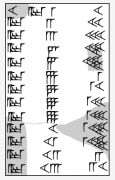
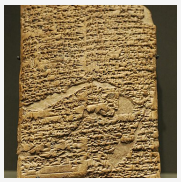
Mathematics in Mesopotamia









- Setting the Stage
- Cuneiform Writing: Birth, Decipherment, and Surviving Texts
- Mesopotamian Mathematics in a slide-nutshell
- More about base 60
- Calculation of Areas
- An amazing approximation of $\sqrt{2}$
- Tables of Reciprocals
- Plimpton 322
- Solutions of equations

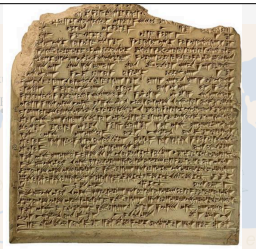
Setting the Stage



Seven wonder of the Ancient World - Kandi, CC BY-SA 3.0 <https://creativecommons.org/licenses/by-sa/3.0>, via Wikimedia Commons

Civilizations 3000 years' span

- Sumerian
- Babylonian
- Assyrian
- Akkadian



Tablet Inscribed in Babylonian with a Ritual for the Observances of Eclipses-The Morgan Library and Museum

Cuneiform Writing on Clay Tablets

- Wedge-shaped "cuneiform" script, written by trained scribes.
- Hundreds of thousands of tablets have survived.
- Most are **administrative records** (memos, receipts, wages).
- Others preserve:
  - Law codes
  - Magical rituals
  - Military campaigns and battles
  - Myths, including the Epic of Gilgamesh
  - Math!

Seven wonders of the Ancient World - Kandi, CC BY-SA 3.0 <https://creativecommons.org/licenses/by-sa/3.0>, via Wikimedia Commons



Pick one mathematical idea from the video that you use every day. Describe how you use it.

Ancient Mesopotamia 101- National Geographic
https://youtu.be/xVf5kZA0HtQ?si=PK4TY11m37V3GYK5

Administration of surplus.

- The fertile Mesopotamian land produced **agricultural surplus,**
- That surplus was **variable** (floods, droughts, seasonal cycles).
- There was a **need to track quantities over time**: how much grain was stored, owed, distributed, or lost.

That practical pressure "pushes" abstraction

- **Numbers** to represent quantities independent of the objects themselves.
- **Calendars** to anticipate floods, planting, and harvest cycles.
- **Writing** to record obligations and inventories beyond human memory.

Abstraction emerges from the need to coordinate resources across time, people, and uncertainty.

A praise poem of King Šulgi

I am a king, offspring begotten by a king and borne by a queen.
I, Šulgi the noble, have been blessed with a favorable destiny right from the womb.
When I was small, I was at the academy, where I learned the scribal art from the tablets of Sumer and Akkad.
None of the nobles could write on clay as I could.
There where people regularly went for tutelage in the scribal art, **I qualified fully in subtraction, addition, reckoning and accounting.**
The fair Nanibgal, Nisaba, provided me amply with **knowledge and comprehension.**
I am an experienced scribe who does not neglect a thing.

The poem links accounting and calculation with "knowledge and comprehension" and presents them as royal achievements. What does this suggest about how Mesopotamian culture understood the value of mathematics?

A praise poem of Šulgi

I am a king, offspring begotten by a king and borne by a queen.
I, Šulgi the noble, have been blessed with a favorable destiny right from the womb.
When I was small, I was at the academy, where I learned the scribal art from the tablets of Sumer and Akkad.
None of the nobles could write on clay as I could.
There where people regularly went for tutelage in the scribal art, **I qualified fully in subtraction, addition, reckoning and accounting.**
The fair Nanibgal, Nisaba, provided me amply with **knowledge and comprehension.**
I am an experienced scribe who does not neglect a thing.

The fair Nanibgal, Nisaba" signifies one goddess, patron of Sumerian scribes.
Nanibgal meant Lady of Great Wisdom"
King Šulgi of Ur (reigned 2000 BCE)
Electronic Text Corpus of Sumerian Literature, "A praise poem of Shulgi (Shulgi B)": etcsl.orinst.ox.ac.uk/section2/tr24202.htm

Cuneiform Writing: Birth, Decipherment, and Surviving Texts

Mesopotamian Accounting Tokens (4000–3300 BCE)



(Image by Denise Schmandt-Besserat and the University of Pennsylvania Museum of Archaeology and Anthropology, University of Pennsylvania, Philadelphia.)

Tokens Meaning Examples

- Cone represents a measure of grain
- Tetrahedron represents a unit of work.

Envelopes:

- Tokens were stored in clay envelopes for verification.
- Engraved with tokens content



Image Denise Schmandt-Besserat and Musée du Louvre, Département des Antiquités Orientales, Paris.)

From Accounting Tokens to Writing (3300–3200 BCE)



Image Denise Schmandt-Besserat and Musée du Louvre, Département des Antiquités Orientales, Paris.)

Each circular impression → one large measure of grain; each wedge → smaller measure of grain



(Image by by Denise Schmandt-Besserat and Musée du Louvre, Département des Antiquités Orientales, Paris.))

3D tokens → 2D impressions → first writing in Mesopotamia. Impressions evolved into cuneiform script.

Writing was invented independently in: Mesopotamia, Egypt, China, Mesoamerica. Possibly also Indus Valley Civilization (Harappan) - c. 2600-1900 BCE.

Behistun Inscription -the "cuneiform Rosetta stone"

- written in three different cuneiform languages: Old Persian, Elamite, and Babylonian.
- Studied and copied by Sir Henry Rawlinson, a British army officer, beginning in 1835.



