://doi.org/10.2307/3617049>



Royal Belgian Institute of Natural Sciences



From the Boston University Website  
<https://www.bu.edu/africa/outreach/teachingresources/outreach-teachingresources-mathematics/>

The Lebombo bone (top) is the oldest known mathematical artifact. It is a tally stick with 29 distinct notches that were deliberately cut into a baboon's fibula. It was discovered within the Border Cave in the Lebombo Mountains of Eswatini. The Lebombo bone (bottom) resembles a calendar stick still used in Namibia. See more about these artifacts under "Other Resources" below.

Mathematical Treasure: Ishango Bone  
MAA website  
<https://www.maa.org/press/periodicals/convergence/mathematical-treasure-ishango-bone>



From the Boston University Website  
<https://www.bu.edu/africa/outreach/teachingresources/outreach-teachingresources-mathematics/>

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Why do you think I showed these two photos? Can you extract a conclusion to apply to your research in this class?

## Reindeer antler of Little Salt Spring, Florida , about 8000BCE



- suggests that the marks were made systematically.*
- 29 incisions
  - very regular
  - about five millimeters long
  - the spacing between them is consistent

- smaller etchings in one-to-one alignment with the larger incisions and next to them,
- suggest that quantities were ticked off along that progression.*

The reindeer antler of Little Salt Spring, Florida, with a colleague's hand for scale. Photograph by Caleb Everett.

Source: Numbers and the Making of Us: Counting and the Course of Human Cultures, by Caleb Everett., 2017, Harvard University Press

## Blanchard "calendar" bone



Photo: Don Hitchcock 2015  
Source: Facsimile, Musée d'Archeologie Nationale et Domaine, St-Germain-en-Laye

This bone suggests that its creator(s) were deliberately representing numerical concepts materially.

The farther we delve into the past, the more we find mathematics entangled with accounting, surveying, astronomy, and the general administration of empires. **Mathematics arises wherever people think about the physical world or about the world of ideas embodied in laws and even theology.** It grows like a plant, from a seed that germinates and later ramifies to produce roots, branches, leaves, flowers, and fruit. It is constantly growing.  
Roger Cooke - The history of Mathematics - Third Edition

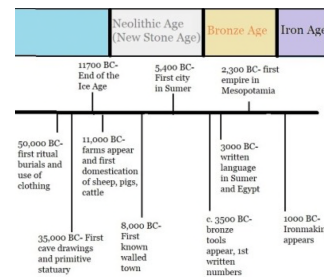
Photo Credit Seawright, Caroline. (2014). ARCSPAL Essay: Ochre and the African middle stone age record. - 70,000 years old - found in Blombos Cave in South Africa



Recall: Ishango bone, 20,000BCE

Can you see any math in these photos?

Reproductions one of Lascaux artworks by Jack Versloot (about 17,000 years old)





# Administrative stuff

## Course Website and Wooclap

- We are going to use the interactive platform Wooclap for questions, polls and surveys.
- You will be able to answer the questions in this platform from a web browser or a smart phone app.

### Course Website

<https://www.math.stonybrook.edu/~moira/courses/mat336-sp2024>

Note: A quick way to find the course website is googling my name, go to my website and find there the link for the course.

## The schedule

### Schedule for MAT 336 - Spring 2024 - Lecture 1

- The list of topics of this schedule is tentative and will be updated with the progress of the class.
- Please post a comment in the Discussion Form in Brightspace if you find a mistake, broken link or something confusing.
- The suggested articles for presentations and paper are [here](#).
- The presentation topics will be assigned before the second week of class.
- Each assignment of the schedule is listed on its due date.

Date	#	Topics	Quizzes and Assignments Due Dates	Presentations
Tu 1/23	1	Beginnings of mathematics, info about the course		
Th 1/25	2	Beginnings of mathematics, info about the course	HW0 Due Reading: <a href="#">On the origin of numbers</a>	
Tu 1/30	3	Beginnings of mathematics / Number systems	Reading: <a href="#">Number systems</a>	
Th 2/1	4	Number systems	HW1 due (Sources, beginnings)	
Tu 2/6	5	Mathematics in Ancient Egypt	Reading : <a href="#">These notes</a> from the beginning until Question 2 in Section 1.1.2. (Of course you are welcome to read them all...)	
Th 2/8	6	Mathematics in Ancient Egypt	HW2 due (Number systems)	
Tu 2/13	7	Mathematics in Ancient Egypt /Mesopotamia	Reading: <a href="#">This text</a> from the beginning to Section 1.5 Plimpton 322 (as usual, you are encouraged to read it all.)	

### What

### % of the grade

Homework 0

1%

Homework Assignments

20%

Quizzes

20%

Wooclap Answers

20%

Presentation

14%

Paper

15%

Paper preparation

(Anotated bibliography for the paper, baby draft, draft, abstract, outline)

10%

## Topics

- The beginning of mathematics
- Number systems
- Sources for studying history.
- Ancient Egypt
- Ancient Mesopotamia
- Around the world
- Hellenic Mathematics
- Ancient and Medieval China
- Ancient and Medieval India
- Ancient and Medieval Islamic world
- European Renaissance
- Calculus
- Selected topics of modern mathematics

If you have a special interest in a math history topic, let me know. We might be able to cover it.

