Mesoamerica

Maya and Inca

Incas

What does Machu Picchu reveal about how the Inca organized people and knowledge across hard terrain? Include one detail from the video.

About the Incas

Empire Scale
· **Where/When:** Andean highlands, 1400s–1500s; empire spanned approximately 2,500 miles (~4,000 km)
, **Fall:** Spanish invasion (mid-1500s), civil war, and disease ended imperial rule

Organization & Communication
· **Statecraft:** Central authority; **Quechua** as lingua franca
· **Record-keeping: Quipus**; no **alphabetic or syllabic** writing system
· **Infrastructure:** ~30,000 km (~18,600 mi) road network, **chaski** relay runners, suspension bridges

Inca empire in 1500

Early Morning in Machu   Pichu
Pedro Szekely at https://www.flickr.com/photos/pedrosz/

About the Incas

By https://clevelandart.org/art/1957.136, CC0, https://commons.wikimedia.org/w/index.php?curid=78063088

Technology & Resources
- **Engineering:** Earthquake-resistant walls; agricultural terraces and irrigation. Built precisely without wheels, steel, iron, or mortar
- **Agriculture:** potatoes, maize, quinoa
- **Logistics:** "Vertical archipelago" across altitudes; llamas; state for redistribution

Knowledge & Culture
- **Beliefs & knowledge:** Sun cult; astronomy and ritual calendars; sacred geography
- Textiles, music, dance, and cultural traditions **still alive**.

About the Chasquis (or Chaskis)

- **Chaski (chasqui):** Inka relay messengers who ran the road network.
- **What they carried:** quipus, memorized oral messages, and small packets.
- **How it worked:** short sprints between waystations, ~10–15 km (6–9 mi) each.
- **Training/selection:** primarily young men with exceptional endurance and local terrain knowledge.
- **Communication:** Inca administration used quipus and oral reporting.

Stretch of Inka road through the upper Amazon. Near Chachapoyas, Peru, 2014. Photo:Inge Schjellerup

A chaski: Felipe Guaman Poma de Ayala (1535–1616). Pen and ink drawing published in The First New Chronical and Good Government (1615). Royal Library, Copenhagen GKS 2232 4°

About the Chasquis (or Chaskis)

A chasqui was a messenger of the Inca empire.

These 'chasques' were employed in this kingdom. They were the sons of curacas [local chiefs] who were loyal. Each runner had a white feather sunshade on his head which he wore so that the next chasque would see him at a distance. The chasque also carried a trumpet called 'putoto' [shell] so that the next runner would be ready. The arms they carried were the 'champi' [a star-headed club] and a 'uaraca' [sling].

These chasques were paid by the Inca, and they got their food from his storehouses. Each 'churo chasque' [chasqui carrying a shell] was stationed at intervals of one-half league [2.5 kilometres (1.6 mi)] so he could run the distance quickly. They say that they could bring a snail alive from the New Kingdom of Granada [northern part of south America] to the Inca in Cuzco.

—Felipe Guaman Poma de Ayala, 1615

What is a quipu?

Threads That Speak: How The Inca Used Strings to Communicate | National Geographic
https://youtu.be/AmPyz1kCbOw

More about incas and quipus

The largest number so far discovered on a quipu is **97,357**.

The ordinary judges gave a monthly account of the sentences they imposed to their superiors, and they in turn reported to their immediate superiors, and so on finally to the Inca or those of his Supreme Council. The method of making these reports was by means of knots, made of various colours, where knots of such and such colours denote that such and such crimes had been punished. Smaller threads attached to thicker cords were of different colours to signify the precise nature of the punishment that had been inflicted.

Inca Garcilaso de la Vega ~ 1600

Quipu
By Claus Ableiter nur hochgeladen aus enWiki - enWiki, hochgeladen von User Lyndsaruell; siehe http://en.wikipedia.org/wiki/Image:Inca_Quipu.jpg, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=2986739

yupana and is presumed to be the counting board of the Incas.
https://mathshistory.st-andrews.ac.uk/HistTopics/Inca_mathematics/

Incas used a positional number system and record their numbers in chords like the one on the photo. What number is shown on the left chord? (Units on the bottom)

Portion of a quipu
https://mathshistory.st-andrews.ac.uk/HistTopics/Inca_mathematics/

5
8
6

Quipu
By Pi3.124 - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=69539294

Quipus

Quechua was the main language of the Inca Empire.

