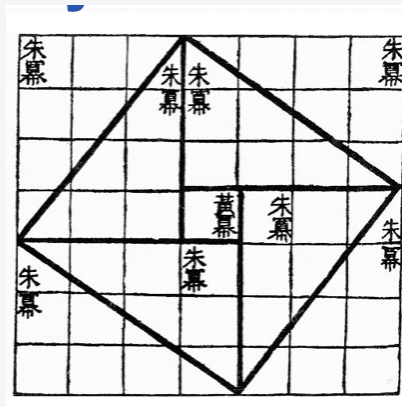
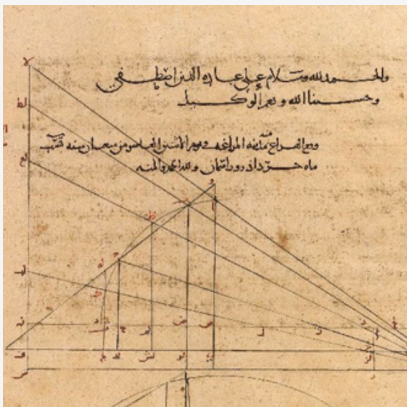
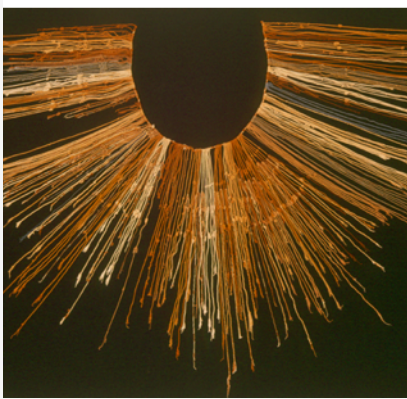


Counting and Counting Numbers

What is Counting and How Do We Do It?

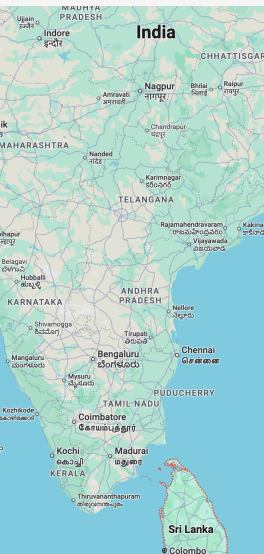


- What is counting (definitions, one-to-one correspondence)
- Recording methods (tally systems)
- Human brain and counting, connection between fingers and numbers (parietal lobe)
- Approximate number sense and object tracking

Examples of counting in different societies

Example: Veddas of Sri Lanka counting

- Lived by eating fruit and turners of jungle plants.
- Had few words to deal with numbers. Some of these words (translated) were: a single, a couple, another one and many.
- To count, say, a pile of coconuts, a Vedda collected a heap of sticks. Then,
 - For each coconut in the pile, she took a stick.
 - Each time she took a stick, she said “another one”
 - When finished, she pointed out the pile of sticks she took and said “That many”.