## Topics (cont.)

- The beginning of mathematics
- Number systems
- Sources for studying history.
- Ancient Egypt
- Ancient Mesopotamia
- Around the world
- Hellenic Mathematics
- Ancient and Medieval China
- Ancient and Medieval India
- Ancient and Medieval Islamic world
- European Renaissance
- Selected topics of modern mathematics

We will go deeper than wider

If you have a special interest in a math history topic, let me know. We might be able to cover it.

## Topics (cont.)

- We will discuss how mathematics has developed in various cultures around the world.
- There is an extremely large number of cultures
- We will have to **choose** a few sample cultures to represent the whole.
- The criteria for these choices is based, among other reasons on pedagogical arguments, on which culture have best-recorded mathematical history and accessible documentation to work with, weight of this culture in the mathematics we do today.

## More about me and this course

- The main point of Wooclap questions, forms, deadlines and class activities is encouraging you to think and learn, not to evaluate. All this activities also help me to gauge your understanding.
- Constructive feedback is welcomed by me, your instructor.
- Feel free (and encouraged!) to discuss with me any classroom dynamics issue that affects you.

There are no dumb questions

**Summary**

**What is  
counting?**

~~**What is  
counting?**~~

**What do you mean  
by counting?**

**What is counting?  
Explain it to  
somebody who does  
not know what it is.**

By counting one (usually) means a process to determine the number of objects in a set.

Counting involves establishing a **one-to-one correspondence** between two sets.

- One set (let's call it the *standard set*) is a list of symbols, objects, etc.
- The other set is variable.



Image credit: <https://creativekindergartenblog.com/one-to-one-correspondence-intervention-for-kindergarten/>

If the standard set is composed of symbols, then each symbol

- has to be different from the others.
- has a prescribed order of use.

# Counting

Body parts as numerals, by Geoffrey Saxe. <https://www.jstor.org/stable/pdf/1129244.pdf>

Tally  
From Latin talea 'twig, cutting'.  
(same root as tailor)

- Objects: sticks, rocks shells - Abacus
- fingers, toes
- assigning labels, language

# Counting

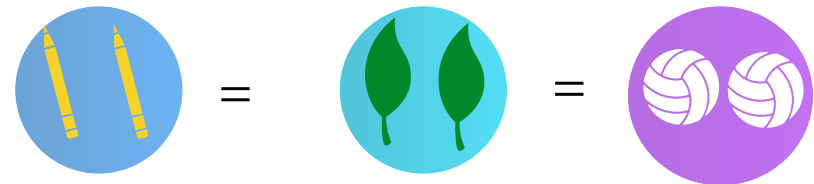
Find out how many  
Record how many

## About counting

- Almost (if not all) societies have/had some awareness of number.
- Counting is establishing a one-to-one correspondence between two sets.
- Counting is linked to fingers. Later
  - Making scratches
  - Using objects (sticks, pebbles, shells)
  - Tying knots.
- The concept of number, although basic is very hard to define rigorously.
- After counting, a need to record arises.



Image credit: <https://creativekindergartenblog.com/one-to-one-correspondence-intervention-for-kindergarten/>



# How and when humans started to count?

# Approximate number sense and object tracking system.

## Approximate number abilities and number sense

Two systems in humans and some animals

### Approximate number system

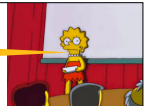
- non-verbal
- representing and comparing numerosities
- ratio is key.

**Number sense:** Mechanism for keeping track of individual objects

- independent of numerical ratio
- recognizing (not counting!) up to four

## The presentation

Engage your audience!



- The **goal is to teach something** to the class.
- It is strongly encouraged **included a learning activity** for the class to help. (you can have a few extra minutes in this case)
- There will be a few minutes of questions afterwards. It is totally fine if you cannot to answer on the spot, and come back later with the response.
- Notes to help your memory are fine. However, the presentation cannot consist only of reading.
- Speaking in public can be scary, but **we will be a kind, supporting audience**, rooting for you.
- 150 of words!!! at most in the slides (that is, about 15 per slide)
- If you need to break any of the rules to give a better presentation, discuss it with me beforehand.
- You are welcome to make an appointment with me to do a rehearsal of your presentation.

## Communications spelled out

- **ALL** your course-related questions should be posted in the General Questions forum in the course Discussion board in Blackboard, so everyone can benefit.
- If you need to email me about something private that you do not feel comfortable to ask in Brightspace discussion forum please start your letter with a greeting and use complete English sentences.
- If you can answer or offer relevant comments to a question in the Discussion Forum, you are enthusiastically encouraged to do so.
- Come to see me in my office hours! (or make an appointment if you cannot make them)
- Any question about grades for homework and quizzes must be directed to the corresponding grader. Be aware that while the graders are in charge of the actual grading, I am the one designing the questions and determining how long quizzes are.

## Math Point

- Addition to the syllabus: A "math point" is a concrete, purely mathematical aspect of the topic which is mastered by the student. This math point can be, for instance, the solution of a problem, or the proof of a statement. It does not have to be the "whole" mathematical aspect of the topic. Examples of math point are:
  - Combinations and binomial coefficients as explained in Pascal's "Traite du triangle arithmetique".
  - Liu Hui and Tsu Keng-chih 's calculation of the Volume of a Sphere
  - Analogues of Pythagorean Theorem with the areas of different shapes, for instance squares, triangles and trapezoids.
  - Desargues Theorem
  - The paper will go over the specific logical problem in question and explain the mathematical and logical structure behind solving it.

