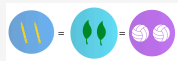


The very beginning of mathematics



On These slides

- What is mathematics?

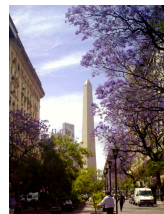
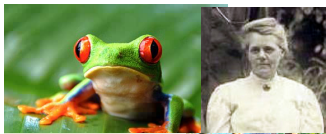
Sources: How do we know what we know in math history.

- Early math sources
- The Ishango bone. Interpretations.

First meeting

- What is mathematics?
- The Ishango bone and its interpretations
- The use of AI
- The use of the Web
- How can we deduce information from bones?
- More about prehistoric mathematics
- About teaching and learning
- What is counting?
- Counting and the brain
- Approximate number system - Object tracking system
- What is a natural (or counting) number? (And why is this such a hard question)
- Examples of counting in different societies
- History of mathematics hidden in language
- Number systems
- Additive number systems
- Ciphored or alphabetic number systems
- Multiplicative number systems
- Positional number systems
- Two examples of positional number systems: Maya and Babylonia
- Sources?
- Review and comparison

Who am I?



I was born in Buenos Aires, Argentina

I studied "licenciatura" in Mathematics at the University of Buenos Aires



Questions?

Instagram @moirart3 or at

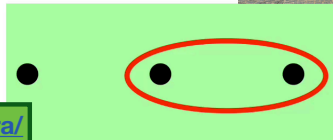
I did my PhD at the Autònoma University of Barcelona



University, New York, US



<http://www.math.stonybrook.edu/~moira/>



Important: log in to the platform with your SB email

Write down your first name and something you like about math.

What is mathematics?

- Make groups of four students and go to a blackboard.
- Introduce yourself to the members of your group and exchange coordinates for future communication.
- Write down a definition of mathematics that you agree with (do not use sources other than your group to maximize your learning).
- You have 5 minutes.

What is mathematics?

What is mathematics?

What do you mean by mathematics?

All science requires mathematics. The knowledge of mathematical things is almost innate in us! This is the easiest of sciences, a fact which is obvious in that no ones brain's rejects it; for laymen and people who are utterly illiterate know how to count and reckon.



Roger Bacon. Stipple engraving, 1786.
Wellcome Library no. 696i
[https://wellcomecollection.org/works/nj73cnnm?
wellcomeImagesUrl=/indexplus/image/V0000285.html](https://wellcomecollection.org/works/nj73cnnm?wellcomeImagesUrl=/indexplus/image/V0000285.html)

Roger Bacon (1214–1294), English philosopher and scientist

Galileo Galilei

~1600

Philosophy is written in that **great book which ever lies before our eyes — I mean the universe** — but we cannot understand it if we do not first learn the language and grasp the symbols, in which it is written.



Justus Sustermans - Portrait of Galileo Galilei, 1636
Public Domain

This book is written in the mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth.

“What we need are notions, not notations.”

- I wrote that sentence in “About the proof of Wilson's theorem,”
Disquisitiones Arithmeticae (1801), Article 76

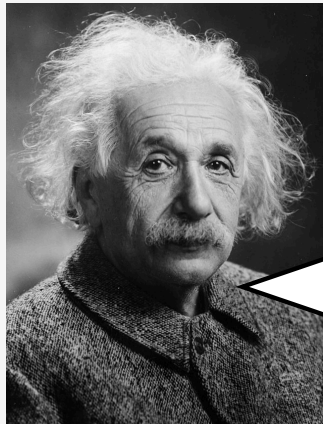
Carl Friedrich Gauss

~1800

Mathematicians stand on each other's shoulders.



Portrait of the mathematician and philosopher Carl Friedrich Gauss (1777-1855) - Public domain



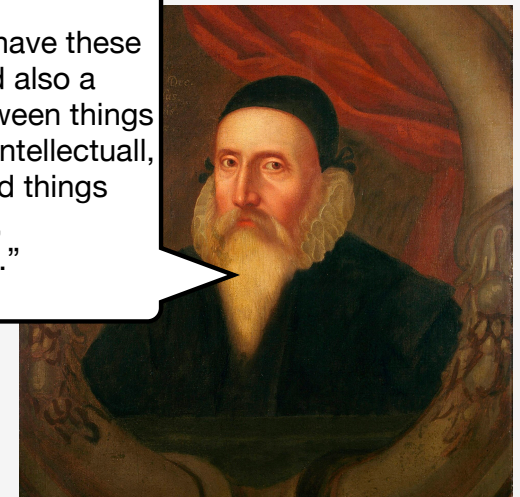
Photograph by Orren Jack Turner, Princeton, N.J. - Public domain

“Pure mathematics is, in its way, the poetry of logical ideas. One seeks the most general ideas of operation which will bring together in simple, logical and unified form the largest possible circle of formal relationships. In this effort toward logical beauty spiritual formulas are discovered necessary for the deeper penetration into the laws of nature.”