Veddas count does not use number-words



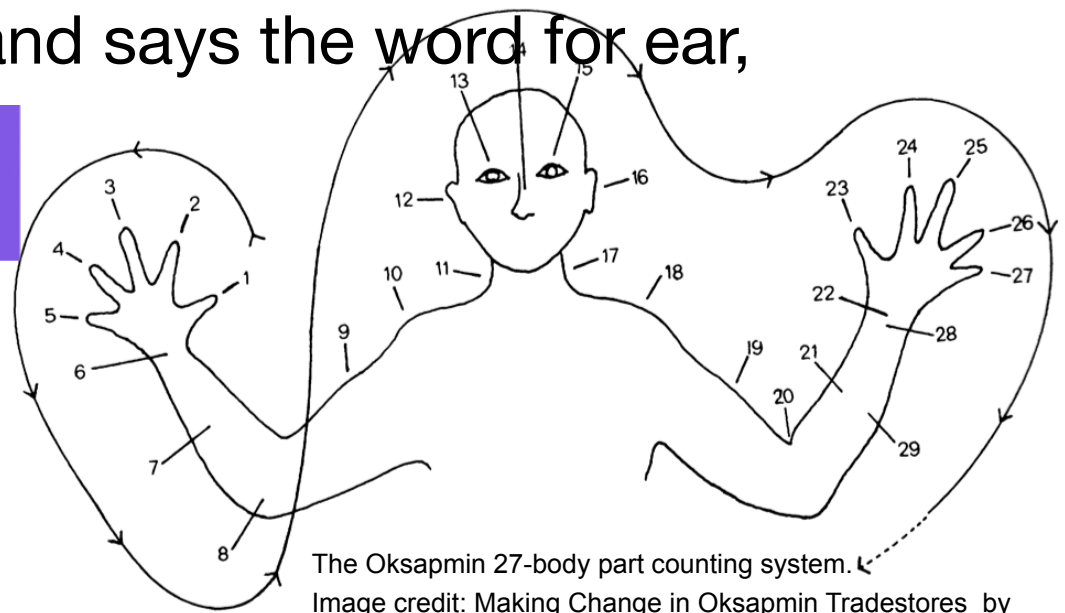
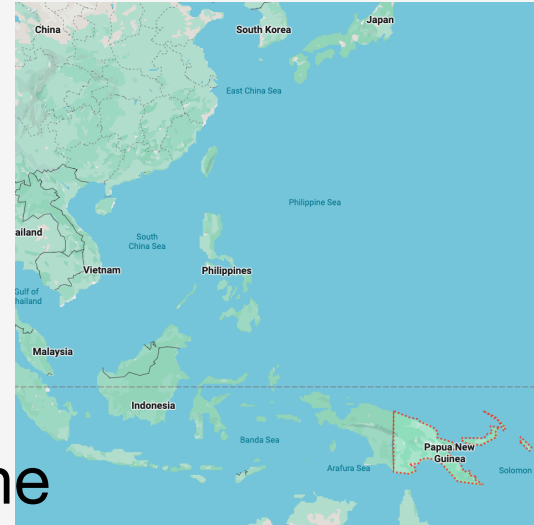
<https://sites.utexas.edu/dsb/files/2021/02/Oneness-Twoness-Threeness-How-Ancient-Accountants-Invented-Numbers.pdf>

Example: Oksapmin from Papua New Guinea

- Begins with the thumb on one hand
- Enumerates 27 places around the body,
- Ends on the little finger of the opposite hand.
- To indicate a particular number, one points to the appropriate body part and says the body part name.
- Example: to indicate the number 12, one points to the ear which is the 12th body part and says the word for ear, “nata”.