**Write down  
something you  
would like to know  
about the course.  
(Or just write that  
you have nothing  
to ask).**

## Last meeting



From the Boston University Website  
<https://www.bu.edu/africa/outreach/teachingresources/outreach-teachingresources-mathematics/>

Mathematical Treasure: Ishango Bone  
MAA website  
<https://www.maa.org/press/periodicals/convergence/mathematical-treasure-ishango-bone>

The Lebombo bone (top) is the oldest known mathematical artifact. It is a tally stick with 29 distinct notches that were deliberately cut into a baboon's fibula. It was discovered within the Border Cave in the Lebombo Mountains of Eswatini. The Lebombo bone (bottom) resembles a calendar stick still used in Namibia. See more about these artifacts under "Other Resources" below.

- Websites are not necessarily reliable (even if they are educational)
- In this course we will work with peer reviewed articles and one or two websites (that I think are reliable)
- No system is perfect, but peer reviewed is the best we have in this area.

# Discussion on the sources to use in this course.

## Primary sources

A **primary source** is an original, firsthand, or direct piece of evidence or material that provides information about a particular topic or event.

- Often they are created during the time period which is being studied (correspondence, diaries, newspapers, government documents, art)
- They can also be produced later by eyewitnesses or participants (memoirs, oral histories).
- We are often going to work with transcriptions and translation of primary sources.

Primary sources include

- traditional print and manuscript texts
- non-written works
- social media item

Some examples of the primary sources we will discuss are

- the Rhind Papyrus,
- the Rosetta Stone,
- Plimpton 322 (a Mesopotamian mathematical tablet),
- Mayan Stella E,
- the Nine Chapters of the Mathematical Art and
- the works of Euler.

**Give a concrete example of a primary source. State a topic that can be studied using this primary source.**

<https://research.library.gsu.edu/HIST2110Primary>

**Give a concrete example of a mathematical primary source. State a topic that can be studied using this document and how to find the text (a translation if the primary source is not in English) Feel free to use Google, Google Scholar, the Stony Brook library website, AI (artificial intelligence)... but make sure that your example exists, and is appropriate.**

YOU

Give a concrete example of a mathematical primary source. State a topic that can be studied using this document and how to find the text (a translation if the primary source is not in English)

[Link](#) to my account in open AI

Give an example (in the form of a bibliography ) of a primary source about mathematics in Ancient Egypt

**Title:** Rhind Mathematical Papyrus

**Author:** Unknown (Attributed to Ahmes)

**Date:** Around 1650 BC

**Language:** Ancient Egyptian

**Description:** A mathematical papyrus containing a collection of mathematical problems and solutions, covering topics such as arithmetic, fractions, geometry, and practical applications. It provides insights into the mathematical knowledge and practices of ancient Egyptians.

**Bibliographic Entry:**

Ahmes (attributed author). "Rhind Mathematical Papyrus." Translated by Arnold Buffum Chace, The Mathematical Association of America, 1927.

**Additional Notes:**

Arnold Buffum Chace's translation of the Rhind Mathematical Papyrus includes a commentary that explains the mathematical content and its historical context. The papyrus is also known as the Ahmes Papyrus, named after the scribe to whom it is attributed. It's worth noting that there

ChatGPT  
2023

Give an example (in the form of a bibliography ) of a primary source about mathematics in Ancient Mesopotamia

**Title:** Plimpton 322

**Author:** Unknown

**Date:** Around 1800 BC

**Language:** Sumerian

**Description:** A cuneiform tablet known as Plimpton 322, containing a collection of numerical tables and calculations, including Pythagorean triples. It offers insights into the mathematical knowledge and computational methods of ancient Mesopotamia.

**Bibliographic Entry:**

Robson, Eleanor, and Daniel Mansfield. "Babylonian Mathematics: Plimpton 322." The Mathematical Gazette, vol. 92, no. 523, 2008, pp. 193-197.

**Additional Notes:**

The Plimpton 322 tablet is one of the most famous mathematical artifacts from ancient Mesopotamia. It contains numerical tables that have been interpreted as evidence of advanced mathematical understanding, including the application of Pythagorean triples. The article by Robson and Mansfield provides analysis and insights into the tablet's mathematical content and its significance. Please note that there are other scholarly works and interpretations re  
Plimpton 322 available as well

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Give an example (in teh form of bibliography) of a primary source for the mathematics of Ancient mesopotamia

ChatGPT

An example of a primary source for the mathematics of Ancient Mesopotamia could be a cuneiform tablet containing mathematical calculations. Here's a hypothetical bibliography entry for such a primary source:

Author: Unknown

Title: Mathematical Cuneiform Tablet from Ancient Mesopotamia

Date: Circa 1800 BCE

Location: [Specify the museum or collection where the tablet is housed]

In this case, the primary source is the actual cuneiform tablet that contains mathematical notations and calculations from the Ancient Mesopotamian civilization. Researchers and historians would analyze the content on the tablet to understand the mathematical techniques and practices of that time.