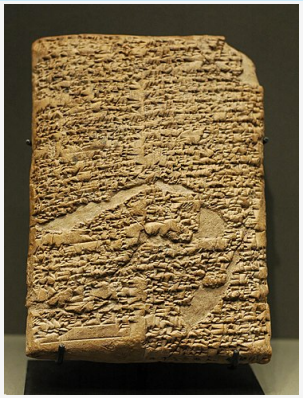
The Behistun Inscription, a large rock relief showing Darius the Great of the Achaemenid Empire punishing conspirators. Wikipedia Korosh.091 - Own work



Location of the Behistun complex

Code of Hammurabi about 1700 BC

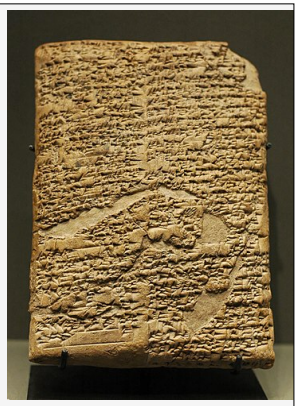
"an eye for an eye, a tooth for a tooth" (lex talionis)





How does a written law code (like Hammurabi's) change the nature of justice compared to an oral tradition?

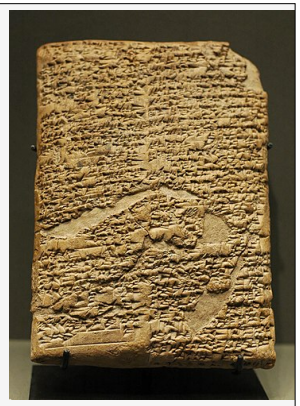




Code of Hammurabi about 1700 BC

"an eye for an eye, a tooth for a tooth" (lex talionis)

How does a written law code (like Hammurabi's) change the nature of justice compared to an oral tradition?





- **standardizes rules,**
- **reduces local variability,**
- **shifts authority from memory to text.**

Epic of Gilgamesh

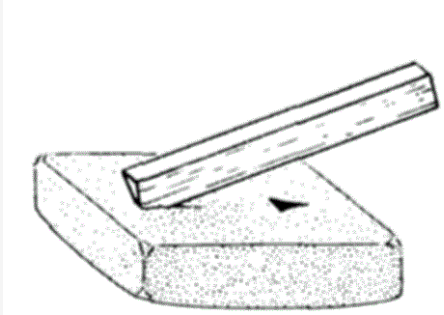


Ancient Assyrian statue currently in the Louvre, possibly representing Gilgamesh

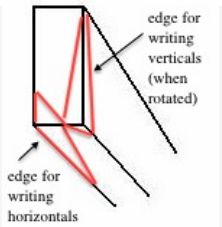
Neo-Assyrian clay tablet. Epic of Gilgamesh, Tablet 11: Story of the Flood. Known as the "Flood Tablet" From the Library of Ashurbanipal, 7th century BC. British Museum



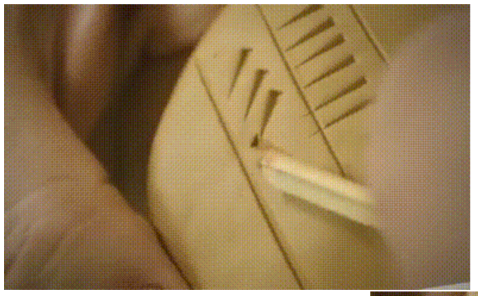
How to write in cuneiform



Impression of cuneiform symbols on clay tablets. (Redrawn from Neugebauer3



http://writingcuneiform.blogspot.com/2012/10/5-making-basic-wedges.html





Cuneiform writing

Clips from https://cuneiform.neocities.org/CWT/Figures/2_keilschrift_hwc.gif



Cuneiform writing: How it was done

https://cuneiform.neocities.org/CWT/Figures/2_keilschrift_hwc.gif

Mesopotamian Mathematics in a slide-nutshell

Overview of Mesopotamian Mathematics



Mathematical tablets are mainly:

Tables and Problems

Examples

- Multiplication tables (by ***some*** numbers).
  Reciprocals tables
- Square roots tables
- **Pythagorean triples? Plimpton 322.**

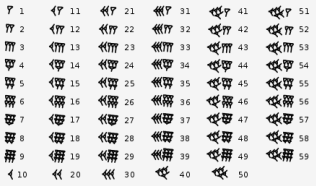
- Rough work (solution of problems)
- Problems - Verbal techniques

Note: No tables of addition



Overview of Mesopotamian Mathematics

- **Scribes** used **positional** number system with base 60 for computations.
- **Scribes** also used other number systems.



positional number system with no zero!

Calendar



Algorithms

- Determine square roots.
- Solution of linear equations (with one or two unknowns)
- Solution of certain quadratic equations (often related to architecture and building)

Overview of Mesopotamian Mathematics



Plimpton 322

Possibly the most famous tablet; understanding it requires resisting modern labels.

Scribes' Shortcut: Multiply by This

- The **defining component** of an equilateral triangle was the **side** and the **coefficient** for the **height** was 7/8.
- The **defining component** of a **circle** was the **circumference**. **Coefficients:**
  - Diameter was 1/3=$(0;20)_{60}$
  - Area 1/12= $(0;5)_{60}$

Decoder key:
defining component = the given measure you start from
coefficient = the multiplier that converts the measure into something else.

You are now a Mesopotamian scribe. Answer at least one question.
a. Given a circle's circumference c and the coefficients for the diameter and the area, state which multiplications you would perform to compute its diameter and its area.
b. Given an equilateral triangle's side s and the coefficient for the height, state which multiplications you would perform to compute the height and area of the triangle.

Scribes' Shortcut: Multiply by This

- The **defining component** of an equilateral triangle was the **side** and the **coefficient** for the **height** was 7/8.
- The **defining component** of a **circle** was the **circumference**. **Coefficients:**
  - Diameter was 1/3=$(0;20)_{60}$
  - Area 1/12= $(0;5)_{60}$

Decoder key:
defining component = the given measure you start from
coefficient = the multiplier that converts the measure into something else.

You are now a Mesopotamian scribe.
a. Given a circle's circumference c and the coefficients for the diameter and the area, state which multiplications you would perform to compute its diameter and its area.
b. Given an equilateral triangle's side s and the coefficient for the height, state which multiplications you would perform to compute the height and area of the triangle.

Part (a) - Circle with circumference c:

- To find diameter: Multiply c by 1/3
- To find area: Multiply c by c, then multiply the result by 1/12

Part (b) - Equilateral triangle with side s:

- To find height: Multiply s by 7/8 (to justify, use Pythagorean theorem and approximate. In modern terms, height^2 = s^2 -$(s/2)^2$.