Albert Einstein, Obituary for Emmy Noether (5 May 1935).

Albert Einstein

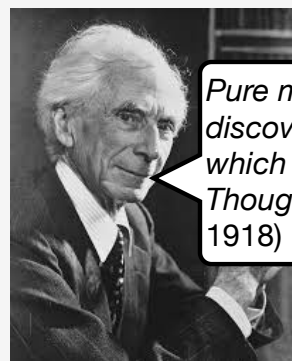
“A marvellous newtrality have these things mathematicall, and also a strange participation between things supernaturall, immortall, intellectuall, simple and indivisible, and things naturall, mortall, sensible, compounded and divisible.”



It seems that this portrait was painted when Dee was 67.

John Dee, 1527-1608. English mathematician and astrologer.

Preface to his edition of Euclid's *Elements*, 1570

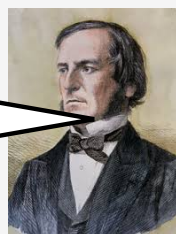


This file is made available under the Creative Commons CC0 1.0 Universal Public Domain Dedication

Bertrand Russell.

*Pure mathematics was discovered by Boole, in a work which he called *The Laws of Thought* (1854)'. (I wrote this in 1918)*

I am George Boole



George Boole, mathematician, 1815-1864 - Public Domain

What is mathematics?

In the 1950s, Sawyer described mathematics as the “classification and study of all possible patterns”.

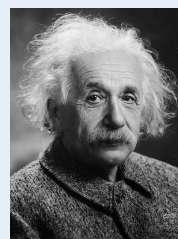
He explained that pattern was meant “to cover almost any kind of regularity that can be recognized by the mind”.

W. W. Sawyer, *Prelude to Mathematics*, 1955

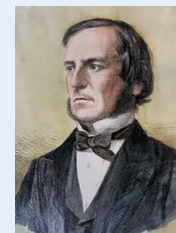
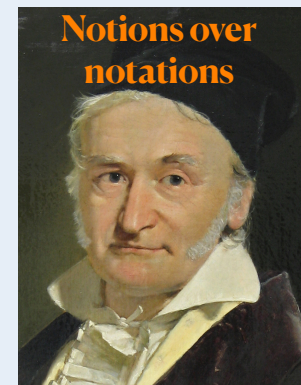
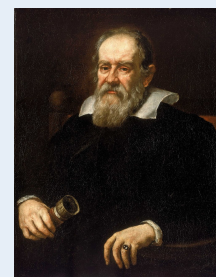
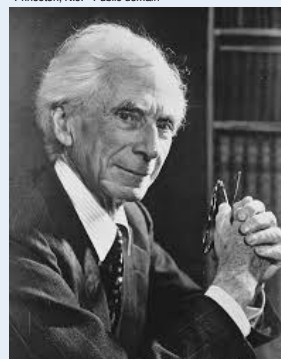
What do you mean by mathematics?

Study of concepts related to number, space, shapes, patterns, and structure, combined with logical reasoning.

Do you find a pattern?



Photograph by Orren Jack Turner, Princeton, N.J. - Public domain





I have said that the love of study is the passion most necessary to our happiness. It is an unfailing resource against misfortunes, it is an unending source of pleasures,...

Madame du Châtelet ~1700

Gabrielle Émilie Le Tonnelier de Breteuil, marquise du Châtelet (1706-1749), French mathematician and physicist - [Maurice Quentin de La Tour](#)

J'ai dit que l'amour de l'étude étoit la passion la plus nécessaire à notre bonheur : c'est une ressource sûre contre les malheurs ; c'est une ressource de plaisirs inépuisable,

The Ishango bone and its interpretations

Old Stone Age

- ~2,500,000 BC ● **Stone tools**
- ~1,000,000 BC ● **Fire use** • evidence debated
- ~40,000 BC ● **Symbolic culture** • cave art, ornaments
- ~20,000 BC ● **Ishango bone** • interpretation debated