Quipu (or khipu) is the Quechuan word for "knot".

Portion of a quipu
https://mathshistory.st-andrews.ac.uk/HistTopics/Inca_mathematics/

5
8
6

Quipu
By Pi3.124 - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=69539294

What it is: A set of colored, knotted cords used to record data.
Structure: one **main cord**; attached **pendant** cords; optional **subsidiary** cords; sometimes a **top cord** tied opposite the pendants.
Encoding cues: cord **color**, cord **order/placement**, **knot type**, **knot position**, and **spaces** between knots.
Where numbers are: knots on pendants/subsidiaries (not on the main cord).
Number system: base-10 place value; knot clusters separated by spaces; highest place nearest the main cord.

The Incas also had an excellent knowledge of **arithmetic**, and the way they counted was quite remarkable. Throughout the Empire, all **levies and taxes, both exemptions and claims, were recorded by means of knots made in colored cords. They could add, subtract, and multiply with these knots, and in order to determine what was owed to each village, made their allotments on the basis of little pebbles and grains of corn, in such a way that there could be no mistakes in their calculations.** Special bookkeepers were appointed for each section, whether it had to do with war or peace, such as taking a census of the vassals, or of levies, herds, laws, and ceremonies, and they were able to give an accounting at any moment, their collections of cords and knots being as clear and exact for them as any books. The head accountants, who were in charge of several things at a time, worked with these same means, quite as easily as the others.

Gómez Suárez de Figueroa, (Inca Garcilaso de la Vega) 1600

Mayans

Maya
ProtoplasmaKid - Own work

Machu Pichu
Martin St-Amant (S23678) - Own work

Daniel Schwen - Own work

MONTH: September
DAY: 28
YEAR: 2021
CONVERT -1 Day +1 Day

Long Count Date
13.0.8.16.3

13 baktun
13 X 144,000 days = 1,872,000 days

0 katun
0 X 7,200 days = 0 days

8 tun
8 X 360 days = 2,880 days

16 uinal
16 X 20 days = 320 days

3 k'in
3 X 1 day = 3 days

Tzolk'in Date: 9 Ak'b'al
Haab Date: 1 Yax
Lord of the Night: G8

https://maya.nmai.si.edu/calendar/maya-calendar-converter

Look up "Maya culture" or "Maya script" in Google Images. What do the images suggest about the complexity or priorities of Maya civilization? Consider what you see in their art, writing, and buildings.

Maya: Where and When, Cities and Society

- **Mesoamerica** (Yucatán, Guatemala, Belize, W. Honduras/Chiapas).
- **When**: 2000 BCE → 1600. CE Over **3,000 years**!
- **Independent city-states**: complex social hierarchy.
- **Religion**: Polytheistic, tied to nature **cycles**; priestly rituals and offerings.
- **Architecture**: stepped pyramids, palaces, ballcourts, plazas, inscriptions.

Map from Wikipedia

Map from Wikipedia

Maya's Focus: Astronomy and the Passage of Time

- **Hieroglyphic writing** used for history, ritual, astronomy.
- **Base-20 mathematics** with an independent **zero**.
- **Calendars**: 260-day ritual + 365-day solar; Long Count chronology.
- Observatories and precise **calendrical/astronomical** tracking.

After counting **8** days forward from the dates on the left marking, Venus reemerges as the morning star

Page 7 of the Maya Codex of Mexico

Photograph of a Maya Vase. Unknown author - Art from late Classic c. 600 - 900 AD, per book "The Blood of Kings, Dynasty and Ritual in Maya Art" by Linda Schele, Mary Ellen Miller, Justin Kerr, Kimball Art Museum, Fort Worth, 1986, plate 115 Mechanical reproduction of art more than 1,000 years old. https://commons.wikimedia.org/wiki/File:Maya_vase.jpg

Introduction and Mayan script

One word that captures what surprised you—or what you found remarkable—about the Maya.