Tables in Tablets

- Amazing aspect of the Babylonian's calculating skills: **construction of tables to aid calculation.**
- Babylonians used the formula

$$a.b = (1/2)((a+b)^2 - a^2 - b^2)$$

to make multiplication easier.

- They did not have an algorithm for long division. Instead they based their method on the fact that

$$A \div B = A \times (1/B)$$

and used tables of reciprocals **(1/B)**.





Advanced astronomy - another motivation for developing mathematics

Fragment of a circular clay tablet with depictions of constellations (planisphere). Neo-Assyrian. - British Museum



Concrete impact:
360 degrees angle
60 minutes in an hour
60 seconds in a minute

More about base 60 and how it was used in Mesopotamia

Historians have proposed several theories for the Babylonian use of base 60. Choose one and defend it.

a. Many even splits: 60 breaks evenly into many parts (2, 3, 4, 5, 6, 10, 12, 15, 20, 30), so common fractions come out neat and calculations are easier.

b. Sky counts: About 12 months × 5 "wandering stars" (Mercury, Venus, Mars, Jupiter, Saturn) = 60. (Sun and Moon were not counted as planets.)

c. Shared number for trade: Different places used different ways of counting and measuring; 60 worked as a handy common base for converting and keeping accounts.

d. Finger math: One hand can count to 12 using finger bones; the other hand tracks sets of 12 x 5 → 60.

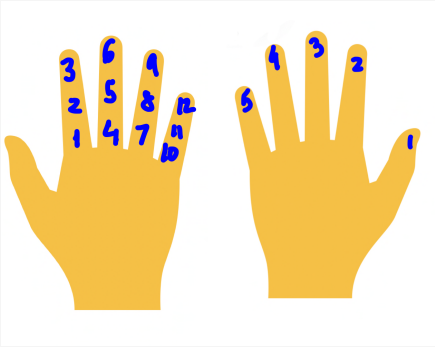


Some of the many theories explaining why base 60.

- Theon of Alexandria (~300CE): 60 is, among all the numbers the most **convenient**, because, being the smallest among all those which have the **most divisors**, it is the easiest to handle.
- More recent theory (~1950CE): **to allow for dividing weights and measures into thirds.**
- Even more recent: Sumerian civilization must have come about through the **joining of two groups**, one of whom had **base 12** for their counting and the other having base **5 or 10.**
- Common measures theory is now widely accepted. This is supported by the fact that there many commodities with different units measures.

There are many theories trying to explain base 60

One can count up to 60 by pointing at one of the twelve parts of the fingers of the left hand with one of the five fingers of the right hand.

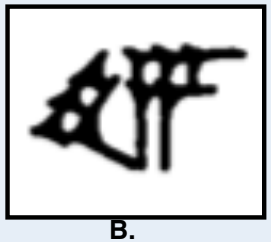


This is not a theory based on concrete evidence. It is included because it is plausible, and related to what we know about how organically number systems develop.

- Assume 1in is 2.5cm (it is 2.54)
- We could define a new unit of measure an M336, of 0.5cm long. So 1in=5 M336
- 1cm=2 M336
- A similar process could have happened to the number system

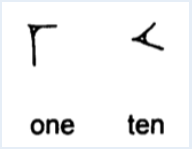
Express numbers A. and B. in Hindu-Arabic numerals. (Hint: The actual numerals, not only those representing 1 and 10 might be helpful)

A.

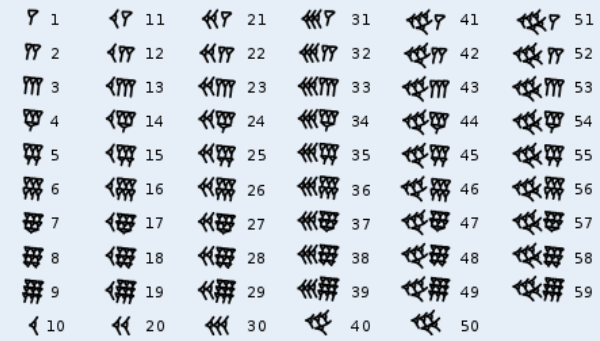


B.



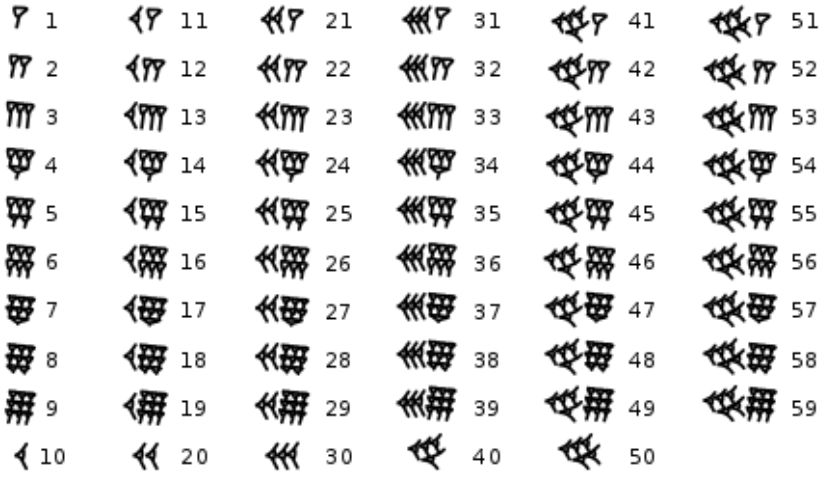


All Mesopotamian numerals



Figures from Robson, Eleanor. "Counting in Cuneiform." Mathematics in school 27.4 (1998): 2-9

All Mesopotamian numerals

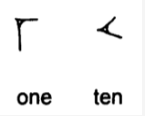




5 meant
$5/60+40/60^2$ or
5+40/60 or
5x60+40 or
$5x60^2+40x60$ or..