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2024

Give a concrete example (in teh form of bibliography) of a primary source for the mathematics of Ancient mesopotamia

ChatGPT

While I cannot provide an actual bibliography entry for a specific cuneiform tablet, I can provide a general example to illustrate how it might look:

Author: Scribe of Ur-Nammu (if known)

Title: Mathematical Cuneiform Tablet: Calculations of Areas and Volumes

Date: Circa 2100 BCE

Location: British Museum, London

In this hypothetical example, the author might be the scribe who created the tablet, and the title indicates the content related to mathematical calculations of areas and volumes. The date reflects the estimated time period, and the location specifies where the tablet is currently housed for further study. Actual citations would provide more detailed information based on the specific tablet being referenced.

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2024



## Examples of primary sources

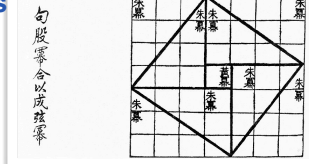
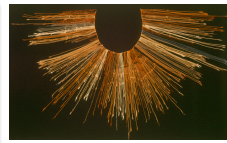
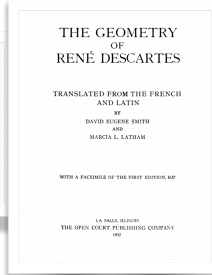
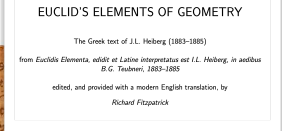


Diagram added by Zhao Shuang (3rd century BCE) to the Zhoubi Suanjing that can be used to prove the Pythagorean Theorem



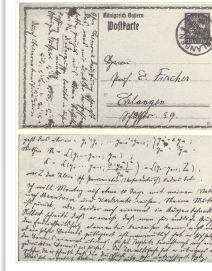
Quipu  
By Claus-Ableiter nur hochgeladen aus enWiki - enWiki, hochgeladen von User Lyndsaruell; siehe [http://en.wikipedia.org/wiki/Image:Inca\\_Quipu.jpg](http://en.wikipedia.org/wiki/Image:Inca_Quipu.jpg), CC BY-SA 3.0, <https://commons.wikimedia.org/w/index.php?curid=2986793>

Section of the Rhind Papyrus-British Museum

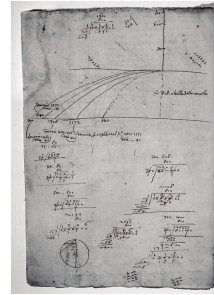


Fragment of what is likely part of a larger papyrus roll from the early years of the current era. Found in Oxyrhynchus (about 110 miles up the Nile from Cairo and 10 miles west of it, Oxyrhynchus at that time was populated by Greek colonists, a remnant of the conquest in about 330 B. C. by Alexander the Great. The fragment contains the statement, in Greek, of Proposition 5 from Book II of Euclid's Elements  
Image Credit: <https://personal.math.ubc.ca/~cass/euclid/papyrus/>

Translation of Descartes' "La Geometrie"

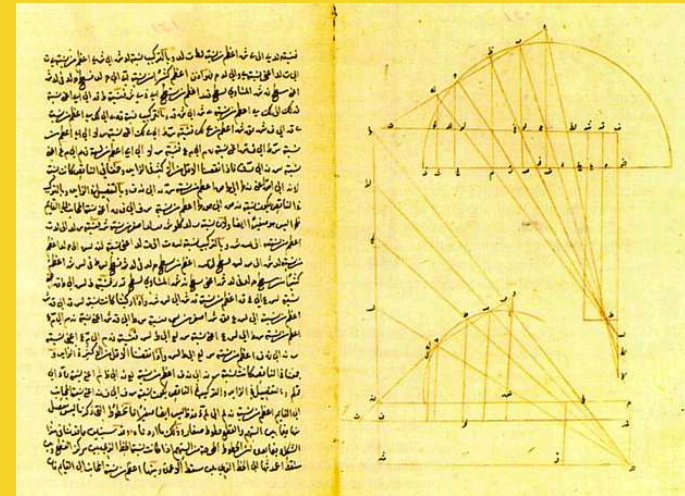


Postcard from Emmy Noether to E. Fischer. Postmarked 10 April 1915. - Public Domain



A page from Galileo's notebooks, showing an experiment such as the one described here. See Stillman Drake, Galileo's Notes on Motion, monograph 5, Annali dell'Istituto e Museo di Storia della Scienza (Florence, 1979), p. 79.

## Is the Arabic translation of the Apollonius book V on Conic Sections. a primary source for some topic? If so, for what topic? If not, why not?



Arabic translation of the Apollonius book V on Conic Sections.

## Secondary sources

**Secondary sources** are works that study a topic through studying, researching, analyzing and interpreting primary sources.

Give a concrete example of a secondary source. State a topic that can be studied using this secondary source.

Secondary sources include books and articles written from a historical perspective (later in time), reviews, textbooks, biographies, and indexes.

In the bibliographies of your paper and presentation, you can **only use relevant secondary sources which are from peer reviewed journals written in English** (One way to find them is through the database linked in the course website). If you are in doubt of your source, consult with me.

Every item in your bibliography should be referenced.

Every fact in your paper that is not common knowledge, should have a reference to an item in your bibliography.

Give a concrete example of a mathematical, peer reviewed. secondary source, State a topic that can be studied using this secondary source. Feel free to use Google, Google Scholar, the Stony Brook library website, , AI (artificial intelligence)... but check it.

Hint: Go to [jstor.org](http://jstor.org) or the SB library (<https://library.stonybrook.edu/>)



## A reliable website?

**Introduction**

Mayan & other Mesoamerican cultures used the vigesimal number system. Vigesimal system based on base 20 and to some extent base 5.

