Slide 1
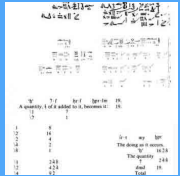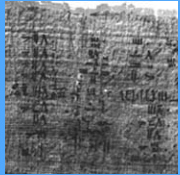
Mathematics in Ancient Egypt

- More about the presentation
- Egypt Overview
- Egyptian Math in a Nutshel
- How do we know what we know
- Scribes and papyri
- The Rosetta Stone
- The Rhind papyrus

1. Multiplication and division
2. Parts (fractions)
3. Method of false position
4. Areas and volumes
   - Area of the circle, π!
   - Volume of the truncated pyramid.
5. As I was going to st Ives…

Slide 2
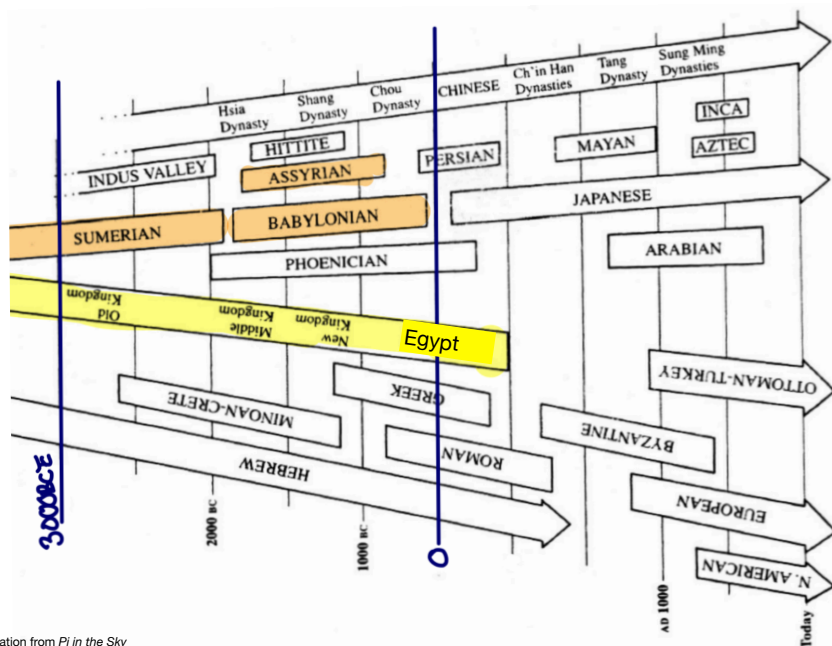
Introduction

Slide 3

Illustration from *Pi in the Sky*
By Wendy Mass Little, Brown Books for Young Readers 2013

Slide 4

Ancient Egypt and Mesopotamia

Common aspects
- Flourished along major rivers.
- Nice climate, fertile lands.
- Developed writing systems.
- Strong centralized government.
- Strong religious life.
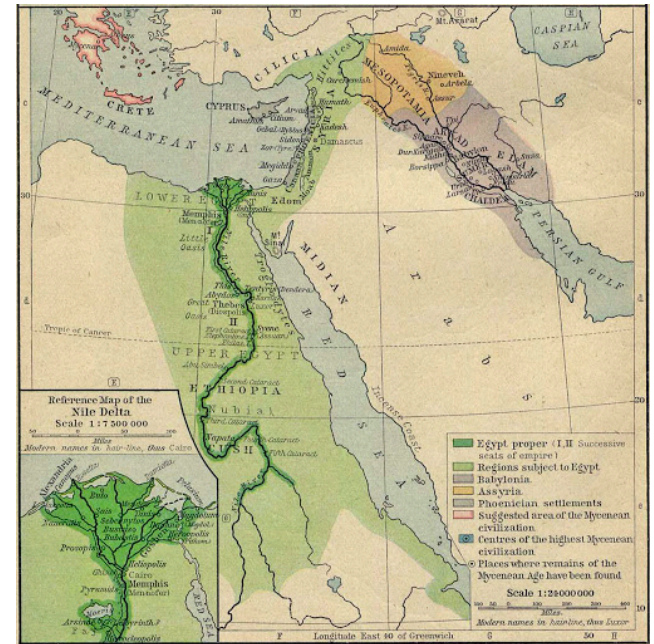
Mathematical Development
- **Motivation**: Administrative needs (taxes, public works, calendar)
- **Focus**: Arithmetic and mensuration (areas, volumes)
- **Evolution**: From practical applications to some abstraction. In later years, perhaps math for its own sake.

Ancient Egypt 101 | National Geographic https://youtu.be/hO1tzmi1V5g

Write down one idea from the video related to mathematics.

http://www.emersonkent.com/map_archive/egypt_syria_mesopotamia_1450_bc.htm

Reference Map of the Nile Delta
Scale 1:7 500 000

Seven wonder of the Ancient World - Kandi, CC BY-SA 3.0 <https://creativecommons.org/licenses/by-sa/3.0>, via Wikimedia Commons

Mesopotamia
- Sargon of Akkad, Akkadian Empire (2350-2295 BC)
- Old-Assyrian Empire of Shamsi-Adad I (1749-1717 BC)
- Babylonian Empire of Hammurabi (1728-1685 BC)
- Neo-Babylonian Empire (625-539 BC)

Egypt
- Border between Lower and Upper Egypt
- Border of the Old Kingdom
- Border of the Middle Kingdom
- Empire of the New Kingdom
- Part of the Empire of Alexander The Great

1.2.2 Egypt and Mesopotamia in ancient times [Map: H. Wesemüller-Kock]

Top-left

| EGYPTIAN AND BABYLONIAN (3000 B.C. to A.D. 260)	GREEK (600 B.C. to A.D. 450)	CHINESE (1030 B.C. to A.D. 1644)	HINDU (200 B.C. to A.D. 1250)
Essentially empirical, or inductive, mathematics Introduction of early numeral systems (decimal and sexagesimal) Simple arithmetic, practical geometry Mathematical tables, collections of mathematical problems Chief primary sources: Moscow (1850 B.C.), Rhind (1650 B.C.), and other Egyptian papyri; Babylonian cuneiform tablets (2100 B.C. to 1600 B.C. and 600 B.C. to A.D. 300)	Significant introduction, then development, of deductive geometry (Thales, 600 B.C.; Pythagoras, 540 B.C.) Start of number theory (Pythagorean School, 540 B.C.) Discovery of incommensurable magnitudes (Pythagorean School, before 340 B.C.) Systematization of deductive logic (Aristotle, 340 B.C.) Axiomatic development of geometry (Euclid, 300 B.C.) Germs of the integral calculus (Archimedes, 225 B.C.) Geometry of conic sections (Apollonius, 225 B.C.)	Largely isolated from the mainstream of mathematical development Decimal numeral system, rod numerals magic squares (from earliest time) Chou-peï, oldest of Chinese mathematical classics (300 B.C.?) Arithmetic in Nine Sections (100 B.C.?) Horner's method (Ch'in Kiu-Shoo, 1247) Pascal's arithmetic triangle, binomial theorem (Chu Shi-kié, 1303) Jesuit missionaries infiltrated China in early 1600s	Introduction of Hindu-Arabic numeral system (before A.D. 250) Negative numbers and invention of zero symbol (early centuries A.D.) Development of early computing algorithms (A.D. 900–1000) Syncopated algebra, indeterminate equations (Brahmagupta, A.D. 628; Bhāskara, A.D. 1150) **ARABIAN (A.D. 650 to 1200)** Preservers of Hindu arithmetic and Greek geometry (encouraged by caliph patrons of learning, such as Harun al-Rashid, A.D. 790)
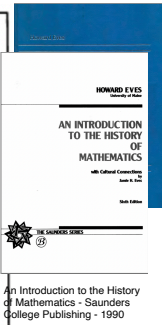
An Introduction to the History of Mathematics - Saunders College Publishing - 1990