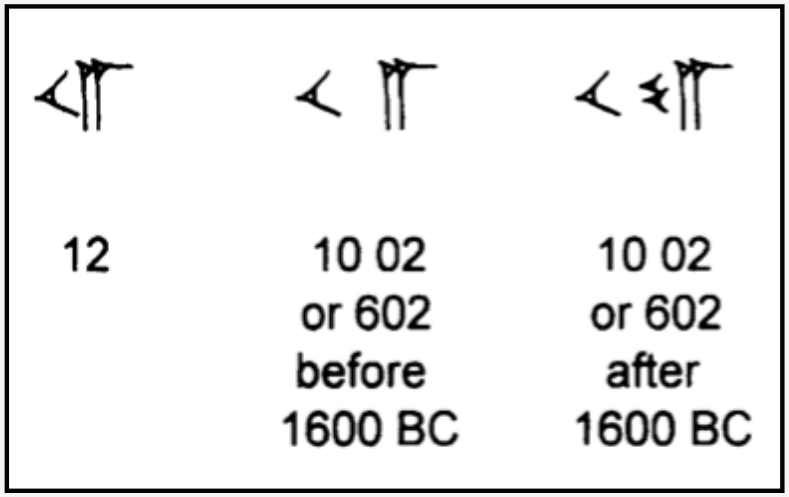
meant
45/60(= 3/4) or
45 or
45x60=2700 or..



How do they know which one? Context!

Figures from Robson, Eleanor. "Counting in Cuneiform." Mathematics in school 27.4 (1998): 2-9

Notations for intermediate zero across time



Figures from Robson, Eleanor. "Counting in Cuneiform." Mathematics in school 27.4 (1998): 2-9

Goal: Write a number **a** between 0 and 1 in base 60. We want numbers c_1, c_2, c_3... such that

$$\mathbf{a} = c_1/60 + c_2/60^2 + c_3/60^3 + \ldots$$

1) Each c_i is an integer
2) $0 \le c_i < 60$

Step 1: $c_1 = \lfloor 60 \cdot \mathbf{a} \rfloor$

$a_1 = 60a - c_1$

Step 2: $c_2 = \lfloor 60 a_1 \rfloor$

$a_2 = 60 a_1 - c_2$

Step n: $c_n = \lfloor 60 a_{n-1} \rfloor$,

$a_n = 60 a_{n-1} - c_n$)

Stop: If we reach i such that $a_i = 0$ the process ends.

Notation: $\lfloor x \rfloor$ denotes the largest integer smaller or equal than x.

Goal: Write a number **a=0.205** between 0 and 1 in base 60. We want numbers c_1, c_2, c_3... such that

$$\mathbf{0.205} = c_1/60 + c_2/60^2 + c_3/60^3 + \ldots$$

1) Each c_i is an integer
2) $0 \le c_i < 60$

Step 1: $c_1 = \lfloor 60 \cdot 0.205 \rfloor = \lfloor 12.3 \rfloor = 12$

$a_1 = 60a - c_1 = 12.3 - 12 = 0.3.$

Step 2: $c_2 = \lfloor 60 a_1 \rfloor = \lfloor 60 \cdot 0.3 \rfloor = \lfloor 18 \rfloor = 18$

$a_2 = 60 a_1 - c_2 = 60 \cdot 0.3 - 18 = 0$

Stop: If we reach i such that $a_i = 0$ the process ends.

Notation: $\lfloor x \rfloor$ denotes the largest integer smaller than or equal than x.

Remark a_1 , a_2 … are numbers (like **a**) between 0 and 1.

Convert at least one of the following fractions into base 60 (sexagesimal). Write down the result in cuneiform. Answer in Vevox using base-60 notation.

A. 1/20
B. 21/80
C. (Optional) 1/7

One Reason of Mesopotamian Influence in Greek Astronomy

The superiority of Babylonian numerical methods has left traces still visible in modern times. The division of the circle into 360 degrees and the division of the hour into 60 minutes and 3600 seconds reflect the unbroken use of the sexagesimal system in their computations by medieval and ancient astronomers. But though the base 60 is the most conspicuous feature of the Babylonian number system, this was by no means essential for its success. The great number of divisors of 60 is certainly very useful in practice, but the **real advantage of its use in the mathematical and astronomical texts lies in the place-value notation**, which is consistently employed in all scientific computations. This gave the Babylonian number system the same advantage over all other ancient systems as our modern place-value notation holds over the Roman numerals. **The importance of this invention can well be compared with that of the alphabet.** Just as the alphabet eliminates the concept of writing as an art to be acquired only after long years of training, so a place-value notation eliminates mere computation as a complex art in itself.

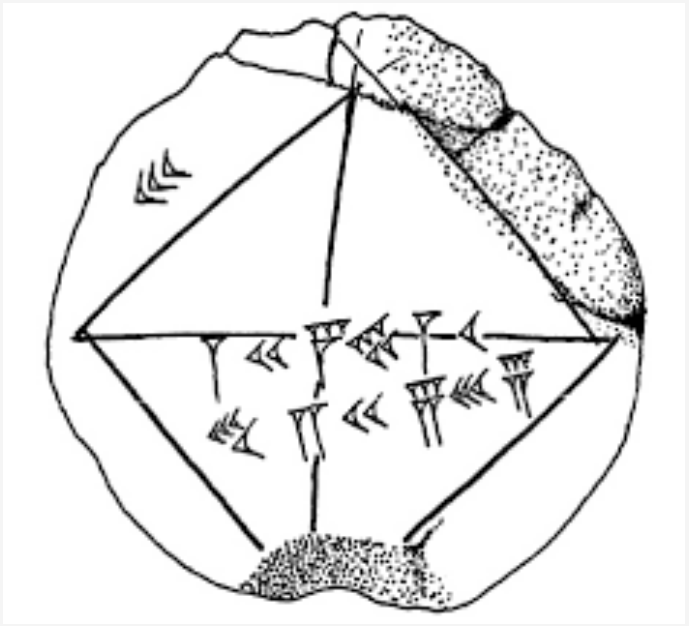
THE HISTORY OF ANCIENT ASTRONOMY : PROBLEMS AND METHODS Author(s): O. Neugebauer, Journal of Near Eastern Studies, 1945, Vol. 4 . https://archive.org/details/O.NeugebauerTheHistoryOfAncientAstronomyProblemsAndMethods1945

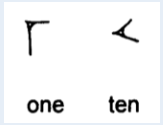
Why does Neugebauer compare place-value notation with the invention of the alphabet? What does he think both inventions made easier?