Theory suggest that it was develop from using the fingers and toes.

Dates back to the 4<sup>th</sup> cent. Approximately 1,000 years more advance than the Europeans

**Diagram B**

Mayan numeral system

20<sup>3</sup> = 8,000 = 40,000  
20<sup>2</sup> = 0 · 400 = 0  
20<sup>1</sup> = 12 · 20 = 240

Mayan number system consist of three symbol. A shell that represent zero, a dot represent one, and a line represent five. One could write up to 19 with the these symbol. Mayans were one of the only civilization that understood the concept of zero from 36 BCE.

**Diagram A**

0 1 5

Mayan Calendar used base 18.

- Mayans number system was used by merchants and in speeches.
- Places values are arrange vertically.
- Numbers can be found on stelae and monuments.
- Mayan may have used a "Abacus"

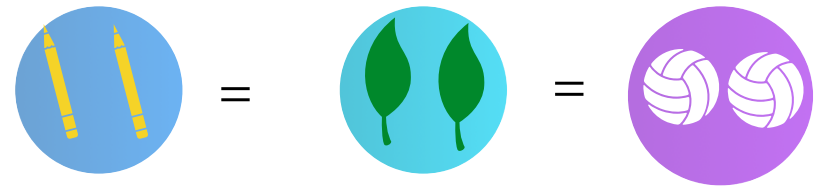
<https://www.slideshare.net/EdicePachikerl/mayan-numeric->

Can these slides be considered a secondary source? Why or why not?



<https://mathed.byu.edu/~williams/Classes/300W2012/PDFs/PPTs/Egyptian%20PPT.pdf>

What is a natural (or counting) number?  
(And why is this such a hard question)





## Russell's paradox

### Other forms

A man of Sevilla is shaved by the Barber of Sevilla if and only if the man does not shave himself.

Does the barber shave himself? Why or why not?



Proposed by Bertrand Russell  
More about this paradox [here](#).

Consider S the set of all sets.  
So,  $\emptyset$  (the empty set) belongs to S.  
The set containing the empty  $\{\emptyset\}$  set belongs to S.  
And so on.  
Define  $R = \{x \text{ in } S : x \text{ does not belong to } x\}$ .

Does R belong to R? Why or why not?

## The beginnings of a rigorous definition of (counting) number

- **Cantor** (late 1800s):
  - Beginnings of set theory.
  - Defined "two infinite sets have the same *power* if the elements of the sets can be put into **one-to-one correspondence**."
- **Frege** (~1900) applied that this idea of the one-to-one correspondence of elements to **finite** sets to **define cardinality**.
  - Two finite sets are said to have the same **cardinal number**—that is, to be equal—if the elements in either class can be put into one-to-one

Example: Cardinality of set of fingers in



is the set of all sets that can be put in one to one correspondence with the fingers in this hand.

Boyer, C. B., & Merzbach, U. C. (2011). "A History of Mathematics."

## Russell's paradox

Define  $R = \{x : x \text{ does not belong to } x\}$ .  
then R belongs to R iff R does not belong to R.

- Russell wrote to Frege explaining his paradox in 1902.
- Russell's paradox showed that the axioms Frege was using to formalize his logic were inconsistent.
- Russell's letter arrived just as the second volume of Frege's *Grundgesetze der Arithmetik (The Basic Laws of Arithmetic, 1893, 1903)* was in press.
- Frege eventually felt forced to abandon many of his views about logic and mathematics.

Thus, giving a rigorous definition of "number" is quite difficult

Stanford Encyclopedia of Philosophy

<https://plato.stanford.edu/entries/russell-paradox/#HOTP>

## Bertrand Russell.



Stanford Encyclopedia of  
Philosophy

[https://plato.stanford.edu/entries/  
russell-paradox/#HOTP](https://plato.stanford.edu/entries/russell-paradox/#HOTP)

As I think about acts of integrity and grace, I realise that there is nothing in my knowledge to compare with Frege's dedication to truth. His entire life's work was on the verge of completion, much of his work had been ignored to the benefit of men infinitely less capable, his second volume was about to be published, and upon finding that his fundamental assumption was in error, he responded with intellectual pleasure clearly submerging any feelings of personal disappointment. It was almost superhuman and a telling indication of that of which men are capable if their dedication is to creative work and knowledge instead of cruder efforts to dominate and be known. (Quoted in van Heijenoort (1967), 127)

## Review about counting

- Almost (if not all) societies have/had some awareness of number, even if **not all can define precisely what counting is**. Counting is establishing a **one-to-one correspondence** between two sets.
- Counting is closely **linked to fingers**.
- The concept of number, although basic is **very hard to define rigorously**. (Recall the barber's paradox)
- After counting, a need to record the result of the counting arises. There are different ways of **keeping records** of "how many", for instance
  - Making scratches
  - Using objects (sticks, pebbles, shells)
  - Tying knots.



Image credit: [https://creativekindergartenblog.com/  
one-to-one-correspondence-  
intervention-for-kindergarten/](https://creativekindergartenblog.com/one-to-one-correspondence-intervention-for-kindergarten/)

# Counting in different societies

Is it possible to  
count without words  
denoting numbers? If  
so, how? If not, why?