Ancient Maya 101 | National Geographic https://youtu.be/Q6eBJjdca14

The Mayas

- had a **written language**, which help us learn about their mathematics.
- records were preserved in **bark paper** or carved in **stone**.
- priestly class who studied mathematics and astronomy.
- Remarkable development between 250 to 900 of our era.
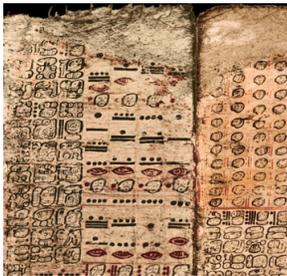
Mayan page in tree bark
From "Cracking the Mayan code"
https://www.pbs.org/wgbh/nova/mayacode/

Mayan writing carved on stone
From "Cracking the Mayan code"
https://www.pbs.org/wgbh/nova/mayacode/

The Monuments of Yaxchilan: Lintel 14 - photograph by Jorge Pérez de Lara

Breaking the Maya Code #2: The Maya Script

https://youtu.be/1t5FALvtMQg

Image of the page from Diego de Landa's 16thC. manuscript, Relación de las Cosas de Yucatán, in which he describes the famous "de Landa alphabet". This "alphabet" shows the letters of the Spanish language and the Maya hieroglyphics symbols which were supposed to correspond with them (according to de Landa's interpretation).

Constantine Rafinesque

Breaking the Maya Code #3: The Maya Books. https://youtu.be/GgCf3Q7qooQ. (3.09 minutes)

What type of Mayan hieroglyphs were deciphered first?

Mayan Calendars

Two (or three, depending on how we count) calendars

Calendar Round (cyclic)
- Tzolk'in: 260-day sacred cycle (13×20)
- Haab: 365-day civil/solar calendar

Long Count: day count since creation; fixes dates.

Describe the Maya calendars. Optional: name a math structure that relates to them.
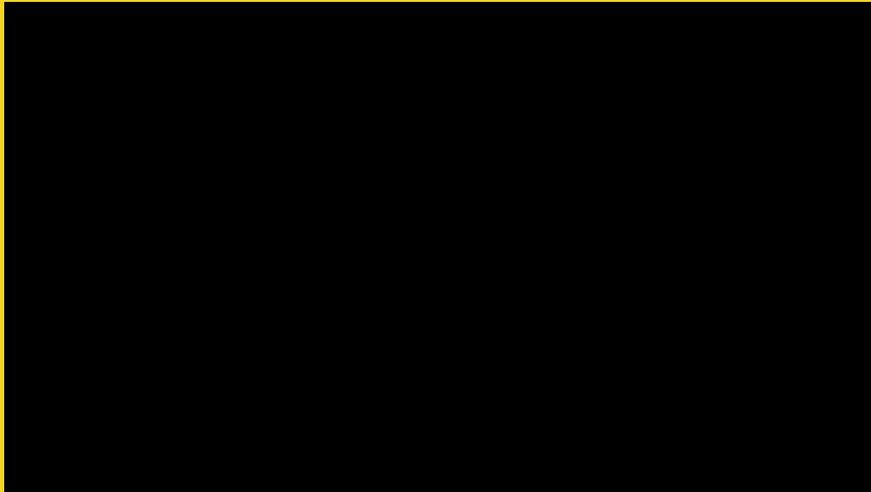
Breaking the Maya Code #4: The Maya Calendar https://youtu.be/qhWItvjk9Yg (2.18minutes)
Breaking the Maya Code by Night Fire films (extract) https://vimeo.com/ondemand/115559/223534703
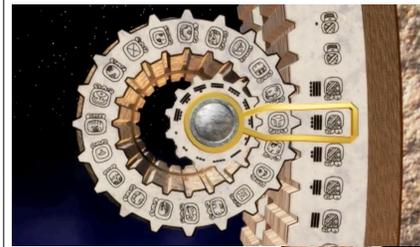
Round calendar
- The two smaller wheels represent the Tzolk'in or sacred calendar.
- The largest wheel represents the farmers or Haab calendar

Modular arithmetic!

The wheels are a modern way of representing the calendars, not a Mayan representation.

Part 2 of. Breaking the Maya Code #4: The Maya Calendar https://youtu.be/qhWItvjk9Yg

1. What do the large numbers in the Dresden Codex represent?
2. (Optional) Explain the Maya Long Count—what it measures and how it works.

Long count calendar
Long count calendar
Long count calendar
Long count calendar
Long count calendar
Tzolk'in Calendar
Haab calendar
Lord of the night

Based on astronomical observations, the Maya developed an elaborate system of calendars.