This way of counting is called **body counting**

Forms of body counting were quite spread around the globe as late as the 1800s

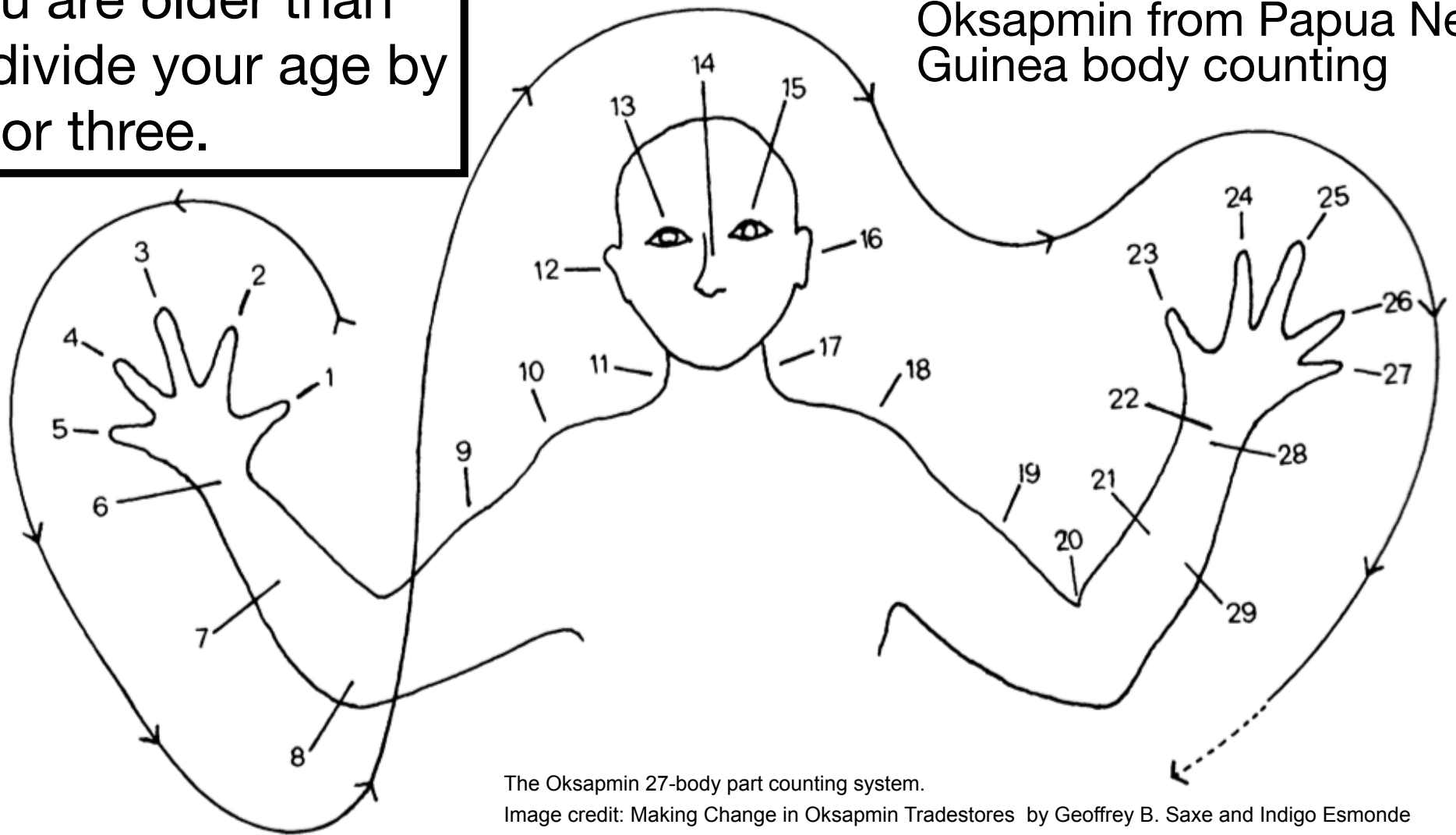


The Oksapmin 27-body part counting system.
Image credit: Making Change in Oksapmin Tradestores by Geoffrey B. Saxe and Indigo Esmonde

"What is your age in Oksapmin body counting? Use the corresponding body part to describe it in English. For example, if you are five years old, you would write 'pinky finger,' and if you are twenty-seven, you would write 'other pinky finger'."

If you are older than 29, divide your age by two or three.

Oksapmin from Papua New Guinea body counting



Who am I?



I was born in
Buenos
Aires
Ar

I studied “licenciatura” in
Mathematics at the
University of Buenos Aires

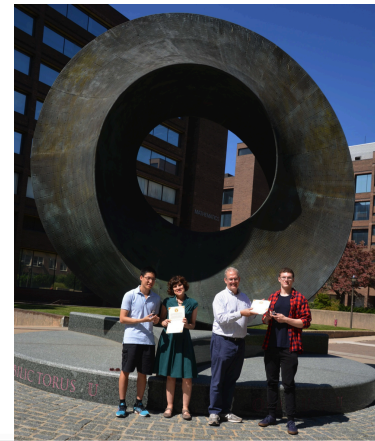


Questions?