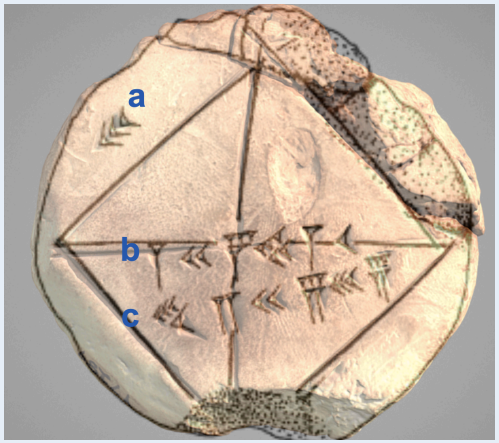
An amazing computation











Picture from Yale Collection
Drawing by Eleanor Robson

1.What are the values of a, b and c?
2.Educated guess: What do you think is the purpose of this tablet?

√2 Babylonian clay tablet (circa 1800–1600 BCE)

- Likely "written" **by a student.**
- Evidence suggests its **author copied** some of the results from an existing table of values and did not compute them himself/herself?.



Picture from Yale Collection
Drawing by Eleanor Robson

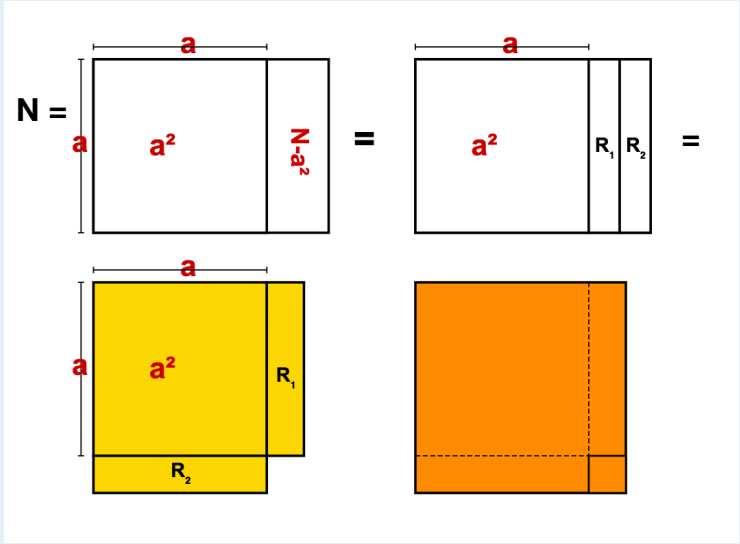
A digital replica to hold in your digital hands

https://sketchfab.com/3d-models/the-diagonal-of-a-square-tablet-605c9d9573d14b52b8880e14c826e133?utm_medium=embed&utm_campaign=share-popup&utm_content=605c9d9573d14b52b8880e14c826e133

- The value demonstrates one of the **greatest known computational accuracy** obtained in the ancient world.

Some examples of notable computational accuracy in the Ancient World

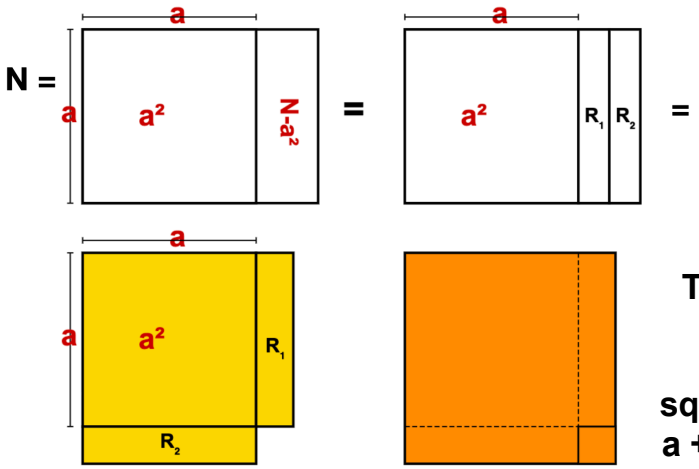
	Who and Where	When	Value	Accuracy
√2	Babylonian	1800–1600 BCE	$(1;24,51,10)_{60}$	error ≈ 6.0×10^{-7} → **6 decimals**
√2	Śulba Sūtras - India	800–500 BCE	1 + 1/3 + 1/(3·4) – 1/(3·4·34)	error ≈ 2.1×10^{-6} → **5 decimals**
π	Archimedes - Greece	3rd c. BCE	223/71 < π < 22/7 (Greek ciphered)	**≈2–3 decimals** bounds
π	Ptolemy - Alexandria	2nd c. CE	$(3;8,30)_{60}$	error ≈ 7.4×10^{-5} → **3 decimals**
π	Zu Chongzhi - China	5th c. CE	355/113	error ≈ 2.7×10^{-7} → **6 decimals**



What is the area of the yellow figure? What is the area of the orange figure? (Express both areas in terms of N and a)

Problem: Given a square of area N, find its side

- **Start with a good guess**: Pick **a** where a^2 is close to N (but $a^2 < N$)



The side of the new (orange) square is a' = a + (N-a²)/(2a)

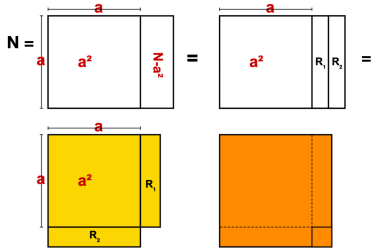
Problem: Given a square of area N, find its side

Babylonian approximation of the square root of a number N.

- **Start with a good guess**: Pick **a** where $\mathbf{a^2}$ is close to **N** (and $\mathbf{a^2 < N}$)
- **Form a rectangle**: Create a rectangle with area **N** and one side of length **a.** (The other side must have length **N/a**)
- **Goal**: Cut up the rectangle and rearrange the pieces to form "almost" **a** square closer to area **N**.This gives a better approximation to **√N**

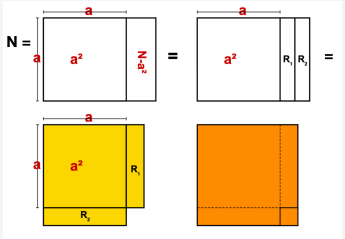
The side of the new (orange) square is a' = a + (N-a²)/(2a)

Does this algorithm ring a bell?