HOWARD EVES — University of Maine — AN INTRODUCTION TO THE HISTORY OF MATHEMATICS with Cultural Connections — Sixth Edition — THE SAUNDERS SERIES

Note the point of view on this table

Top-right

EGYPTIAN AND BABYLONIAN (3000 B.C. to A.D. 260)	GREEK (600 B.C. to A.D. 450)	CHINESE (1030 B.C. to A.D. 1644)
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Bottom-left

Naqada Tablets - 4th millennium BCE
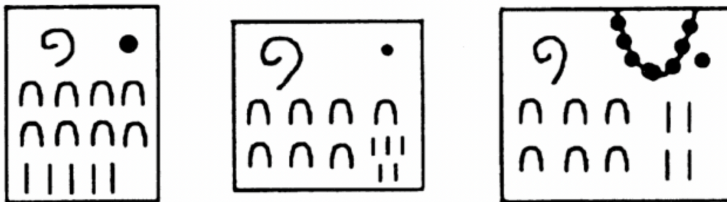
The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook By Annette Imhausen, Eleanor Robson, Victor J. Katz, Victor J. Katz, Annette Imhausen · 2007

Other objects from Naqada

https://brittlebooks.library.illinois.edu/brittlebooks_open/Books2009-08/petrwi0001naqbal/petrwi0001naqbal.pdf

Bottom-right

Ancient Egyptian Mathematics in a very small nutshell

Sources & Context:
- Very few surviving sources (papyrus is fragile!)
- Practical problems:
  - Example: area and volume
  - Example: fair division of loaves of bread.

Key Mathematical Features
- Linear equations solved by "false position" method
- Doubling/halving as basic arithmetic operations
- Egyptian fractions: written as sums of unit fractions (1/n)
- Surprisingly accurate π approximation.
- Some Problems with theoretical interest
  - Adding $7 + 7^2 + 7^3 \ldots + 7^5$
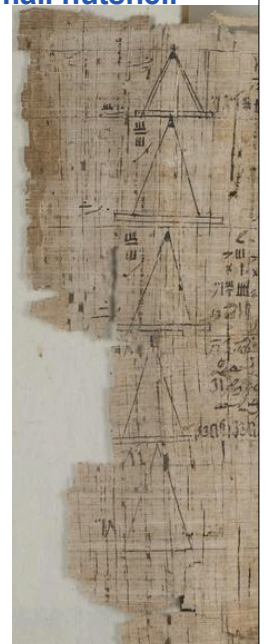
Ancient Egyptian Mathematics in a very small nutshell

Sources & Context:
- Problems and solutions to concrete algebraic and geometric problems
  - finding the area or volume of certain shapes,
  - fair division of loaves of bread.
  - feeding animals and storage of grain
  - solutions of linear equations with one unknown - false position.

Key Mathematical Features
- Some Problems with theoretical interest
  - Adding $7 + 7^2 + 7^3 \ldots + 7^5$
- Examples (as opposed to rules); how (as opposed to why)
- Doubling and halving were the basic arithmetic operations.
- Two number systems: hieroglyphic and a ciphered (used for different purposes).
- Intriguingly good approximation to π
- Fractions were written as a sum of *parts* (fractions of form 1/n).
- Development of calendar.

Two number (and writing) systems in Ancient Egypt

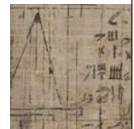### Hieroglyphic numerals

Egyptian numeral hieroglyphs

Example of a hieroglyphic number from a tomb inscription.

Hieratic numerals

Hieroglyphic and Hieratic
- Two parallel systems used for around 2000 years
- **Hieroglyphic**: Formal script carved or painted in stone monuments
- **Hieratic**: Cursive script written on papyrus for daily use
- Both evolved over time
- Mathematical texts primarily used hieratic (practical documents)

Credit tables: https://mathshistory.st-andrews.ac.uk/HistTopics/Egyptian_numerals/

Rhind Papyrus Section
British Museum

How was geometry invented, according to Herodotus?

"It was this king, moreover, who divided the country among all the Egyptians by giving each an equal square parcel of land, and made this his source of revenue, appointing the payment of a yearly tax. And any man who was robbed by the river of a part of his land would come to Sesostris and declare what had befallen him; then the king would send men to look into it and measure the space by which the land was diminished, so that thereafter it should pay in proportion to the tax originally imposed. **Perhaps this was the way in which the art of measuring land (geometry) was invented, and passed afterwards into Greece**"

Herodotus (~400BC) was an ancient Greek historian who was born in Halicarnassus in the Persian Empire (modern-day Bodrum, Turkey).


Bust of Herodotus. 2nd century AD. Roman copy after a Greek original. On display along the portico of the Stoa of Attalus, which houses the Ancient Agora Museum in Athens.