Mayan Calendars

- Round calendar
  - Tzolk'in calendar
  - Haab calendar
- Long count

https://maya.nmai.si.edu/calendar/maya-calendar-converter

Today in Mayan Calendars

MONTH: **September** DAY: **29** YEAR: **2025**

CONVERT -1 Day +1 Day

Long Count Date
13.0.12.17.5

13 baktun
13 X 144,000 days = 1,872,000 days

0 katun
0 X 7,200 days = 0 days

12 tun
12 X 360 days = 4,320 days

17 uinal
17 X 20 days = 340 days

5 k'in
5 X 1 day = 5 days

Tzolk'in Date: 2 Chikchan
Haab Date: 3 Yax
**Lord of the Night:** G

Round calendar (Haab +Tzolk'in)

Haab Calendar

- A solar calendar used to guide farming and civic life

- 18 "months" of 20 days each + 5 Uayeb days (unlucky/nameless)

- Each "month" had its own name (e.g., Pop, Wo, Sip)

- Days were numbered from 0 to 19 (e.g., 0 Pop, 1 Pop, … 19 Pop, 0 Wo…)

The months of the Haab Calendar

1. Pop
2. Wo
3. Sip
4. Sotz'
5. Sek
6. Xul
7. Yaxk'in
8. Mol
9. Ch'en
10. Yax
11. Sak
12. Keh
13. Mak
14. K'ank'in
15. Muwan
16. Pax
17. K'ayab
18. Kumk'u
19. Wayeb (the short 5-day unlucky month)

Why does the Haab have 365 days —just like ours—even though it developed independently?
Hint: What must farmers and astronomers track every year?

Uayeb translates to *"That which has no name"* — a time of caution and reduced activity.
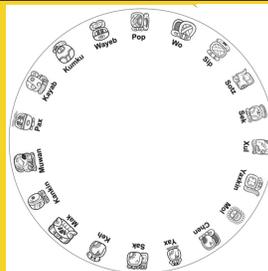
Tzolk'in or Sacred Calendar

- 20 day signs
- 13 sequential numbers

Each date is specified by a pair **(a, Db),** where

- $1 \leq a \leq 13$

- $1 \leq D_b \leq 20$ (We'll use numbers instead of day signs. Imix = 1, Ik=2, etc)

- The Tzolk'in cycle begins with **(1, 1),** continues with **(2,2), (3,3), (13,13), (1,14), (2,15)** and so on.

Day Signs

1. Imix
2. Ik'
3. Ak'bal
4. K'an
5. Chikchan
6. Kimi
7. Manik'
8. Lamat
9. Muluk
10. Ok
11. Chuwen
12. Eb'
13. Ben
14. Ix
15. Men
16. Kib'
17. Kab'an
18. Etz'nab'
19. Kawak
20. Ajaw

1. How many unique dates are in the Tzolk'in calendar?
(Hint: Each day is specified by a pair, see above)
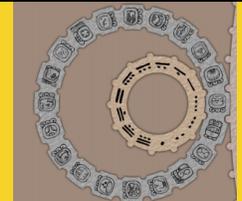**2. (Optional) If today is 10 K'an (that is, (10,4)), what will the day be 5 days later?**

Image from Breaking the Maya Code

A Maya representation of the Tzolk'in from the Madrid Codex. Time is represented by 260 dots marking a path or a journey of twenty days and thirteen numbers.

Source: Smithonian, National Museum of the American Indian
https://maya.nmai.si.edu/sites/default/files/resources/The%20Maya%20Calendar%20System.pdf

---

If the Haab and Tzolk'in start together (e.g., 0 Pop, 1 Imix), how many days until they realign on the same date again?

- A Haab year contains 365 days.
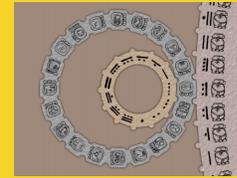- A Tzolk'in cycle contains 260 days.

Image from Breaking the Maya Code

A contemporary representation of the Calendar Round, interlocking the Tzolk'in (left) with the Haab (right).
Source: Smithonian, National Museum of the American Indian
https://maya.nmai.si.edu/sites/default/files/resources/The%20Maya%20Calendar%20System.pdf

The Haab and Tzolk'in together form the Calendar Round.