## Example: Veddas of Sri Lanka counting

- Lived by eating fruit and turners of jungle plants.
- Had few words to deal with numbers. Some of these words (translated) were: a single, a couple, another one and many.
- To count, say, a pile of coconuts, a Vedda collected a heap of sticks. Then,
  - For each coconut in the pile, she took a stick.
  - Each time she took a stick, she said “another one”

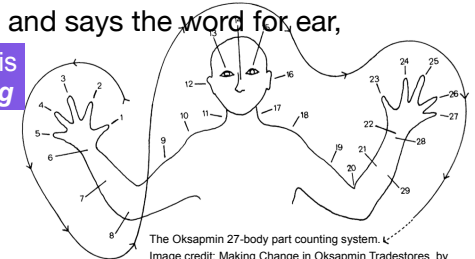
When finished, she pointed out the pile of sticks she took and said “That many”.



Veddas count does not use number-words

## Example: Oksapmin from Papua New Guinea

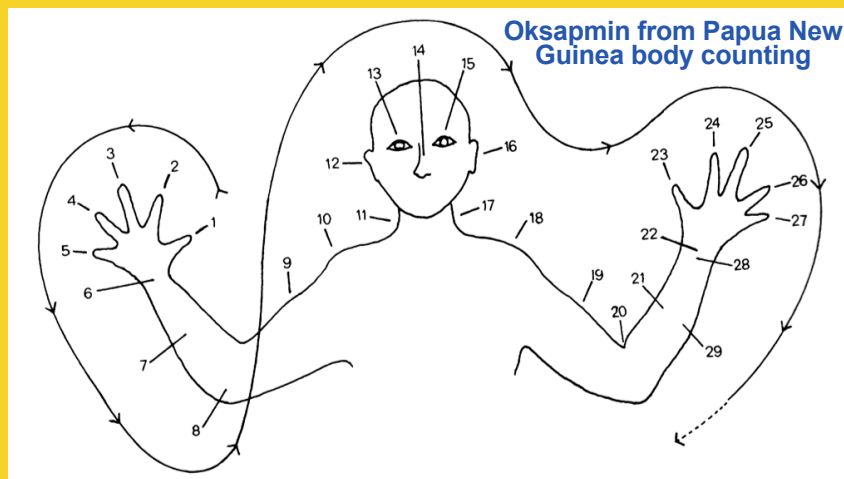
- Begins with the thumb on one hand
- Enumerates 27 places around the body,
- Ends on the little finger of the opposite hand.
- To indicate a particular number, one points to the appropriate body part and says the body part name.
- Example: to indicate the number 12, one points to the ear which is the 12th body part and says the word for ear, “nata”. This way of counting is called **body counting**



The Oksapmin 27-body part counting system. Image credit: Making Change in Oksapmin Tradestores by Geoffrey B. Saxe and Indigo Esmonde

Forms of body counting were quite spread around the globe as late as the 1800s

What is your age in Oksapmin body counting? (In English, of course. Thus, if you are five years old, you would write “pinky finger”, and if you are twenty seven, you wrote “other pinky finger”)



The Oksapmin 27-body part counting system. Image credit: Making Change in Oksapmin Tradestores by Geoffrey B. Saxe and Indigo Esmonde

<https://twitter.com/britishlibrary/status/1301807524904083456>



Illustrations of hands labelled with figures on the fingers to calculate the date of Easter and perform mathematical calculations (British Library, Egerton MS 3314, f. 73r, detail)

Forms of body counting were quite spread around the globe as late as the 1800s

## Example: Nivkh people from lower Amur River - Russia

	Means <i>two</i> of the following
Mex	trees, sticks, pencils and some other long things
Met	leaves or pieces of textiles or other flat items.
Mik	berries, balls and other round things.

Example: **Met** can mean, for instance, “**two leaves**”.

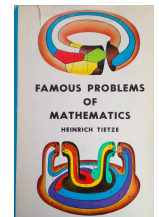
Is there any noun in English similar to those of the Nivkh people? If so, write it down.

## A way of counting

A group of friends, let's call them A, B and C want to count a large number of people.

1. The people starts walking before them. Each time a person walks by, A raises a finger (and keeps it raised)
2. When all fingers of A are raised, B raises one finger and A lowers all their fingers. The procedure continues repeating appropriately 1. and 2.
3. A some point, all fingers of B are raised. Then C raises one finger and B lowers all their fingers.
4. And so on.

According to Tietze, this form of counting was used by a tribe from South Africa

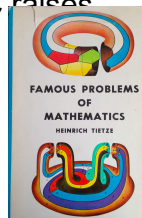


## Counting in a planet of beings with only one hand, with three fingers in that lonely only hand

A group of friends, let's call them A, B and C want to count a large number of people.

1. The people starts walking before them. Each time a person walks by, A raises a finger (and keeps it raised)
2. When all fingers of A are raised, B raises one finger and A lowers all their fingers. The procedure continues repeating appropriately 1. and 2.
3. A some point, all fingers of B are raised. Then C raises one finger and B lowers all their fingers.
4. And so on.

According to Tietze, this form of counting was used by a tribe from South Africa



**Mathematics is, of course, a part of culture. Every people inherits from its predecessors or contemporary neighbors along with ways of cooking marrying, worshiping, etc., ways of counting, calculating, and whatever else mathematics does. . . . Whether a people counts by fives, tens, twelves or twenties; whether it has no words for cardinal numbers beyond 5, or possesses the most modern and highly developed mathematical conceptions, their mathematical behavior is determined by the mathematical culture which possesses them.**

Leslie A. White, *The Science of Culture* (New York: Grove Press, 1949), p. 286.