… Cheops became king over them and brought them to every kind of evil: (….) he then bade all the Egyptians work for him. So some were appointed to draw stones from the stone-quarries in the Arabian mountains to the Nile, and others he ordered to receive the stones after they had been carried over the river in boats, and to draw them to those which are called the Libyan mountains; and **they worked by a hundred thousand men at a time, for each three months continually**. Of this oppression there passed ten years (..) For this they said, the ten years were spent, and for the underground he caused to be made as **sepulchral chambers for himself** in an island, having conducted thither a channel from the Nile. **For the making of the pyramid itself there passed a period of twenty years.**
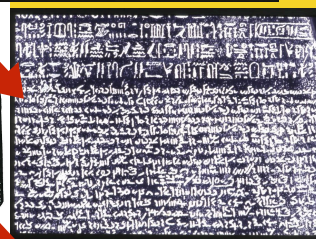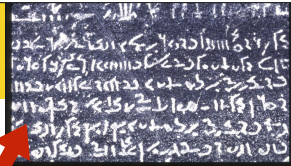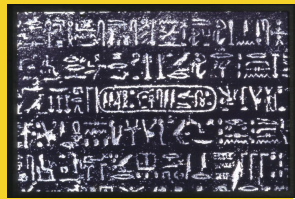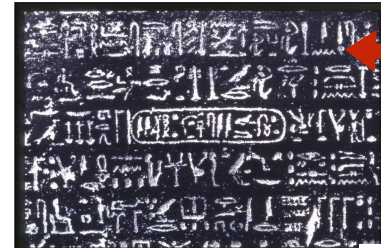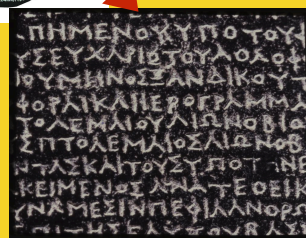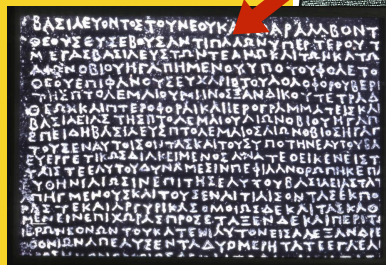
The Rosetta Stone
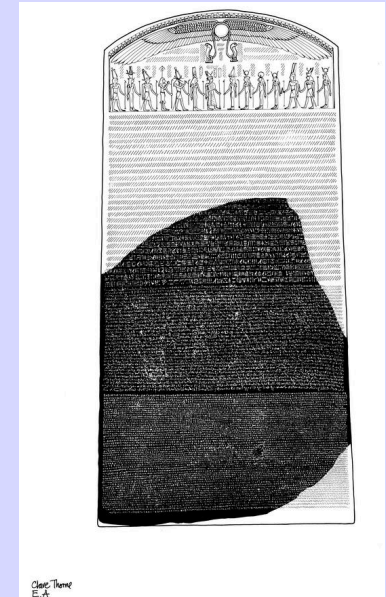
The Rosetta stone
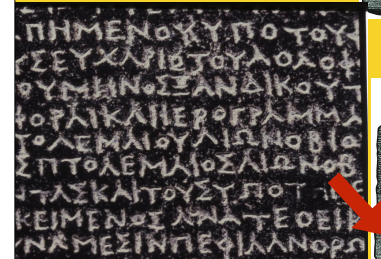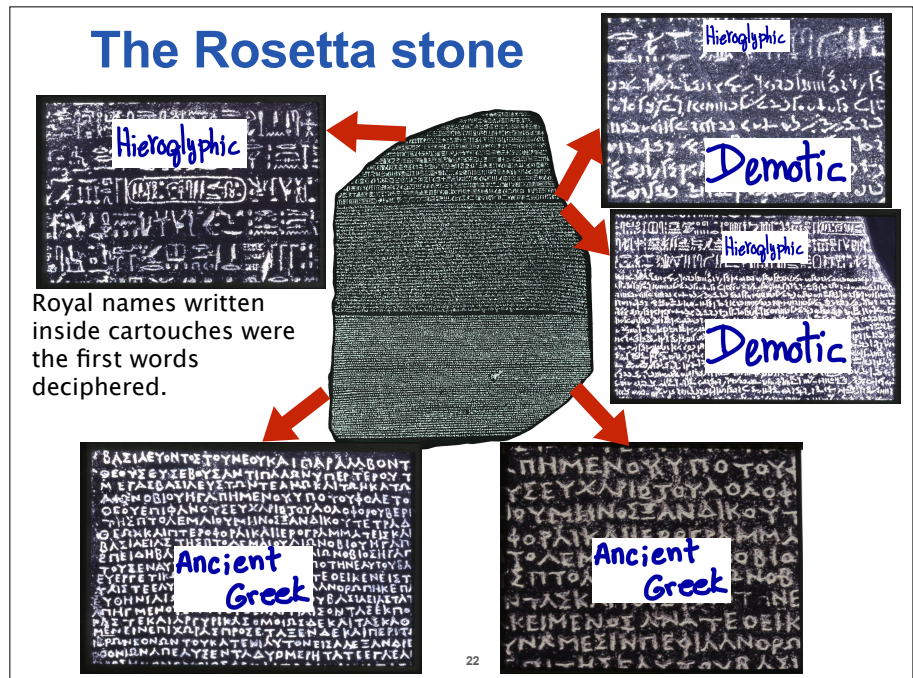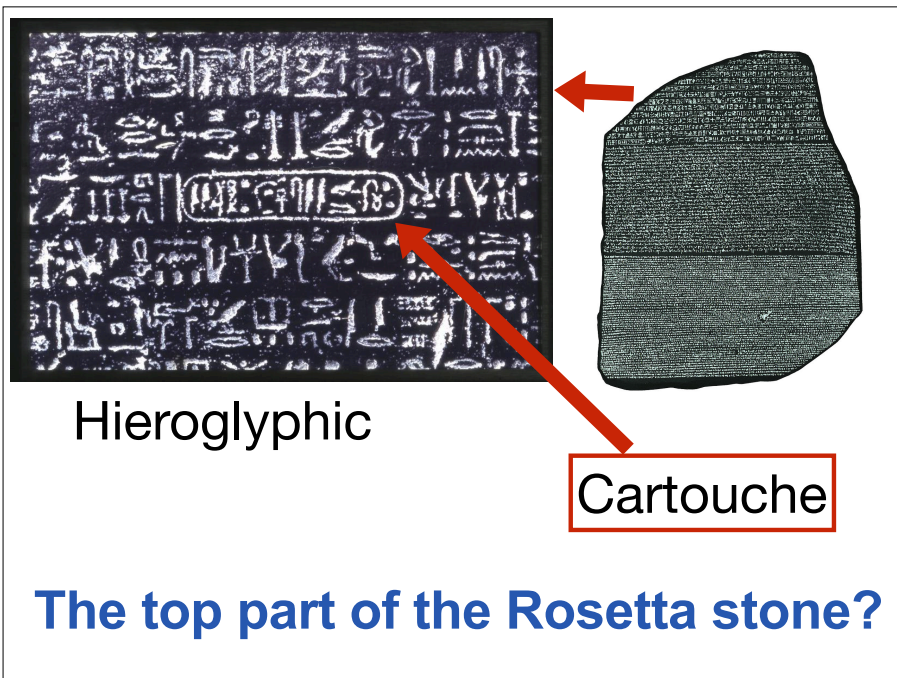
What languages are used in the Rosetta stone?

1(top)

2(middle)

3(bottom)

What languages are used in the Rosetta stone?

19

Hieroglyphic

Cartouche

The top part of the Rosetta stone?

The Rosetta stone

Hieroglyphic

Hieroglyphic

Demotic

Hieroglyphic

Demotic

Royal names written inside cartouches were the first words deciphered.

Ancient Greek

Ancient Greek

Ancient Egypt Numbers

Some numbers in hieroglyphics

Hieratic

In their long history, Egyptians created more than one system of writing. Some of these systems were used simultaneously.

Hieratic Forms

Demotic Forms

Demotic

Rough History of the Rosetta Stone

- Made in 196 BC, on the first anniversary of the coronation of king Ptolemy V, by then a teenager.
- It's a decree issued by Egyptian priests, ostensibly to mark the coronation and to declare Ptolemy's new status as a living god - divinity went with the job of being a pharaoh
- It was the result of hard political negotiations with his extremely powerful Egyptian priests.
- Survived unread through two thousand years of further foreign occupations - Romans, Byzantines, Persians, Muslim Arabs and Ottoman Turks, all had stretches of rule in Egypt.
- A French invasion (which was not only military but intellectual) found it in the town of Rosetta (now el Rashid) in 1799.
- The French took it as cultural trophy of war. But Napoleon was defeated, and in 1801 the terms of the Treaty of Alexandria, signed by the French, British and Egyptian generals, included the handing over of antiquities - and the Rosetta Stone was one of them.
- on the broken side, you can see that in fact there are four. Because there, stenciled on in English, you can read: "CAPTURED BY THE BRITISH ARMY IN 1801; PRESENTED BY KING GEORGE III".

https://www.bbc.co.uk/programmes/articles/3dtD0Nr8tJPgH7pXwt302Ys/episode-transcript-episode-33-rosetta-stone

Decipherment of the Rosetta Stone

- It was hoped that the Egyptian text could be deciphered through its Greek translation.
- Phonetic glyphs in a **cartouche** containing the name of an Egyptian king of foreign origin, Ptolemy V.
- In the early 1820s Champollion compared Ptolemy's cartouche with others and realised the **hieroglyphic script was a mixture of phonetic and ideographic elements.**
- Young, meanwhile, largely **deciphered demotic** using the Rosetta Stone in combination with other Greek and demotic parallel texts.
- New progress was made in the second quarter of the 1800s.