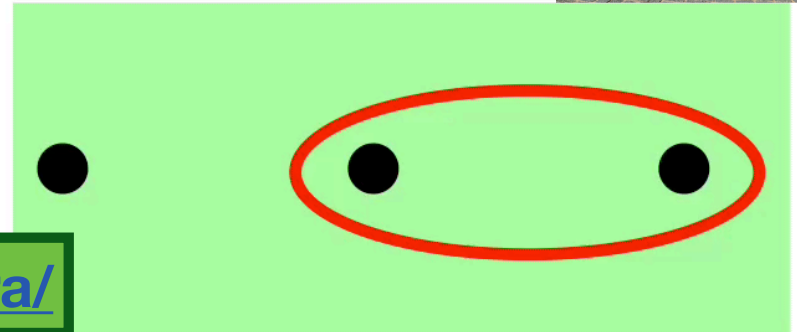
Instagram @moirart3 or at

I did my PhD at the Autònoma University
of Barcelona

University, New
York, US



<http://www.math.stonybrook.edu/~moira/>



Example: Nivkh people from lower Amur River - Russia

	Means <i>two</i> of the following
Mex	trees, sticks, pencils and some other long things
Merax	leaves or pieces of textiles or other flat items.
Mik	berries, balls and other round things.



Based on the oral accounts of Mamiya Rinzō, Sadatsugu Murakami depicted the portrait of the Smerenkul Jurchen (Nivkh people) in Hokui Bunka Yowa (Supplementary Tales of the Northern Barbarians' Boundaries).

Example: “**two leaves**” is expressed as “merax x” where x is the word for leaves.

Is there any noun in English similar to those of the Nivkh people? If so, write it down.

Table 1-2-1

objects_being_counted	one	two	three	four	five	ten
one-dimensional objects	nex	mex	ceX	nux	thox	mxox
two-dimensional objects	nrax	merax	crax	nrX	thorax	mxorax
three-dimensional objects	nik	mik	ceX	nax	thox	mxox
sledges	niř	miř	ceř	nař	thoř	mxon
boats	nim	mim	cem	nəm	thom	mxon
generic	naqř	meqř	caqř	nakř	thoqř	mxoqř
fishnet cells	niu	miu	ceu	nuu	thou	mxou
fishnet strips	neřqe	meřqe	ceřqe	nařqe	thořqe	?
fishnets and fish-spears	nvor	mevor	cfor	nvur	thovor	mxovor
special fishnets	neo	meo	ceo	nəu	thou	mxou
poles for making fish-spears	nla	mel	cla	nly	thola	mxola
poles for drying fish	nesk	mesk	cesk	nask	thosk	?
boards for building boats	nec	mec	cec	nac	thoc	mxoec
families	niřn	miřn	ceřn	nařn	thořn	mxoniřn
generations	nesvax	mesvax	cesvax	nasvax	thosvax	?
places	navř	mevř	cavř	navř	thovř	mxovř
day's rests on one's way	nix	mix	ceX	nax	thox	?
humans	nenn	menn	caqř	nařy	thory	

Concrete counting (numerosity)

Concrete counting: Counting systems tied to what you're counting. So the number is not fully independent of the thing counted.

The counting word tells you what kind of thing is being counted (shape, animacy, role, material, etc.), and the noun tells you which thing.

Examples

- Various Amazonian languages have separate number words for different object types.
- Some Australian Aboriginal languages count people vs. animals vs. objects differently.