Babylonian approximation of the square root of a number N when $a^2 < N$

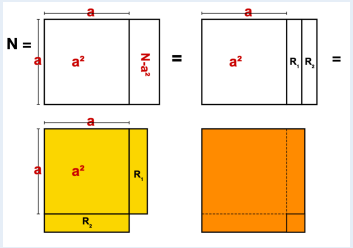
1. Find a such that a^2 is close to N and $a^2 < N$.
2. The yellow shape has area N = $a^2 + (N - a^2)$.
3. The "leftover area $(N - a^2)$ is split into two equal rectangles, each of area $(N - a^2)/2$. So their width is $(N - a^2)/(2a)$
4. Since a^2 is close to N, the area of the yellow shape (N) is close to the area of the orange square."They differ only by the small corner square of side
5. Therefore,√N is close to the side length of the orange square.
6. The orange square's side length is $a+ (N - a^2)/(2a)$. Thus, $a+ (N - a^2)/(2a)$ is the new approximation to √N.

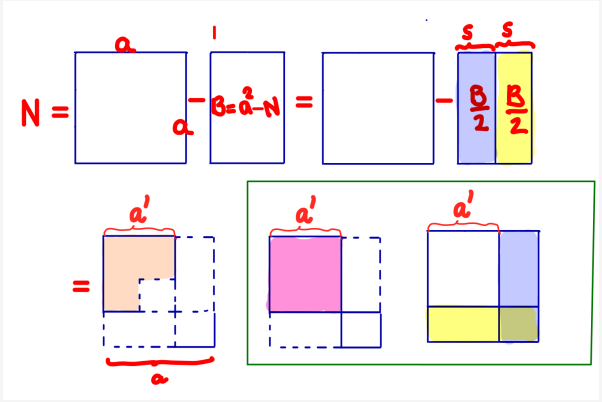


Using the algorithm, compute an approximation of √2 starting with a=1

- **Start with a good guess**: Pick **a** where **a²** is close to **N** (but **a² < N**)
- **Form a rectangle**: Create a rectangle with area **N** and one side of length **a.** (The other side must have length **N/a**)
- **Goal**: Cut up the rectangle and rearrange the pieces to form "almost" **a** square closer to area **N**.This gives a better approximation to **√N**

The side of the new (orange) square is a' = a + (N-a²)/(2a)





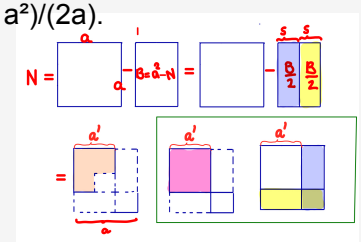
Babylonian method to find square roots, using an approximation a with a²>N.

Given **N** and **a** such that:
- N>0, a>0
- a²>N
- a² is close to N (thus a is close to √N)
- Hint: Write **s** in terms of **N** and **a**

Following the figures, find a' such that (in terms of a and N) such that (a')² is even closer N than a² (so a' is even closer than a to √N)

Babylonian approximation of the square root of a number N when a² > N

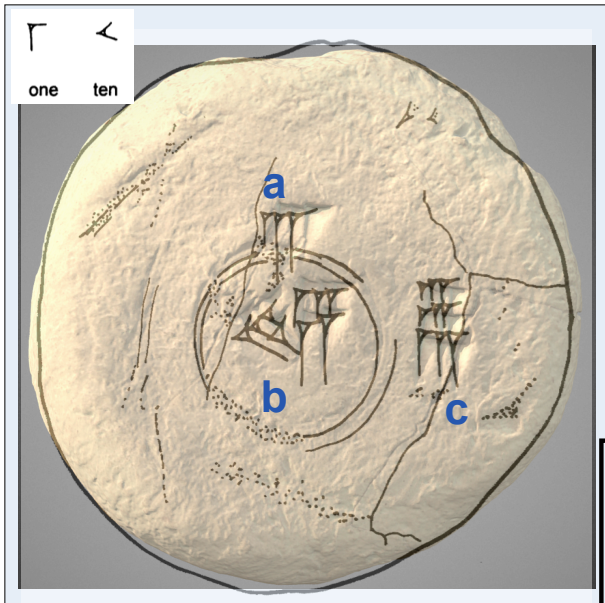
1. Pick a so that a² is close to N and a² > N.
2. The yellow shape has area N = a² − (a² − N).
3. Split the excess area (a² − N) into two equal rectangles, each of area (a² − N)/2, so their width is (a² − N)/(2a).
4. Remove one strip from the right and one from the bottom of the a × a square to get the orange shape. Since the two stripes share a small square, we remove it too.
5. The small missing corner has side (a² − N)/(2a), so the orange area is close to N.
6. Therefore √N is close to the side of the orange square.
7. The orange square's side is a − (a² − N)/(2a).
   Thus the new approximation is a′ = a − (a² − N)/(2a) = a + (N − a²)/(2a).



Calculation of Areas



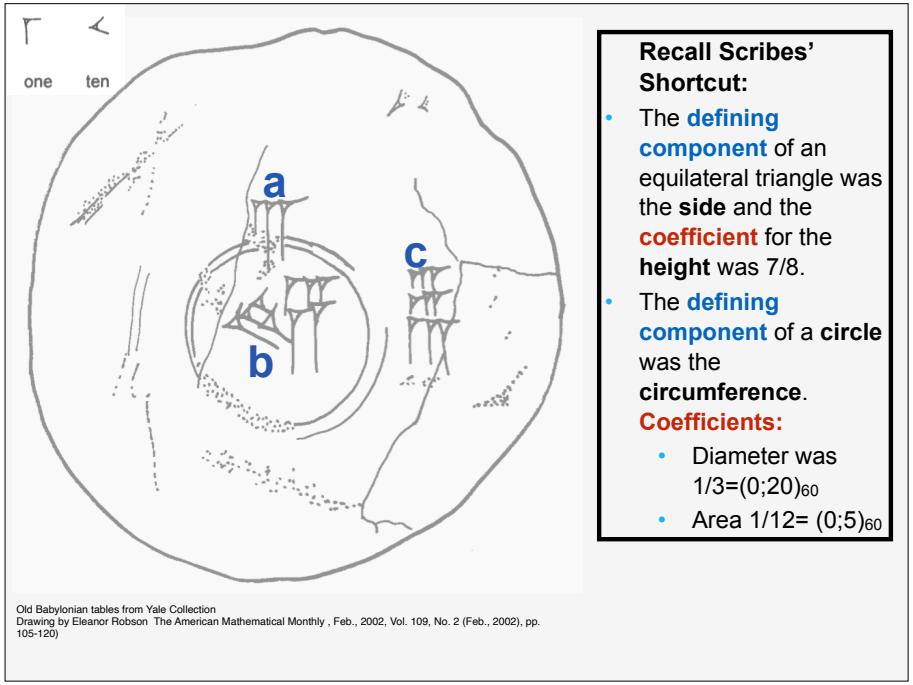
https://sketchfab.com/3d-models/the-area-of-a-circle-tablet-16f3e288b7be485297ddd0945fd10ece