Men Splitting Papyrus (public domain)
A.D. 1914–1916, original ca. 1479–1458 B.C

This is a photo of Papyrus growing wild along the banks of the Nile River in Uganda. It was taken by Michael Shade in the fall of 2006.

Papyrus
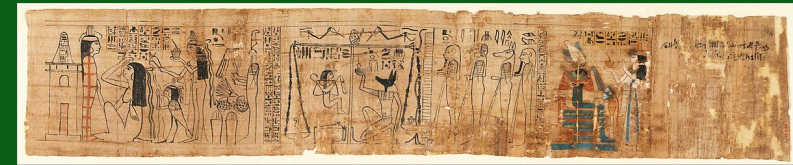
Book of the Dead Papyrus of Tiyeca. 975–945 B.C. Discovered tucked inside her hollow wooden "Osiris figure" (now Cairo JE 49164) this papyrus was designed to assist Tiye in her successful transition from death to eternal life. - (Public domain)

Papyri, reading, writing and math

- Papyri are fragile
- Papyri were expensive (labor intensive production)
- Only about 10 mathematical papyri have survived

- Mostly fragments, except for Rhind and Moscow papyri
- Scholars think that only **about 1 or 2% of the population was able to read and write.**

Aristotle writes (Metaphysics):
"Thus the mathematical sciences originated in the neighborhood of Egypt, because there the priestly class was allowed leisure."

Scribes

Funerary model of a granary with a scribe recording amounts of grain stored or issued, Egypt, Middle Kingdom (about 2055–1795 BC).

Funerary model of a granary - British Museum

Scribes – specially trained in reading, writing and calculating – were essential in the organization and running of Ancient Egypt.

29

The seated scribe, about 2500 BCE, Louvre Museum

image https://commons.wikimedia.org/
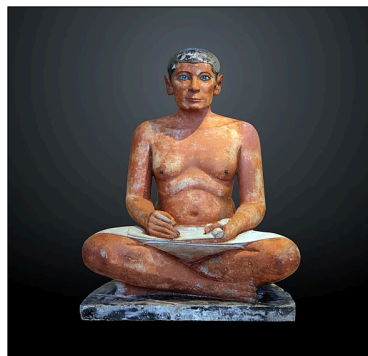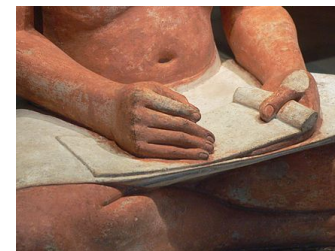
Very often, in tombs of high officials, the tomb owner is shown as a **inspector in sciences of accounting cattle or product**, and sometimes several scribes are depicted working together as a group. Several models depict the filling of granaries, and a **scribe is always present to record the respective quantities.**

Scribes were, among other functions, accountants.

Katz, Victor J., ed. The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook. Princeton University Press, 2007.30

A fragment from Papyrus Anastasi - A fictional letter, which forms part of a debate between two scribes

You are told: "**Empty the magazine that has been loaded with sand** under the monument for your lord—may he live, prosper, and be healthy—which has been brought from the Red Mountain. **It makes 30 cubits stretched upon the ground with a width of 20 cubits,** passing chambers filled with sand from the riverbank. The **walls of its chambers have a breadth of 4 to 4 to 4 cubits.** It has a **height of 50 cubits in total.** [...] You are commanded to find out what is before it. **How many men will it take to remove it in 6 hours if their minds are apt?** Their desire to remove it will be small if (a break at) noon does not come. You shall give the troops a break to receive their cakes, in order to establish the monument in its place. One wishes to see it beautiful.

The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook By Annette Imhausen, Eleanor Robson, Victor J. Katz, Victor J. Katz, Annette Imhausen · 2007, page 11
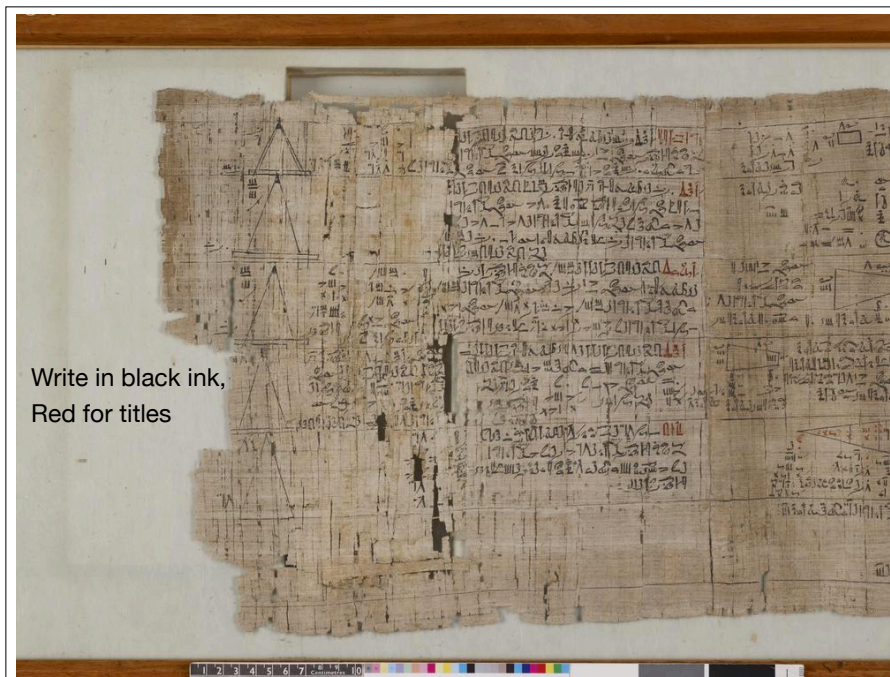
The and Moscow Rhind Papyri

The Rhind papyrus

Fragment

13 inches high and 18 feet long!

https://www.britishmuseum.org/collection/object/Y_EA10057

Write in black ink,
Red for titles

Rhind Mathematical Papyrus

35

Image from the British Museum
https://www.britishmuseum.org/collection/image/366145001