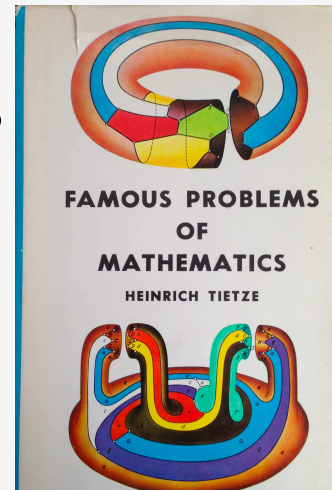
A way of counting

A group of friends, let's call them A, B and C want to count a large number of people.

1. The people starts walking before them. Each time a person walks by, A raises a finger (and keeps it raised)
2. When all fingers of A are raised, B raises one finger and A lowers all their fingers. The procedure continues repeating appropriately 1. and 2.
3. At some point, all fingers of B are raised. Then C raises one finger and B lowers all their fingers.
4. And so on.

This type of counting was probably common but is hard to document.

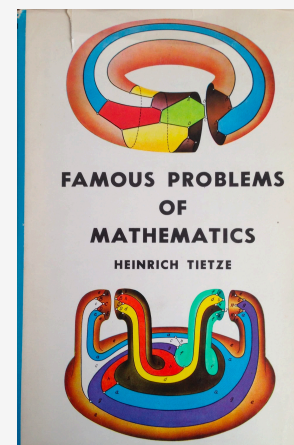
According to Tietze, this form of counting was used by a tribe from South Africa

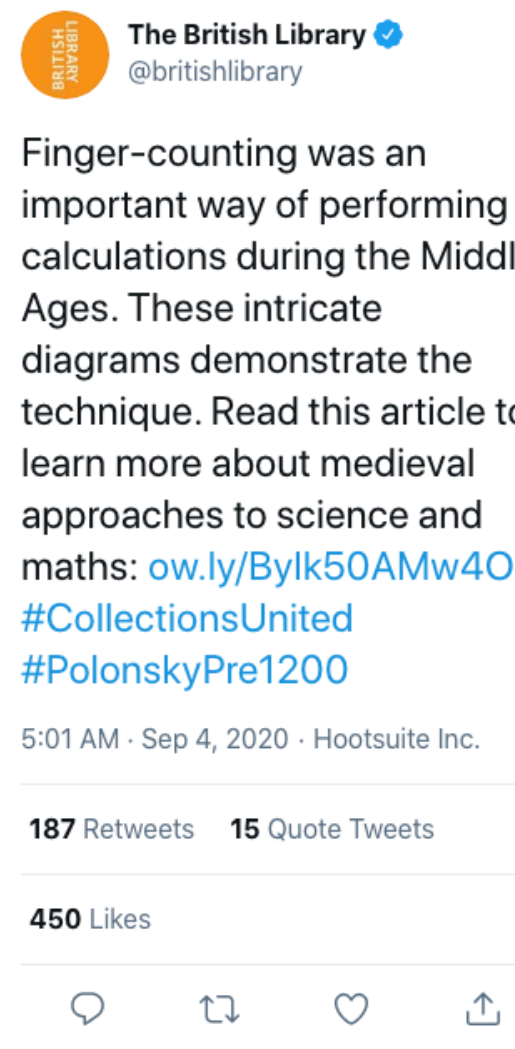


Counting in a planet of beings with only one hand, with three fingers in that lonely only hand

A group of friends, let's call them A, B and C want to count a large number of people.

1. The people starts walking before them. Each time a person walks by, A raises a finger (and keeps it raised)
2. When all fingers of A are raised, B raises one finger and A lowers all their fingers. The procedure continues repeating appropriately 1. and 2.
3. At some point, all fingers of B are raised. Then C raises one finger and B lowers all their fingers.
4. And so on.