# History of mathematics hidden in language

## Early word numbers

English	Gothic	Latin	Ancient Greek	Welsh	Sanskrit	Basque
one	ains	unus	heis	un	eka	bat
two	twai	duo	dyo	dau	dva	biga
three	threis	tres	treis	tn	tri	hirur
four	fidwor	quattuor	tettares	pedwar	catur	laur
five	fimf	quinque	pente	pump	panca	bortz
six	saihs	sex	hex	chwech	sad	sei
seven	sibun	septem	hepta	saith	sapta	zazpi
eight	ahtau	octo	okto	wyth	asta	zortzi
nine	niun	novem	ennea	aw	nava	bederatzi
ten	taihun	decem	deka	deg	dasa	hamar
eleven	ainlif	undecim	hendeka	un ar ddeg	ekaadasha	hamaika
twelve	twalif	duodecim	dodeka	deuddeg	dvaadashan	hamabi
twenty	twaitigjus	viginti	eikosi	ugain	vimsatih	hogoi

Source: Barrow-Green, June, Jeremy Gray, and Robin Wilson. The History of Mathematics: A Source-Based Approach: Volume 1. Vol. 45. American Mathematical Soc., 2019.

A counting system is quite a complex construction, with different historical structures overlaying one another

English	Gothic	Latin	Ancient Greek	Welsh	Sanskrit	Basque
one	ains	unus	heis	un	eka	bat
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three	threis	tres	treis	tn	tri	hirur
four	fidwor	quattuor	tettares	pedwar	catur	laur
five	fimf	quinque	pente	pump	panca	bortz
six	saihs	sex	hex	chwech	sad	sei
seven	sibun	septem	hepta	saith	sapta	zazpi
eight	ahtau	octo	okto	wyth	asta	zortzi
nine	niun	novem	ennea	aw	nava	bederatzi
ten	taihun	decem	deka	deg	dasa	hamar
eleven	ainlif	undecim	hendeka	un ar ddeg	ekaadasha	hamaika
twelve	twalif	duodecim	dodeka	deuddeg	dvaadashan	hamabi
twenty	twaitigjus	viginti	eikosi	ugain	vimsatih	hogoi

## Early word numbers

- Our words 'eleven' and 'twelve' seem quite close to their Gothic equivalents, but not to anything else here: these Gothic words mean something like 'one left' and 'two left', in the sense of 'left over'.
- Since **this construction occurs in most other old northern European languages**, we may infer that at an A counting system is quite a complex construction, with different historical structures overlaying one another there were **northern European tribes whose counting words went up only to 'ten'**. In order to count to 'eleven' and 'twelve', when needed, they took away ten and counted what was left.

Source: Barrow-Green, June, Jeremy Gray, and Robin Wilson. The History of Mathematics: A Source-Based Approach: Volume 1. Vol. 45. American Mathematical Soc., 2019.

## Early word numbers

English	Gothic	Latin	Ancient Greek	Welsh	Sanskrit	Basque
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four	fidwor	quattuor	tettares	pedwar	catur	laur
five	fimf	quinque	pente	pump	panca	bortz
six	saihs	sex	hex	chwech	sad	sei
seven	sibun	septem	hepta	saith	sapta	zazpi
eight	ahtau	octo	okto	wyth	asta	zortzi
nine	niun	novem	ennea	aw	nava	bederatzi
ten	taihun	decem	deka	deg	dasa	hamar
eleven	ainlif	undecim	hendeka	un ar ddeg	ekaadasha	hamaika
twelve	twalif	duodecim	dodeka	deuddeg	dvaadashan	hamabi
twenty	twaitigjus	viginti	eikosi	ugain	vimsatih	hogoi

Source: Barrow-Green, June, Jeremy Gray, and Robin Wilson. The History of Mathematics: A Source-Based Approach: Volume 1. Vol. 45. American Mathematical Soc., 2019.

- Other traces of a twenty-base can be found in the Bible: 'three score and ten' means 70:(Bible, Psalm 90, verse 10: "The days of our years are three score and ten.)

Can you remember a historical occurrence of the word "score"? (Where score mean 20)

- In French, some higher numbers are formed differently.
- The word for 20 ('vingt') seems independent of the words for 2 ('deux') and 10 ('dix'),
- It is used in some higher number words such as 'quatre-vingts-sept' (four-twenties-seven) for 87.
- So French counting words have traces of a twenty-base, as well as a ten-base: an example of the latter is 'dix-huit' (ten-eight) for 18.

Source: Barrow-Green, June, Jeremy Gray, and Robin Wilson. The History of Mathematics: A Source-Based Approach: Volume 1. Vol. 45. American Mathematical Soc., 2019.



Four score and seven years ago our fathers brought forth on this continent, a new nation, conceived in Liberty, and dedicated to the proposition that all men are created equal.

Now we are engaged in a great civil war, testing whether that nation, or any nation so conceived and so dedicated, can long endure. We are met on a great battle-field of that war. We have come to dedicate a portion of that field, as a final resting place for those who here gave their lives that that nation might live. It is altogether fitting and proper that we should do this.