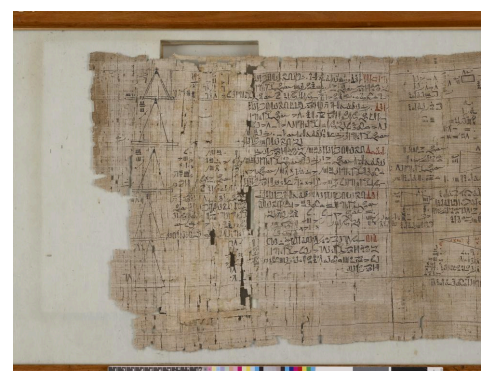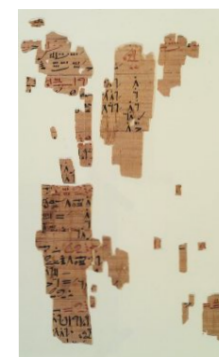*Fragments of Rhind Mathematical Papyrus*, ca. 1493-1481 B.C.E. Papyrus, ink, Largest Fragment: 6 5/16 x 3 3/8 in. (16 x 8.5 cm). Brooklyn Museum, Charles Edwin Wilbour Fund, 37.1784Ea-b (Photo: Brooklyn Museum, 37.1784E_negA_bw_IMLS.jpg)

Rhind papyrus recent history

- It was acquired by the Scottish lawyer A.H. Rhind in Thebes in about 1858.
- Evidence indicate that these fragments were found in a chamber of a ruined building
- The two sections in the British Museum were linked by a missing section about 18 cm long; the original may have been cut in half by modern robbers to increase its sale value.
- Fragments which partly fill this gap were identified in 1922, in the collection of the **New York Historical Society,** which had acquired them from Edwin Smith. Smith also acquired a surgical papyrus of about the same date as the Rhind Papyrus, suggesting that these two documents could have come from a cache of early New Kingdom manuscripts.

Adapted Fromm the British Museum curator's comments about the Ahmes or Rhind Papyrus

https://www.britishmuseum.org/collection/object/Y_EA10058

What did we discussed about Egypt?
Write down something important and/or interesting that you remember.

The Rhind papyrus beginning

Accurate reckoning for inquiring into things, insight into all that exists, knowledge of all obscure secrets. This book was copied in regnal year 33, month 4 of Akhet, under the majesty of the King of Upper and Lower Egypt, Awserre, given life, from an ancient copy made in the time of the King of Upper and Lower Egypt Nimaatre. The scribe Ahmose writes this copy.

thorough study of all things, insight into all that exists, knowledge of all obscure secrets.

- "Ahmes" or "Ahmose " is writing it about 1550 BC but that he had copied it from "ancient writings" (1800 BC or earlier).
- "Ahmes" or "Ahmose " is the earliest personal name known to us on the history of mathematics.
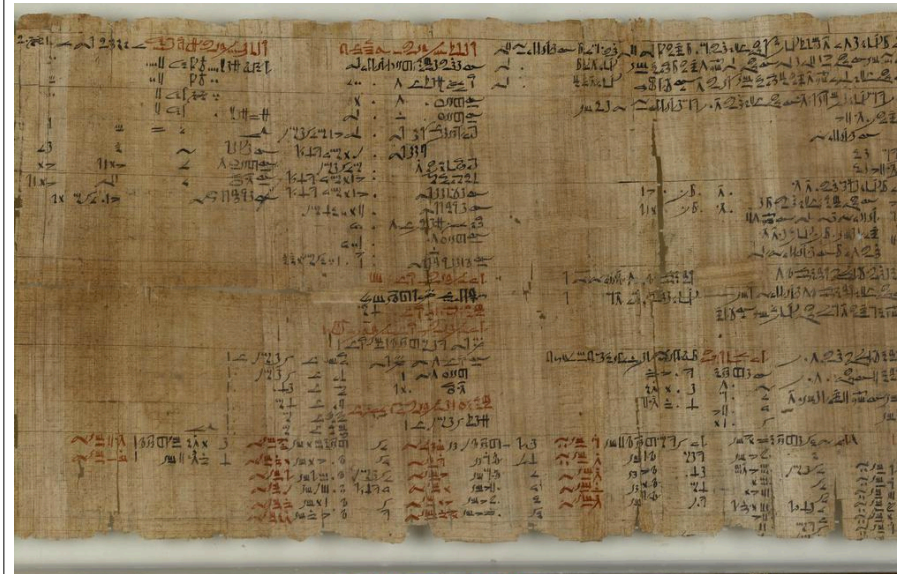
Three types of problem

- pure mathematical problems teaching basic techniques
- practical problems, which contain an additional layer of knowledge from their respective practical setting
- non-utilitarian problems, which are phrased with a pseudo-daily life setting without having a practical application (only very few examples extant)

  - No symbols (like + or -)
  - No variables (like x)
  - Algorithmic: a list of concrete instructions to solve them

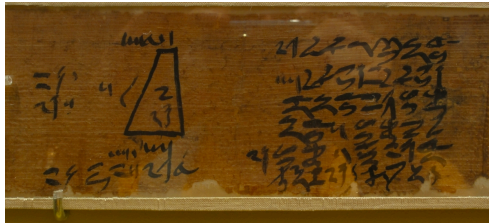The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook By Annette Imhausen, Eleanor Robson, Victor J. Katz, Victor J. Katz, Annette Imhausen · 2007

- Table 2/n as a sum of parts, n=3 to 103

- divisions of a certain number of loaves of bread among 10 men

- addition of fractions, summing up to 1.

- solution of linear equations (but not as we understand them)

- unequal distribution of goods and other problems

- find the volume of cylindrical and rectangular granaries.

- show how to compute an assortment of areas

- slopes (of pyramids.)

- multiplications of fractions.

- Value, fair exchange and feeding

The Rhind Papyrus (in present language)

The Moscow Papyrus

Moscow Mathematical PapyrusnnnFotography by Carles Dorce
https://themathematicaltourist.wordpress.com/2012/10/19/moscow-mathematical-papyrus/

14th problem of the Moscow Mathematical Papyrus

Dimensions of the Moscow Papyrus
Length: 5.5 metres (18 ft)
Width: 3.8 to 7.6 cm (1.5 to 3 in)

Multiplication in Ancient Egypt

Explain what it means to multiply a positive integer A by another positive integer B. (If you prefer, you may explain it with an example, say A=20 and B=45)

Ancient Egyptian multiplication

EXAMPLE: A=45, B=20

Find the product of A and B

- Set up two columns: write 1 in the left column and B in the right column.
- Double both numbers until the left column exceeds A.
- Mark the rows in the left column whose values sum to A, and mark the corresponding right-column values (hint: start from the bottom row).
- Add the marked right-column values; their sum is the product A·B.

A.B	
1	B
2	2B
4	4B
...	...
2^n	$2^n B$

n is the largest integer such that $2^n < A$

46

Ancient Egyptian multiplication

EXAMPLE: A=45, B=20

Find the product of A and B

- Set up two columns: write 1 in the left column and B in the right column.
- Double both numbers until the left column exceeds A.
- Mark the rows in the left column whose values sum to A, and mark the corresponding right-column values (hint: start from the bottom row).
- Add the marked right-column values; their sum is the product A·B.

A.B			
1	B	1	20
2	2B	2	40
4	4B	4	80
...	...	8	160
2^n	$2^n B$	16	320
		32	640

n is the largest integer such that $2^n < A$

STOP

20.45 = 640+160+80+20 = 900

Mark all the numbers in the 1-column that add up to B??