Illustrations of hands labelled with figures on the fingers to calculate the date of Easter and perform mathematical calculations (British Library, Egerton MS 3314, f. 73r, detail)

Forms of body counting were quite spread around the globe as late as the 1800s

Navajo Number Names: Fingers and Mathematics

Number	Navajo	Approximate Translation	Arithmetic
1	Łá	Little finger	1
2	Náka	Ring finger	2
3	Tátł'j	Middle finger	3
4	Dij	Index finger	4
5	Ashdla'	The hand is finished	5
6	Łáts'jį	One past the hand	5 + 1
7	Tso	Two past the hand	5 + 2
8	Tsoo t'áá	Three past the hand	5 + 3
9	Náhąą	Four past the hand	5 + 4
10	Benáá	Both hands finished	5 + 5
11	Benáá lá	Ten and one	10 + 1
15	Benáá ashdla'	Ten and five	10 + 5
20	Náhąą benáá	Two tens	2 × 10

Review about counting

- Almost (if not all) societies have/had some awareness of number.
- Counting is establishing a **one-to-one correspondence** between two sets.
- Counting is closely **linked to fingers**.
- The concept of number, although basic is **very hard to define rigorously**. (Recall the barber's paradox)
- After counting, a need to record the result of the counting arises. There are different ways of **keeping records** of “how many”, for instance
 - Making scratches
 - Using objects (sticks, pebbles, shells)
 - Tying knots.



Image credit: <https://creativekindergartenblog.com/one-to-one-correspondence-intervention-for-kindergarten/>

History of mathematics hidden in language

How can we explore the origins of numbers?

Some approaches:

- Comparing past and present number systems
- Analyzing notations in ancient number systems
- Investigating the origins and evolution of number words

Example: Evidence in ordinal numbers:

- "First" = before all others
- "Second" = from Latin *secundus*, "following", not related to *duo* ("two")
- "Third, fourth, fifth..." = systematic counting

Early word numbers: Observe patterns

Number	English	Gothic	Latin	Ancient Greek	Welsh	Sanskrit	Basque
1	one	ains	unus	heis	un	eka	bat
2	two	twai	duo	dyo	dau	dva	biga
3	three	threis	tres	treis	tri	tri	hirur
4	four	fidwor	quattuor	tettares	pedwar	catur	laur
5	five	fimf	quinque	pente	pump	panca	bortz
6	six	saihs	sex	hex	chwech	sad	sei
7	seven	sibun	septem	hepta	saith	sapta	zazpi
8	eight	ahtau	octo	okto	wyth	asta	zortzi
9	nine	niun	novem	ennea	naw	nava	bederatzi
10	ten	taihun	decem	deka	deg	dasa	hamar

A counting system is quite a complex construction, with different historical structures overlaying one another

Describe all patterns you can find in this table

Number	English	Gothic	Latin	Ancient Greek	Welsh	Sanskrit	Basque
11	eleven	ainlif	undecim	hendeka	un ar ddeg	ekaadasha	hamaika
12	twelve	twalif	duodecim	dodeka	deudddeg	dvaadashan	hamabi
13	thirteen	pritehund	tredecim	treiskaideka	tri ar ddeg	trayodasha	hamahiru
14	fourteen	fidwartehund	quattuordecim	tessareshkaideka	pedwar ar ddeg	chaturdasha	hamalau
15	fifteen	fimftehund	quindecim	pentekaideka	pymtheg	panchadasha	hamabost
16	sixteen	saihstehund	sedecim	hexkaideka	un ar bymtheg	shodasha	hamasei
17	seventeen	sibuntehund	septendecim	heptakaideka	dau ar bymtheg	saptadasha	hamazazpi
18	eighteen	ahtautehund	duodeviginti	oktokaideka	deunaw	ashtadasha	hemezortzi
19	nineteen	niuntehund	undeviginti	enneakaideka	pedwar ar bymtheg	navadasha	hemeretzi
20	twenty	twaitigjus	viginti	eikosi	ugain	vimsatih	hogoi