But, in a larger sense, we can not dedicate -- we can not consecrate -- we can not hallow -- this ground. The brave men, living and dead, who struggled here, have consecrated it, far above our poor power to add or detract. The world will little note, nor long remember what we say here, but it can never forget what they did here. It is for us the living, rather, to be dedicated here to the unfinished work which they who fought here have thus far so nobly advanced. It is rather for us to be here dedicated to the great task remaining before us -- that from these honored dead we take increased devotion to that cause for which they gave the last full measure of devotion -- that we here highly resolve that these dead shall not have died in vain -- that this nation, under God, shall have a new birth of freedom -- and that government of the people, by the people, for the people, shall not perish from the earth.

**Abraham Lincoln**  
**The Gettysburg Address**  
**Gettysburg, Pennsylvania**  
**November 19, 1863**

English	Gothic	Latin	Ancient Greek	Welsh	Sanskrit	Basque
one	ains	unus	heis	un	eka	bat
two	twai	duo	dyo	dau	dva	biga
three	threis	tres	treis	tri	tri	hiru
four	fiduor	quattuor	tettares	pedwar	catu	laur
five	finf	quinque	pentē	pump	panca	bortz
six	saith	sex	hex	chwech	sad	sei
seven	silban	septem	hepta	siith	sapta	zazpi
eight	abtau	octo	okto	wyth	asta	zortzi
nine	ninn	novem	ennea	aw	nava	bederazi
ten	salban	decem	deka	deg	dasa	hamar
eleven	ainlf	undecim	hendeka	un ar ddeg	ekadasha	hamaika
twelve	swalf	duodecim	dodeka	deuddeg	dvadashan	hamahi
twenty	twaitigais	viginti	eikosi	again	vimsath	bogai

Source: Barrow-Green, June, Jeremy Gray, and Robin Wilson. The History of Mathematics: A Source-Based Approach: Volume 1. Vol. 45. American Mathematical Soc., 2019.

We can even form a hypothesis about **the age** of our counting system, as seen in its number words up to 'nine' or 'ten'. There are several similarities between the words that become more apparent when you say them aloud. The most striking example, because it is so unexpected, is how close to Latin, or even to Gothic and English, the Sanskrit number words can be made to sound.

**It thus seems plausible that Sanskrit and the Western languages (except for Basque, which seems rather different) may have had a common root, which from the given dating was before or during the second millennium BC.**

## Early word numbers

Since **Latin, Greek, Welsh, and Sanskrit** seem to have much the **same word for 'ten'**, we can reasonably infer that the **counting system went up at least to ten** by this time.

- To justify this hypothesis we clearly need to examine much more evidence of languages old and new and other historical information relating to the movement of tribes and peoples and cultural influences.

Source: Barrow-Green, June, Jeremy Gray, and Robin Wilson. The History of Mathematics: A Source-Based Approach: Volume 1. Vol. 45. American Mathematical Soc., 2019.

## Early word numbers

- General agreement: **English, Russian, Persian, Afghan, and most European languages are descended from an original Indo-European language spoken possibly around 4000 BC.**
- Similarities in the words for 100** suggest that there was a counting word for 'hundred' in the original Indo-European languages.
- Counting by twenties**, which can be seen residually in French and Welsh, seems to have been the practice in languages such as Basque, which were spoken in Europe before the spread of Indo-European.
- This shows that we can begin to build up some knowledge about the past by piecing together several different sorts of evidence. **It seems that our spoken counting system may be from 5000 to 8000 years old.**


Source: Barrow-Green, June, Jeremy Gray, and Robin Wilson. The History of Mathematics: A Source-Based Approach: Volume 1. Vol. 45. American Mathematical Soc., 2019.

## From Two to Three, a gigantic step

- It appears that language contains evidence that it was a struggle to pass beyond two.
  - Three* is often associated with *many*:
    - three* and *through*
    - trois*, and *très*, (in French)
  - After one and two, the ordinals were formed "third, fourth, fifth,..." Formerly, first kept its meaning, which is before all the others, while we find second used for "the other" or "the one that comes after", compare the Latin "secundus" [from sequi, secutus to follow]. It is difficult to draw a distinction between second and twice. Let us go on further, leaving the few for the many.

“The same way you can feed your body junk food, we are feeding our mind junk information. We need an information diet. People should think carefully about the amounts and the quality of the information they take in.”

Yuval Noah Harari



contrast  
with

“I hear and I forget  
I see and I remember.  
I do and i understand”

Confucius



**Active learning increases student performance in science, engineering, and mathematics**

Scott Freeman<sup>a,1</sup>, Sarah L. Eddy<sup>a</sup>, Miles McDonough<sup>a</sup>, Michelle K. Smith<sup>b</sup>, Nnadozie Okoroafor<sup>a</sup>, Hannah Jordt<sup>a</sup>, and Mary Pat Wenderoth<sup>a</sup>

<sup>a</sup>Department of Biology, University of Washington, Seattle, WA 98195; and <sup>b</sup>School of Biology and Ecology, University of Maine, Orono, ME 04469

Edited\* by Bruce Alberts, University of California, San Francisco, CA, and approved April 15, 2014 (received for review October 8, 2013)

To test the hypothesis that lecturing maximizes learning and course performance, we metaanalyzed 225 studies that reported data on examination scores or failure rates when comparing student learning interventions varied widely in intensity and implementation, and included approaches as diverse as occasional group

<https://www.pnas.org/content/111/23/8410>