The Calendar Round:
Haab + Tzolk'in

Image from Breaking the Maya Code

- **Haab + Tzolk'in realign every 18,980 days**
  (= 52 Haab years = 73 Tzolk'in cycles = LCM(260,365)).

- **Calendar Round date**: a pair (Tzolk'in day, Haab day) that uniquely identifies any event within 52 years.

- **Beyond 52 years**: the **Long Count** records longer spans.

- The **52-year return** marked major cultural celebrations and ceremonies.

- Reaching one's birth date again was considered a full life cycle. A person who reaches 52 years of age, attains the special wisdom of an elder.

Long count calendar

Slide 1 (top-left)

What numbers are shown above glyphs a–d, (optional e)?

a b
c d
e

Images from the documentary Breaking the Mayan Code https://nightfirefilms.org/breaking-the-maya-code/

Slide 2 (top-right)

9 x 400 years   14 x 20 years

19 years   8 months

0 days

The long count calendar counts how much time elapsed since the creation date.

Images from the documentary Breaking the Mayan Code https://nightfirefilms.org/breaking-the-maya-code/

Slide 3 (bottom-left)

Long count calendar

A long count calendar date it is written as 2.3.4.5.6.

Reading the Maya Calendar
https://maya.nmai.si.edu/calendar/maya-calendar-converter

- **Long Count** = elapsed days since the creation date.
- **Units:**
  - kin = 1 day
  - uinal = 20 kin (≈ 20 days ~ "month")
  - tun = 18 uinal = 360 d (≈ "year" minus 5 days)
  - k'atun = 20 tun = 7,200 d (≈ 19.7 years)
  - bak'tun = 20 k'atun = 144,000 d (≈ 394.3 years)
- Example shown: 9.14.19.8.0 = 9 bak'tun, 14 k'atun, 19 tun, 8 uinal, 0 kin
  (= 1,404,760 days ≈ 3,847.3 years)

9 x 400 years   14 x 20 years
19 years   8 months
0 days

Slide 4 (bottom-right)

**How many years are in a bak'tun?
Assume 365-day years; calculators allowed.**

- **Long Count** = elapsed days since the creation date.
- **Units:**
  - kin = 1 day
  - uinal = 20 kin
  - tun = 18 uinal
  - k'atun = 20 tun
  - bak'tun = 20 k'atun
- **Example**: 9.14.19.8.0 = 9 bak'tun, 14 k'atun, 19 tun, 8 uinal, 0 kin = 1,403,800 days ≈ 3,846.0 years (using 365-day years) after the creation date.

- A Long Count date and a Calendar Round date were used by the Maya to place mythical and historical events in chronological order.
- An era (full cycle) is composed of 13 bak'tuns.
- Note that this is similar to the Gregorian calendar system that counts days, months, years, centuries and millennia.

Slide 1 (top-left)

What is the day after the date below in the Long Count calendar? Write it as *bak'tun.k'atun.tun.uinal.kin*

bak'tun

k'atun

tun

uinal

kin

Teams of 3

- kin = 1 day
- uinal = 20 kin
- tun = 18 uinal
- k'atun = 20 tuns
- bak'tun = 20 k'atun

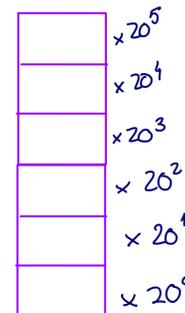## Slide 2 (top-right)

The 2012 "end of the world"?

- **Creation Date in Maya inscriptions:**
  - **Long Count:** 13.0.0.0.0; **Tzolk'in**: 4 Ajaw 8 Kumk'u
  - ~ 11 Aug 3114 BCE.
- **2012 "end of the world" hype: Long Count reached 13.0.0.0.0** (21/23 Dec 2012, correlation-dependent)→ completion of 13 bak'tuns since the creation date.
- Maya texts celebrate such **completions with renewal rites**, and they even compute far-future dates (e.g., 1.0.0.0.0.0, five zeros), showing the count (and the world!) continues.
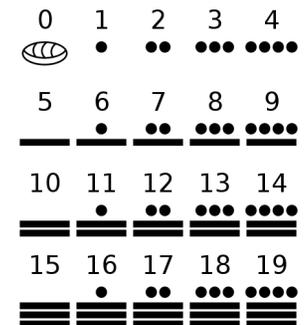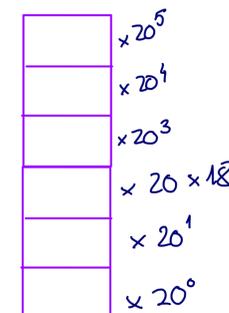
MONTH: September
DAY: 8
YEAR: -3114