Hidden History in Number Words

Num ber	English	Gothic	Latin	Ancient Greek	Welsh	Sanskrit	Basque
11	eleven	ainlif	undecim	hendeka	un ar ddeg	ekaadasha	hamaika
12	twelve	twalif	duodecim	dodeka	deudddeg	dvaadashan	hamabi
13	thirteen	þritehund	tredecim	treiskaideka	tri ar ddeg	trayodash a	hamahiru
14	fourteen	fidwartehund	quattuordecim	tessareskaideka	pedwar ar ddeg	chaturdasha	hamalau
15	fifteen	fimftehund	quindecim	pentekaideka	pymtheg	panchadasha	hamabost
16	sixteen	saihstehund	sedecim	hexkaideka	un ar bymtheg	shodasha	hamasei
17	seventeen	sibuntehund	septendecim	heptakaideka	dau ar bymtheg	saptadasha	hamazazpi
18	eighteen	ahtautehund	duodeviginti	oktokaideka	deunaw	ashtadasha	hemezortzi
19	nineteen	niuntehund	undeviginti	enneakaideka	pedwar ar bymtheg	navadasha	hemeretzi
20	twenty	twaitigjus	viginti	eikosi	ugain	vimsatih	hogoi

- "Eleven" = "one left" – words close in English and Gothic.
- "Twelve" = "two left" – words close in English and Gothic.

We can infer that in here were **northern European tribes whose counting words went up only to 'ten'** then counted leftovers

Hidden History in Number Words

Base-20 Traces

- French: "quatre-vingts-sept" = $4 \times 20 + 7 = 87$
- English (Bible Psalm 90:10): "three score and ten" = $3 \times 20 + 10 = 70$

Mixed systems show historical layers

**Can you remember a historical
occurrence of the word “score”?
(Where score mean 20)**

Four **score** and seven years ago our fathers brought forth on this continent, a new nation, conceived in Liberty, and dedicated to the proposition that all men are created equal.

Now we are engaged in a great civil war, testing whether that nation, or any nation so conceived and so dedicated, can long endure. We are met on a great battle-field of that war. We have come to dedicate a portion of that field, as a final resting place for those who here gave their lives that that nation might live. It is altogether fitting and proper that we should do this.

But, in a larger sense, we can not dedicate -- we can not consecrate -- we can not hallow -- this ground. The brave men, living and dead, who struggled here, have consecrated it, far above our poor power to add or detract. The world will little note, nor long remember what we say here, but it can never forget what they did here. It is for us the living, rather, to be dedicated here to the unfinished work which they who fought here have thus far so nobly advanced. It is rather for us to be here dedicated to the great task remaining before us-- that from these honored dead we take increased devotion to that cause for which they gave the last full measure of devotion -- that we here highly resolve that these dead shall not have died in vain -- that this nation, under God, shall have a new birth of freedom -- and that government of the people, by the people, for the people, shall not perish from the earth.

**Abraham Lincoln The Gettysburg Address
Gettysburg, Pennsylvania November 19, 1863**

How Old Is Our Counting System?

Language family evidence:

- Most European and some Asian languages descend from Indo-European, spoken around 4000 BC.
- Similar words for "hundred" across these languages
- Suggests counting to 100 existed in original language

Before Indo-European:

- Counting by twenties (Basque, traces in French/Welsh) in Europe before Indo-European.
- Conclusion: Or spoken counting systems are 5000-8000 years old

Evolution of the Word "Digital"

Ancient
Latin
"Digitus"
finger/toe

14th-16th C.
Anatomical
Fingers &
toes

17th-18th C.
Mathematical
Numbers
(0-9)

1940s-60s
Computing
Digital vs
analog

1970s-90s
Consumer
Clocks &
cameras

2000s+
Digital Age
Everything
digital

- Originally referred to fingers (from Latin digitus).
- Later came to describe things represented in distinct units (digits), which led to its association with discrete values in contrast to analog (continuous) representations.
- Extended to binary data & computers → Broader use: digital media, communication, transformation.