New Stone Age • Near East

- ~10,000 BC ● **Neolithic begins** • settled agriculture, Near East

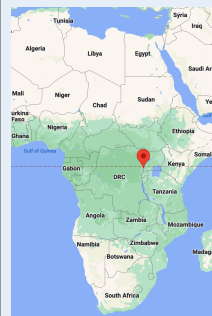
Bronze Age • Near East (dates vary by region)

- ~3,400 BC ● **Earliest written numbers** • used in accounting, Mesopotamia
- ~3,300 BC ● **Bronze Age begins** • Near East

Ishango Bone (About 20,000 BC)

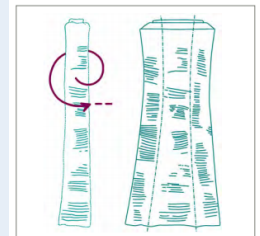
Educated guess: The purpose of the Ishango bone is:

- a. counting animals
- b. serve as a calendar
- c. unknown



<https://www.naturalsciences.be/en/museum/exhibitions-activities/exhibitions/250-years-of-natural-sciences/the-ishango-bone>

11	3	11
	6	
13	4	21
	8	
17	10	19
	5	
19	5	9
	7	
= 60		= 48
		= 60



Interpretations: Ishango Bone

11	3	11
	6	
13	4	21
	8	
17	10	19
	5	
19	5	9
	7	
= 60	= 48	= 60

Jean de Heinzelin (found the bone in 1960) →
 - arithmetical game,
 - counting system based on 10
 - knowledge of multiplication by 2 and prime numbers.

Alexander Marshack (1972) → six-month lunar calendar.

Claudia Zaslavsky (1991) → lunar calendar made by women to keep track of their monthly cycles.

Olivier Keller (2010) → Nothing can be said definitively.

“We do not see things as they are, we see them as we are.”
 Anais Nin, probably quoting from the Talmud.



Are other cultures so different that they are incomprehensible to us?

Let's distinguish conjectures from facts!

“All history is contemporary history.” Benedetto Croce

“As the South African archeologist David Lewis-Williams says of prehistoric art, ‘Meaning is always culturally bound’”

R. Leaky - The origin of humankind

“We do not see things as they are, we see them as we are.”
 Anais Nin, probably quoting from the Talmud.



Are other cultures so different that they are incomprehensible to us?

Challenge of prehistoric archaeology

Interpret the purpose, and meaning of marks which exhibit some regularity (length, spacing, and orientation) makes them visually indistinguishable from one another,



There are a lot of possibilities, and not methods of criteria to decide among them with the desired level of reliability and confidence.

Overmann, K. A. (2023). *The materiality of numbers: Emergence and elaboration from prehistory to present*. Cambridge University Press.

- decorative?
- conventional?
- mnemonic?
- symbolic?
- notational?
- numerical?
- astronomical?
- calendrical?
- musical?
- utilitarian?

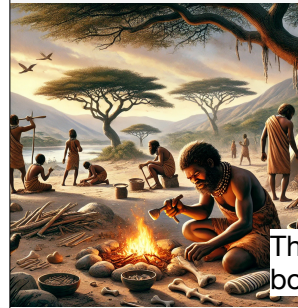
Do these marks represent numbers?

- Assume that prehistoric marks on nonperishable materials like bones were numbers. Then, these numbers
 - most likely would have been realized and elaborated through the use of the **fingers (and toes)** (as the hands/feet are the material form most commonly used today in emerging number systems.)
 - would also likely have been in existence for some period of time – **long enough to motivate not only their being recorded**, but also the use of a **nonperishable** substance for it.
- But with two debated exceptions – the Ishango and Les Pradelles artifacts – **no prehistoric artifact has marks organized in ways that unambiguously identify them as numbers.**
- The earliest notations with **unambiguous numerical organization**, the Mesopotamian impressions, **occur 5000 or 6000 years ago.**
- **The late emergence of unambiguously numerical marks suggests that Palaeolithic marks either did not represent numbers**

Overmann, K. A. (2023). *The materiality of numbers: Emergence and elaboration from prehistory to present*. Cambridge University Press.

The use of AI

Images made by ChatGPT in 2023 of a person carving the Ishango bone



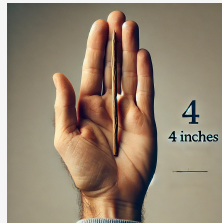
The size of the Ishango bone is about 4 inches



Prompts to ChatGPT and Perplexity in 2025. Make a realistic image of a person carving the Ishango bone.