Long Count Date
0.0.0.0.0

0 baktun
0 X 144,000 days = 0 days
0 katun
0 X 7,200 days = 0 days
0 tun
0 X 360 days = 0 days
0 uinal
0 X 20 days = 0 days
0 kin
0 X 1 day = 0 days

Tzolk'in Date: 4 Ajaw
Haab Date: 17 Sip
Lord of the Night: G9

https://maya.nmai.si.edu/calendar/maya-calendar-converter

Slide 3 (bottom-left)

All calendars again

Slide 4 (bottom-right)

Mayan number system

Write down 365 in the Mayan number system. (numbers are written vertically, lower units at the bottom)

0	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19

Mayan numerals - Wikipedia

Base 20

$\times 20^5$
$\times 20^4$
$\times 20^3$
$\times 20^2$
$\times 20^1$
$\times 20^0$

Modified base 20

$\times 20^5$
$\times 20^4$
$\times 20^3$
$\times 20 \times 18$
$\times 20^1$
$\times 20^0$

Haab Calendar

When days were counted, the third-place unit, instead of being 20.20 was 20.18.
This was probably because there are 360 days in the "regular" part of the 365 days calendar know as the *Haab* and the other 5 days were considered "unlucky" (and so best not included in the count)

Translate a Mayan number on to Hindu-Arabic in two ways (units at the bottom):

1. Assuming Mayan number system was "honest" base 20

2. Assuming the "calendar way" (that is days were counted, the third-place unit, instead of being 20x20 was 20x18).

Base 20

$\times 20^5$
$\times 20^4$
$\times 20^3$
$\times 20^2$
$\times 20^1$
$\times 20^0$

Modified base 20

$\times 20^5$
$\times 20^4$
$\times 20^3$
$\times 20 \times 18$
$\times 20^1$
$\times 20^0$

All calendars

Calendar Round

- **Haab**: 365-day solar.

- **Tzolk'in**: 260-day (13 × 20).

- **Together**: Calendar Round → dates repeat every 52 Haab years.

Long count

- Counts days since a fixed epoch (creation date).

- Written a.b.c.d.e = bak'tun.k'atun.tun.uinal.kin.
  Value in days: 144,000a + 7,200b + 360c + 20d + e.

- Period endings (e.g., 13.0.0.0.0) mark completions; the count continues (texts even project to 1.0.0.0.0.0).

And more…

Codex-Style Vessel CE 700–750.
Two teaching scenes with Itzam—an elderly creator/atlas-like deity—instructing four students

https://kimbellart.org/collection/ap-200404

Itzam ID cues: Aged face; netted headdress with a brush tucked in.

Scene 1: Itzam uses a pointer and a folded codex; a speech thread with **bar-and-dot numerals suggests arithmetical/calendrical calculation**.

Scene 2: Itzam taps the ground while addressing two students; spoken glyphs on a speech thread.

Takeaway: Elite scribal education in mathematics/ calendar lore framed as instruction from a creation deity.

COL, Museum of the American Indian, Heye Foundation
(Drawing by Persis Clarkson [in Clarkson 1978: Fig. 3])

Copyright 1976 by the FOUNDATION FOR
LATIN AMERICAN ANTHROPOLOGICAL RESEARCH

A female
Mayan scribe?

Still debated…

Answer at least two:

1. Name one concrete thing you remember about the Maya.
   (Hint: writing, zero/base-20, pyramids, maize, deities, astronomy)
2. What's the difference between the Tzolk'in and the Haab, and what did each mainly guide?
   (Hint: Tzolk'in = 260 ritual, Haab = 365)
3. What are the Tzolk'in and the Haab, and how do they combine to make the Calendar Round?
   (Hint: two cycles interlock, repeat every 52 Haab years)
4. What does 13.0.0.0.0 mean in the Long Count, and why did some people talk about 2012?
   (Hint: days since creation, 13 bak'tuns completed, correlation sets date)
5. Reflect: What most changed your impression of the Maya—writing, timekeeping, cities, or continuity—and why?