1. Write a, b, c in the Hindu-Arabic number system.
2. What do these numbers represent?

Hint: Consider the arrangement of the numbers with respect to the circle.

Old Babylonian tables from Yale Collection
Drawing by Eleanor Robson The American Mathematical Monthly , Feb., 2002, Vol. 109, No. 2 (Feb., 2002), pp. 105-120)



Old Babylonian tables from Yale Collection
Drawing by Eleanor Robson The American Mathematical Monthly , Feb., 2002, Vol. 109, No. 2 (Feb., 2002), pp. 105-120)

Recall Scribes' Shortcut:

- The **defining component** of an equilateral triangle was the **side** and the **coefficient** for the **height** was 7/8.
- The **defining component** of a **circle** was the **circumference**.

Coefficients:

- Diameter was $1/3=(0;20)_{60}$
- Area $1/12= (0;5)_{60}$

Tablets in Cuneiform Deciphering and finding meaning



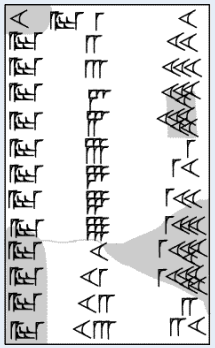
Photographs by Bill Casselman, Peter Damerow, Jöran Friberg.

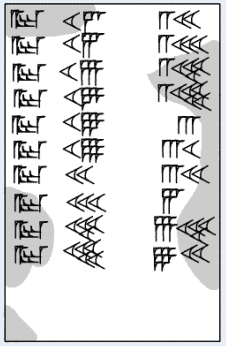
Deciphering and finding meaning. What s this table about?

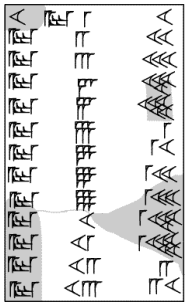
Ask for help if you need it

Please do not use any other tools (except your mind and writing instruments)









Decipher tablet

The first line (partially illegible here) reads

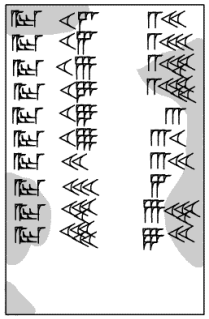
10 a-rà(times) 1 10.

From line 2 on, in this format, the multiplier is left out:

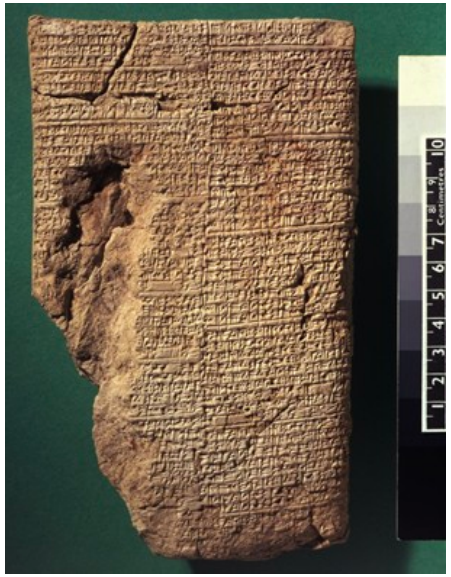
a-rà 2 20

a-rà 3 30…

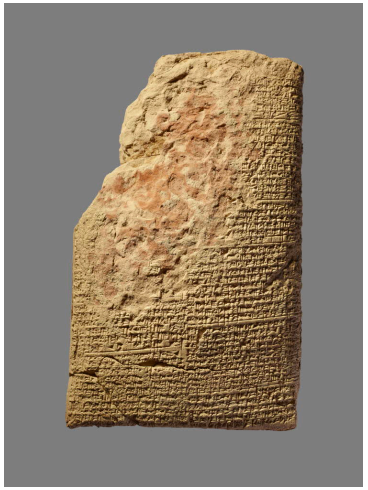




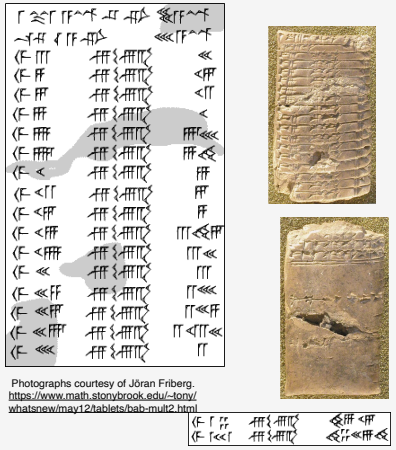
http://www.ams.org/publicoutreach/feature-column/fc-2012-05



Tablet 13901
British Museum
complete the square problems



Babylonian multiplication tables



Photographs courtesy of Jöran Friberg.
https://www.math.stonybrook.edu/~tony/whatsnew/may12/tablets/bab-mult2.html

Tables of Reciprocals

n	1/n
2	0.5
3	0.333333333333333
4	0.25
5	0.2
6	0.166666666666667
7	0.142857142857143
8	0.125
9	0.111111111111111
10	0.1
11	0.0909090909090909
12	0.0833333333333333
13	0.0769230769230769
14	0.0714285714285714
15	0.0666666666666667
16	0.0625
17	0.0588235294117647
18	0.0555555555555556
19	0.0526315789473684
20	0.05
21	0.0476190476190476

n	1/n
22	0.0454545454545455
23	0.0434782608695652
24	0.0416666666666667
25	0.04
26	0.0384615384615385
27	0.037037037037037
28	0.0357142857142857
29	0.0344827586206897
30	0.0333333333333333
31	0.032258064516129
32	0.03125
33	0.0303030303030303
34	0.0294117647058824
35	0.0285714285714286
36	0.0277777777777778
37	0.027027027027027
38	0.0263157894736842
39	0.0256410256410256
40	0.025
41	0.024390243902439

a.Describe the numbers n such the 1/n has a terminating decimal. (The table might help.) b. (Optional) Why does this pattern occur?