The size of the Ishango bone is about 4 inches



ChatGPT



Prompts to ChatGPT and Perplexity in 2025. Does the person have to be alone? Is it possible there were other human beings around?

Perplexity



Claude says: Better detail of the carving process, stronger community feel, but potentially misleading geographic/cultural context

Short hair?

No beard?

ChatGPT



Claude says: Accurate African savanna setting, culturally appropriate context

The size of the Ishango bone is about 4 inches

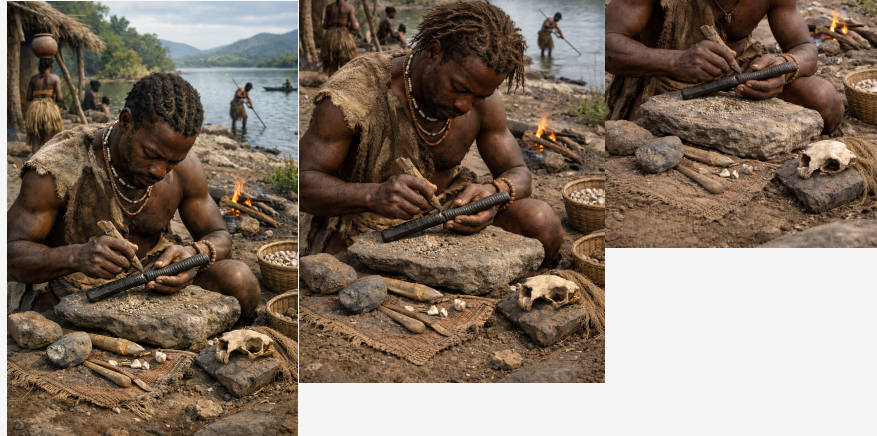


Skin color?

Gender?

Make a plausible image of a person carving the Ishango bone. Include a bit of the environment and their society around this person, if appropriate, I mean, if the picture is possible with these additions

Repeat the image corresponding to a careful reconstruction



Images made by ChatGPT in 2026

The use of the Web



From the Boston University Website
<https://www.bu.edu/africa/outreach/teachingresources/outreach-teachingresources-mathematics/>

The Lebombo bone (top) is the oldest known mathematical artifact. It is a tally stick with 29 distinct notches that were deliberately cut into a baboon's fibula. It was discovered within the Border Cave in the Lebombo Mountains of Eswatini. The Lebombo bone (bottom) resembles a calendar stick still used in Namibia. See more about these artifacts under "Other Resources" below.



Mathematical Treasure: Ishango Bone
MAA website

<https://old.maa.org/press/periodicals/convergence/mathematical-treasure-ishango-bone>



From the Boston University Website
<https://www.bu.edu/africa/outreach/teachingresources/outreach-teachingresources-mathematics/>

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Why do you think I showed these two photos? Can you extract a conclusion to apply to your research in this class?

How can we deduce information from bones?

Reindeer antler of Little Salt Spring, Florida , about 8000BCE



- 29 incisions
- very regular
- about five millimeters long
- the spacing between them is consistent

suggests that the marks were made systematically.

- smaller etchings in one-to-one alignment with the larger incisions and next to them,

suggest that quantities were ticked off along that progression.

Source: *Numbers and the Making of Us: Counting and the Course of Human Cultures*, by Caleb Everett., 2017, Harvard University Press

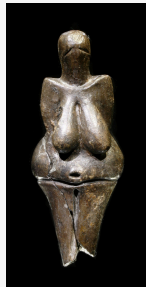
The reindeer antler of Little Salt Spring, Florida, with a colleague's hand for scale. Photograph by Caleb Everett.

Dolní Věstonice (in Czech Republic) archeological site

- From (roughly) 27,000 to 20,000 B.C
- Many archeological artifacts.
- Representations of men, women, and animals, along with personal ornaments, human burials and engravings.

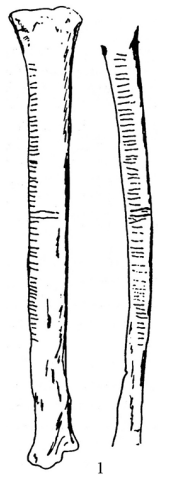


Carved female head from Dolní Věstonice, Krahuletz-Museum (replica)
Source: Wikipedia



Venus of Dolní Věstonice, the earliest discovered use of ceramics (29,000 BCE – 25,000 BCE) by Petr Novák, Wikipedia

Tally Sticks of the Stone Age
Andrej Kapcar
https://www.academia.edu/18709816/Tally_Sticks_of_the_Stone_Age



The Wolf Bone of Dolní Věstonice

Did Neanderthals count?