47

Exercise: Multiply 20 by 45 as we just did, but exchanging the roles of A and B.

Multiply 25 by 26

EXAMPLE:

1	45

ANOTHER EXAMPLE:

Recall that **a number and the representation of the number** in a number system are two different concepts. There are many ways to represent a given number, but each number is "unique".

In a similar way, **multiplication and how multiplication is performed are different concepts** Again, there are many algorithms, that is, many ways to multiply two numbers. But the meaning of multiplication is only one.

what ≠ how

However,

Rhind Mathematical Papyrus, problem 69

.	80
\ 10	800
2	160
\ 4	320
Total	1120

The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook By Annette Imhausen, Eleanor Robson, Victor J. Katz, Victor J. Katz, Annette Imhausen · 2007, page 11

Division in Ancient Egypt

Ancient Egyptian division

Example: Find the quotient of 130 divided by 10 using the Egyptian method. Think of it as solving $10 \times x = 130$.

- Set up two columns: write 1 in the left column and 10 in the right.
- Double both numbers until the right-column value exceeds 130.
- Mark the rows in the right column whose values sum to 130; mark the corresponding rows in the left column.
- Add the marked numbers in the left column. This sum is the quotient $130 \div 10$.

Ancient Egyptian division

Example: Find the quotient of 130 divided by 10 using the Egyptian method. Think of it as solving $10 \times x = 130$.

- Set up two columns: write 1 in the left column and 10 in the right.

- Double both numbers until the right-column value exceeds 130.

- Mark the rows in the right column whose values sum to 130; mark the corresponding rows in the left column.

- Add the marked numbers in the left column. This sum is the quotient $130 \div 10$.

1	10
2	20
4	40
8	80

80< 130 but
2.80=160 >130

130= 80 + 40 +10
130/10= 8+4+1

Ancient Egyptian division

Find the quotient of A divided by B using the Egyptian method. Think of it as solving $B \times x = A$.

- Set up two columns: write 1 in the left column and B in the right.

- Double both numbers until the right-column value exceeds A.

- Mark the rows in the right column whose values sum to A; mark the corresponding rows in the left column.

- Add the marked numbers in the left column. This sum is the quotient $A \div B$.

1	B
2	2B
4	4B
...	...
2^n	$2^n B$

n is the largest integer such that $2^n B < A$

Exercise: Find the quotient of 420 divided by 35

Find the quotient of 561 divided by 17.

EXAMPLE:

ANOTHER EXAMPLE:

The fundamental operations of Egyptian arithmetic are adding and doubling.

Fractions (parts) in Ancient Egypt

Examples of fractions: 1/2, 3/2, 1/3, 2/3, 1/4, 20/501….

Examples of parts: 1/2, 1/3, 1/4, 1/5,…

Ancient Egyptian Fractions: Parts

- A **part** is a fraction of the form 1/n
  - *Examples*: 1/2, 1/3, 1/10.
- Egyptians used
  - whole numbers (1,2,3..),
  - parts (1/2, 1/3, 1/4..), and
  - the special fraction 2/3.
- All other fractions were written as a sum of distinct parts.

Example:
5/6 was written as 1/2 + 1/3 (with no + sign!)

We will write parts as 1/n with + signs.

2/n table from the Rhind Mathematical Papyrus

2/3 = 1/2 + 1/6	2/5 = 1/3 + 1/15	2/7 = 1/4 + 1/28
2/9 = 1/6 + 1/18	2/11 = 1/6 + 1/66	2/13 = 1/8 + 1/52 + 1/104
2/15 = 1/10 + 1/30	2/17 = 1/12 + 1/51 + 1/68	2/19 = 1/12 + 1/76 + 1/114
2/21 = 1/14 + 1/42	2/23 = 1/12 + 1/276	2/25 = 1/15 + 1/75
2/27 = 1/18 + 1/54	2/29 = 1/24 + 1/58 + 1/174 + 1/232	2/31 = 1/20 + 1/124 + 1/155
2/33 = 1/22 + 1/66	2/35 = 1/30 + 1/42	2/37 = 1/24 + 1/111 + 1/296
2/39 = 1/26 + 1/78	2/41 = 1/24 + 1/246 + 1/328	2/43 = 1/42 + 1/86 + 1/129 + 1/301
2/45 = 1/30 + 1/90	2/47 = 1/30 + 1/141 + 1/470	2/49 = 1/28 + 1/196
2/51 = 1/34 + 1/102	2/53 = 1/30 + 1/318 + 1/795	2/55 = 1/30 + 1/330
2/57 = 1/38 + 1/114	2/59 = 1/36 + 1/236 + 1/531	2/61 = 1/40 + 1/244 + 1/488 + 1/610
2/63 = 1/42 + 1/126	2/65 = 1/39 + 1/195	2/67 = 1/40 + 1/335 + 1/536
2/69 = 1/46 + 1/138	2/71 = 1/40 + 1/568 + 1/710	2/73 = 1/60 + 1/219 + 1/292 + 1/365
2/75 = 1/50 + 1/150	2/77 = 1/44 + 1/308	2/79 = 1/60 + 1/237 + 1/316 + 1/790
2/81 = 1/54 + 1/162	2/83 = 1/60 + 1/332 + 1/415 + 1/498	2/85 = 1/51 + 1/255
2/87 = 1/58 + 1/174	2/89 = 1/60 + 1/356 + 1/534 + 1/890	2/91 = 1/70 + 1/130
2/93 = 1/62 + 1/186	2/95 = 1/60 + 1/380 + 1/570	2/97 = 1/56 + 1/679 + 1/776
2/99 = 1/66 + 1/198	2/101 = 1/101 + 1/202 + 1/303 + 1/606	

Image Credit: Wikipedia

2/n	Decomposition		2/n	Decomposition
2/3	1/2 + 1/6		2/21	1/14 + 1/42
2/5	1/3 + 1/15		2/23	1/12 + 1/276
2/7	1/4 + 1/28		2/25	1/15 + 1/75
2/9	1/6 + 1/18		2/27	1/18 + 1/54
2/11	1/6 + 1/66		2/29	1/24 + 1/58 + 1/174 + 1/232
2/13	1/8 + 1/52 + 1/104		2/31	1/20 + 1/124 + 1/155
2/15	1/10 + 1/30		2/33	1/22 + 1/66
2/17	1/12 + 1/51 + 1/68		2/35	1/30 + 1/42
2/19	1/12 + 1/76 + 1/114		2/37	1/24 + 1/111 + 1/296

Slide 61

It is the year 1500 BCE. You are a scribe in Egypt
1. Multiply 1/6 by 17.

2/n	Decomposition
2/3	1/2 + 1/6
2/5	1/3 + 1/15
2/7	1/4 + 1/28
2/9	1/6 + 1/18
2/11	1/6 + 1/66
2/13	1/8 + 1/52 + 1/104
2/15	1/10 + 1/30
2/17	1/12 + 1/51 + 1/68
2/19	1/12 + 1/76 + 1/114

61

Slide 62

It is the year 1500 BCE. You are a scribe in Egypt
1. Multiply 1/6 by 17.