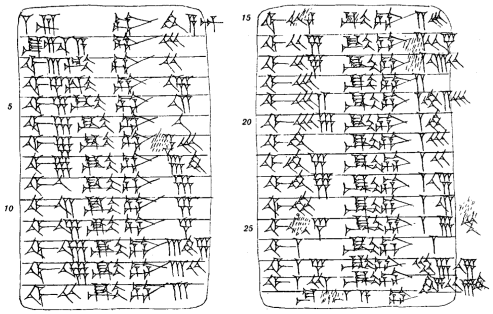
n	1/n
2	0.5
3	0.333333333333333
4	0.25
5	0.2
6	0.166666666666667
7	0.142857142857143
8	0.125
9	0.111111111111111
10	0.1
11	0.0909090909090909
12	0.0833333333333333
13	0.0769230769230769
14	0.0714285714285714
15	0.0666666666666667
16	0.0625
17	0.0588235294117647
18	0.0555555555555556
19	0.0526315789473684
20	0.05
21	0.0476190476190476

n	1/n
22	0.0454545454545455
23	0.0434782608695652
24	0.0416666666666667
25	0.04
26	0.0384615384615385
27	0.037037037037037
28	0.0357142857142857
29	0.0344827586206897
30	0.0333333333333333
31	0.032258064516129
32	0.03125
33	0.0303030303030303
34	0.0294117647058824
35	0.0285714285714286
36	0.0277777777777778
37	0.027027027027027
38	0.0263157894736842
39	0.0256410256410256
40	0.025
41	0.024390243902439

The first numbers n such the 1/n has a terminating decimal.

n	1/n	C= 1/n with integer fraction separator removed	C.n
2	0.5	5	10
4	0.25	25	100
5	0.2	2	10
8	0.125	125	1000
10	0.1	1	10
16	0.0625	625	10000
20	0.05	5	100
25	0.04	4	100
32	0.03125	3125	100000
40	0.025	25	1000
50	0.02	2	100

Integer fraction separator is the decimal point



Tables of reciprocals

From Robson, Eleanor. "Words and pictures: New light on Plimpton 322." The American Mathematical Monthly 109.2 (2002): 105-120.

Two thirds of 1 is 0;40.
Its half is 0;30.
The reciprocal of 2 is 0;30.
The reciprocal of 3 is 0;20.
The reciprocal of 4 is 0;15.
The reciprocal of 5 is 0;12.
The reciprocal of 6 is 0;10.
The reciprocal of 24 is 0;02 30.
The reciprocal of 25 is 0;02 24.
The reciprocal of 27 is 0;02 13 20.
The reciprocal of 30 is 0;02.
The reciprocal of 32 is 0;01 52 30.
The reciprocal of 36 is 0;01 40.
The reciprocal of 40 is 0;01 30.

From Robson, Eleanor. "Words and pictures: New light on Plimpton 322." The American Mathematical Monthly 109.2 (2002): 105-120.

2	30
3	20
4	15
5	12
6	10
8	7,30
9	6,40
10	6
12	5
15	4
16	3,45
18	3,20
20	3
24	2,30
25	2,24

27	2,13,20
30	2
32	1,52,30
36	1,40
40	1,30
45	1,20
48	1,15
50	1,12
54	1,6,40
1	1
1,4	56,15
1,12	50
1,15	48
1,20	45
1,21	44,26,40

What patterns you and/ or neighbor find in this table? (Numbers are in base 60. Hints: It is related to the previous table)



https://www.britishmuseum.org/collection/object/W_Sp-II-49

2	30
3	20
4	15
5	12
6	10
8	7,30
9	6,40
10	6
12	5
15	4
16	3,45
18	3,20
20	3
24	2,30
25	2,24

27	2,13,20
30	2
32	1,52,30
36	1,40
40	1,30
45	1,20
48	1,15
50	1,12
54	1,6,40
1	1
1,4	56,15
1,12	50
1,15	48
1,20	45
1,21	44,26,40

What do you think this table is about? Can you find a pattern? (Numbers are in base 60)

n	B=1/n in sexagesimal with integer-fraction separator removed	n	B decimal	n.B
2	30	2	30	60
3	20	3	20	60
4	15	4	15	60
5	12	5	12	60
6	10	6	10	60
8	7,30	8	450	3600
9	6,40	9	400	3600
10	6	10	6	60
12	5	12	5	60
15	4	15	4	60
16	3,45	16	225	3600
18	3,20	18	200	3600
20	3	20	3	60
24	2,30	24	150	3600
25	2,24	25	144	3600
27	2,13,20	27	8000	216000

Tables of Reciprocals and Regular numbers

Base-10: Reciprocals have finite decimal expansions only when prime factors are 2 and 5

Base-60: Reciprocals have finite sexagesimal expansions only when prime factors are 2, 3, and 5

Regular numbers: Numbers with only prime factors 2, 3, or 5

Why regular numbers mattered: Mesopotamians performed division by multiplying with reciprocals—but only regular numbers have finite sexagesimal representations

Reciprocal tables: Scribes created reference tables containing **only regular numbers** and their reciprocals.





Recall: Tables in Tablets

- Amazing aspect of the Babylonian's calculating skills: **construction of tables to aid calculation.**
- Babylonians used the formula

$$a.b = (1/2)((a+b)^2 - a^2 - b^2)$$

to make multiplication easier.

- They did not have an algorithm for long division. Instead they based their method on the fact that

$$A \,\%\, B = A \times (1/B)$$

and used tables of reciprocals $(1/B)$.