Hyena Bone, about 60,000 years old, found in France



Prehistoric accounting? Markings made on a hyena bone by a Neanderthal might have recorded numerical information.

According to a report published in *Nature*, the femur bone bears nine notches which are 'strikingly similar and approximately parallel, as if they were meant to signify something'.

Barras, C. (2021). How did Neanderthals and other ancient humans learn to count?. *Nature*, 594(7861), 22-25. <https://media.nature.com/original/magazine-assets/d41586-021-01429-6/d41586-021-01429-6.pdf>

More about prehistoric mathematics

The farther we delve into the past, the more we find mathematics entangled with accounting, surveying, astronomy, and the general administration of empires. **Mathematics arises wherever people think about the physical world or about the world of ideas embodied in laws and even theology.** It grows like a plant, from a seed that germinates and later ramifies to produce roots, branches, leaves, flowers, and fruit. It is constantly growing. Roger Cooke - The history of Mathematics - Third Edition

Photo Credit: Seawright, Caroline. (2014). ARCSFAL Essay: Ochre and the African middle stone age record. - 70,000 years old - found in Blombos Cave in South Africa



70,000 years old - found in South Africa

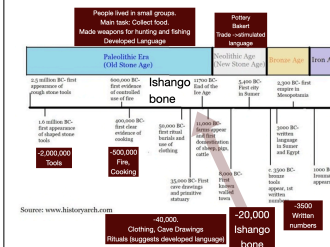
Recall: Ishango bone, 20,000BCE

Can you see any math in these photos?

Reproductions one of Lascaux artworks by Jack Versloot (about 17,000 years old)



17,000 years old, found in France



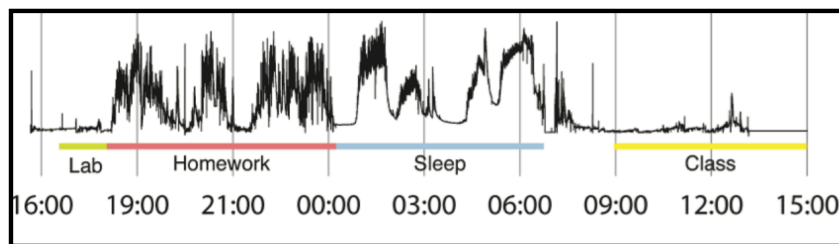
About teaching and learning

Fixed mindset

Growth mindset

Instead of thinking	Try this
I am not good at this	What am I missing?
I can't make it any better	I can improve if I keep trying
I made a mistake. Therefore, I am not smart.	What can I learn from this mistake?
This is too hard. I give up	This might take longer than I expected
The problem is that X is smarter than me	I'll try to learn how X does it.
This is too easy for me	Can I understand this more deeply?

Note: Lectures barely register -
Active learning lights up the brain



Brain on lecture vs. brain on active learning

Table from an old slide of Eric Mazur (<http://ericmazur.com/about.php>)

A picture of a lecture I will
work on **avoiding**

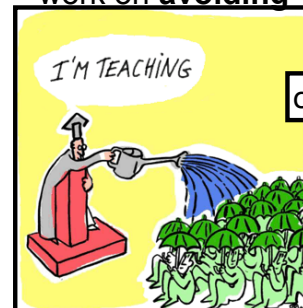


Image credit: Could not find it

contrast with

I hear and I forget

I see and I remember.

I do and I understand

Confucius

All the activities of class are
tools to help you learn.

Active learning increases student performance in science, engineering, and mathematics

Scott Freeman^{a,1}, Sarah L. Eddy^a, Miles McDonough^a, Michelle K. Smith^b, Nnadozie Okoroafor^a, Hannah Jordt^a, and Mary Pat Wenderoth^a

^aDepartment of Biology, University of Washington, Seattle, WA 98195; and ^bSchool of Biology and Ecology, University of Maine, Orono, ME 04469

Edited* by Bruce Alberts, University of California, San Francisco, CA, and approved April 15, 2014 (received for review October 8, 2013)

To test the hypothesis that lecturing maximizes learning and course performance, we metaanalyzed 225 studies that reported data on examination scores or failure rates when comparing student learning interventions varied widely in intensity and implementation, and included approaches as diverse as occasional group

<https://www.pnas.org/content/111/23/8410>

Image created by AI (DALL-E), illustrating something analogous to the use of AI to complete assignments