2/n	Decomposition
2/3	1/2 + 1/6
2/5	1/3 + 1/15
2/7	1/4 + 1/28
2/9	1/6 + 1/18
2/11	1/6 + 1/66
2/13	1/8 + 1/52 + 1/104
2/15	1/10 + 1/30
2/17	1/12 + 1/51 + 1/68
2/19	1/12 + 1/76 + 1/114

1	1/6
2	1/3
4	2/3=1/2+1/6
8	1+1/3
16	2+1/2+1/6

17(1/6)= 2+1/2+1/6+1/6
17(1/6)= 2+1/2+1/3

62

Slide 63

It is the year 1500 BCE. You are a scribe in Egypt

2. Multiply 1/5 by 17.

2/n	Decomposition
2/3	1/2 + 1/6
2/5	1/3 + 1/15
2/7	1/4 + 1/28
2/9	1/6 + 1/18
2/11	1/6 + 1/66
2/13	1/8 + 1/52 + 1/104
2/15	1/10 + 1/30
2/17	1/12 + 1/51 + 1/68
2/19	1/12 + 1/76 + 1/114

63

Slide 64

It is the year 1500 BCE. You are a scribe in Egypt

2. Multiply 1/5 by 17.

2/n	Decomposition
2/3	1/2 + 1/6
2/5	1/3 + 1/15
2/7	1/4 + 1/28
2/9	1/6 + 1/18
2/11	1/6 + 1/66
2/13	1/8 + 1/52 + 1/104
2/15	1/10 + 1/30
2/17	1/12 + 1/51 + 1/68
2/19	1/12 + 1/76 + 1/114

1	1/5
2	2/5=1/3+1/15
4	1/2+1/6+1/10+1/30
8	1+1/3+1/5+1/15
16	2+1/2+1/6+1/3+1/15+1/10+1/30

17(1/5)= 2+1/2+1/3+1/5+1/6+1/10+1/5+1/30

64

Slide 1

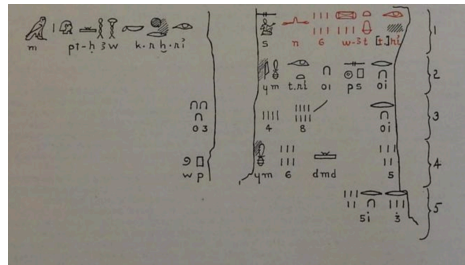**Problem 3 of the Rhind Mathematical Papyrus:**

Copy of Sector of Rhind Papyrus

Divide 6 loaves among 10 men.

https://archive.org/details/arnoldbuffumchaceludlowbullhenryparkermanningtherhindmathematicalpapyrus.volumei/page/n117/mode/1up
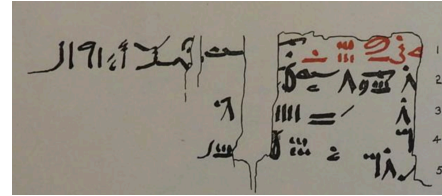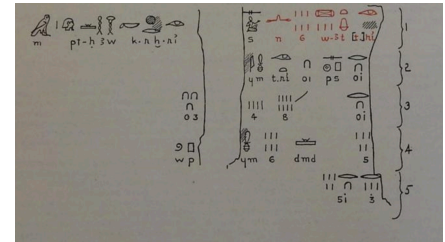
Translation to hieroglyphs

https://archive.org/details/arnoldbuffumchaceludlowbullhenryparkermanningtherhindmathematicalpapyrus.volumei/page/n117/mode/1up

Slide 2

Problem 3 of the Rhind Mathematical Papyrus:

Copy of Sector of Rhind Papyrus

Divide 6 loaves among 10 men.

https://archive.org/details/arnoldbuffumchaceludlowbullhenryparkermanningtherhindmathematicalpapyrus.volumei/page/n117/mode/1up
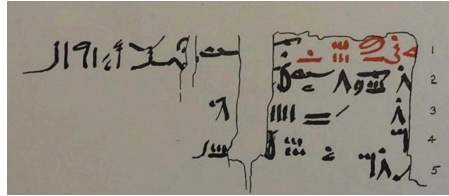
1. Break 5 loaves in two. Each man gets 1/2

2. Break the remaining loaf in 10, each man gets 1/10.

Total: 1/2+1/10. Note **use of the parts!**

Translation to hieroglyphs

https://archive.org/details/arnoldbuffumchaceludlowbullhenryparkermanningtherhindmathematicalpapyrus.volumei/page/n117/mode/1up

Slide 3

Method of false position

Slide 4

Exercise: Find the "Egyptian" quotient of 19 divided by 8

Slide 1 (top-left)
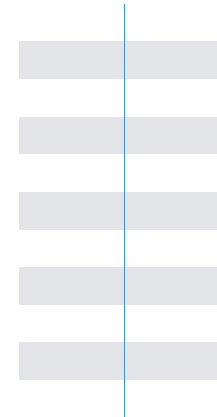
Exercise: Find the "Egyptian" quotient of 19 divided by 8

1	8
2	**16**
1/2	4
1/4	**2**
1/8	**1**

Quotient: 2+1/4+1/8

Slide 2 (top-right)

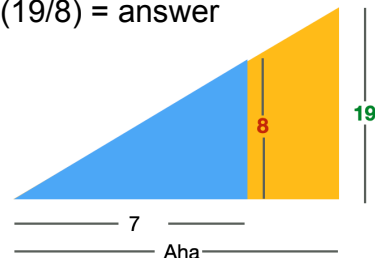## Slide 3 (bottom-left)
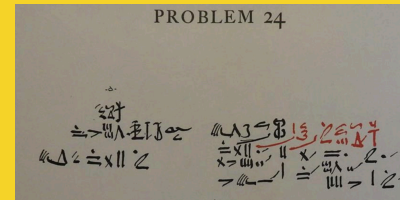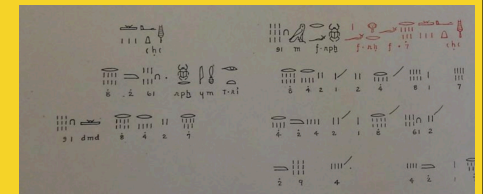
Method of False Position (aha)

Problem 24 of the Rhind Mathematical Papyrus:
A quantity and its 1/7 added together become **19**.
What is the quantity?

- **Step 1:** Pick a convenient number (here: 7)
- **Step 2**: Plug in 7 → 7 + 1/7 of 7 = **8**
- **Step 3**: Find the scale factor that turns **8** into **19** → 19/8.
- **Step 4**: Scale the guess. 7 × (19/8) = answer

> 21st century Hint:
> $7/8 = x/19 \Rightarrow x = 7\cdot(19/8)$

ꜥḥꜣ (aha): Egyptian word for quantity or number.

Slide 4 (bottom-right)

Method of False Position (aha)

Problem 24 of the Rhind Mathematical Papyrus:
A quantity and its 1/7 added together become **19**.
What is the quantity?

Step 1: Assume 7

7	7
1/7	1
Total	8

Problem 24 of the Rhind Mathematical Papyrus:

A quantity and its 1/7 added together become 19. What is the quantity?

Assume 7.

\1	7
\1/7	1
Total	8.

First, let's solve the problem in the "spirit of ancient Egypt" but using modern notation and language.
To obtain 8, the quantity is 7. (We choose 7 because it is easy to compute and then we obtain 8)

1	8
\2	16
1/2	4
\1/4	2
\1/8	1
Total 2 1/4 1/8.	
\1	2 1/4 1/8
\2	4 1/2 1/4
\4	9 1/2

Do it thus: The quantity is 16 1/2 1/8,
1/7 2 1/4 1/8,
Total 19.

To obtain 19, what is the quantity?
We have the ratio,
19/8 = x/7 where x is the number we are trying to find.

By proportionally, we know that to obtain 19, x - the quantity we are looking for- is 7 multiplied by 19/8.
Before we found that 19/8 =2+1/4+1/8

Hence, the answer is
7(2+1/4+1/8)=16+1/2+1/8

Method of false position

Problem 25 of the Rhind Mathematical Papyrus:

A quantity and its 1/2 added together become 16.
What is the quantity?

Method of false position

Problem 31 of the Rhind Mathematical Papyrus:

A quantity, its 2/3, its 1/2, and its 1/7 added together become 33. What is the quantity?

The total is

14 1/4 1/56/ 1/97 1/194 1/388 1/679 1/776,

which multiplied by 1 2/3 1/2 1/7 makes 33.

Method of false position

Ideas of Length, Area and Volume

1. How can you find the length of a segment?
2. What does it mean to "*measure*" a segment?
3. (Optional) What does it mean for two curves to have the same length?

> Answer 1. or 2.
> Hint: use a concrete example.

Take 1 minute to write down your first thoughts, then turn to someone you never talked to before and compare ideas.

---

1. How do you measure a shape? (area or volume)
2. What does "measurement" mean?
3. What does it mean for two shapes to have the same measurements?

> Choose Area or Volume and answer 1, 2, or 3.
> You may use practical examples or procedures.

Take 1 minute to write down your first thoughts, then turn to someone you never talked to before and compare ideas.

The same amount unit squares can be rearranged to occupy both planar shapes

that they are the same size but can be different shapes

What does it mean for two planar shapes to have the same area?

That means we can put the same amount of water or some kind of unit into two planar shapes

They must be congruent.   identical

The measurement of area agrees

Here are some answers from students from previous years. Some right, some wrong, some do not answer the question..can you tell which is which?

Measuring a segment means comparing its length with that of a chosen unit, and finding how many times the unit "fits" into the segment. (The unit may not "fit" into the segment an integer or fractional number of times.)

Similarly, **measuring a planar figure** means finding how many times a given unit of area (and/or fractions of that unit) "fits" into the figure.

Finally, **measuring a solid** means finding how many times a given unit of volume (and/or fractions of that unit) "fits" into the solid.

What do you think the scissor congruence app shows?

https://dmsm.github.io/scissors-congruence/

Scissor CongruenceTheorem (Wallace–Bolyai–Gerwien): If two polygons have the same area, then it is possible to cut one of them into polygonal pieces and rearrange these pieces to form the other.

(Note: The number of pieces is finite here)

Two shapes of a given kind are *scissor congruent* if one can be cut into finitely many pieces of that kind, which can then be rearranged to form the other.

Measurement in Egypt: length, area, volume

- Volumes of
  - Cylindrical containers.
  - Rectangular parallelepipedal containers.
  - Truncated pyramid
- Areas of
  - Rectangles
  - Circles
  - Triangles
  - Trapezoids

- Division of given area of land into equal-sized fields.

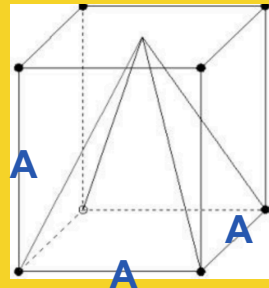- A quantity related to what we now call slope

---

The Moscow Papyrus: Volume of the truncated pyramid

Slide 1:

P = volume of a square pyramid with base side A and height A.
C = volume of a cube with side A (So C = A³).
Question: Which statement is correct?

1. C = P/2
2. P/2 < C < P
3. C = P
4. P < C < 3P
5. C = 3P
6. 3P < C < 4P
7. C ≥ 4P
8. None of the above

A
A
A

Slide 2:

## What is the volume of a pyramid of square base?

Demo

The Great Pyramid of Giza

Slide 3:

We did a demonstration in which we saw that a cube can be filled exactly with three squared pyramids (of square base of same side length and height as the cube).

Does the demonstration constitute a mathematical proof that the volume of the cube is three times the volume of the pyramid?

Slide 4:

Problem 14 of the Moscow Mathematical Papyrus
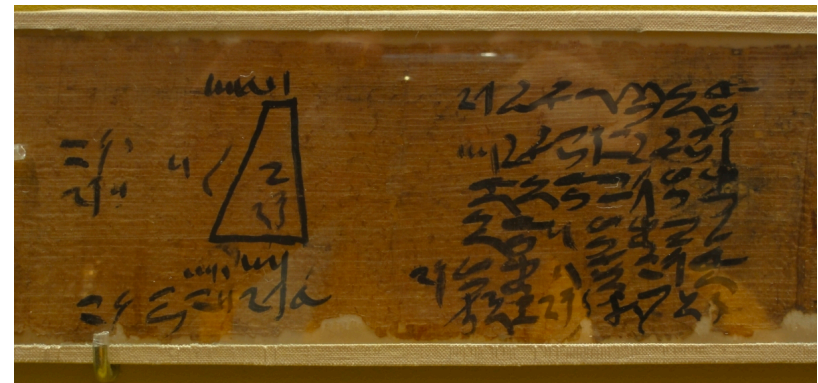
*Carles Dorce photographed problem 14 of the Moscow Mathematical Papyrus*

If someone says to you: a truncated pyramid of 6 for the height and by 4 on the base by 2 on the top. You are two square this 4; the result is 16. You are to double 4; the result is 8. You are to square this 2; the result is 4. You are to add the 16 and the 8 and the 4; the result is 28. You have to take 1/3 of the 6 the result is 2. You have to take 28 two times; the result is 56. Behold, the volume is 56. You will find that this is correct.
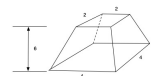
Problem 14 of the Moscow Mathematical Papyrus

1. If someone says to you:
2. a truncated pyramid of **H** for the height and by **A** on the base by **B** on the top.
3. You are two square this **A**; the result is A^2.
4. You are to multiply **B** by **A**; the result is **A.B**.
5. You are to square this **B**; the result is B^2
6. You are to add the A^2 and the **A.B** and the B^2; the result is $A^2 + A.B + B^2$.
7. You have to take 1/3 of the **H** the result is **H**/3
8. You have to multiply $(A^2 + A.B + B^2)$ by **H**/3; the result is $(A^2 + A.B + B^2)$**H**/3. Behold, the volume is $(A^2 + A.B + B^2)$**H**/3.
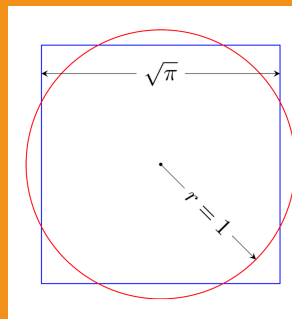9. You will find that this is correct.
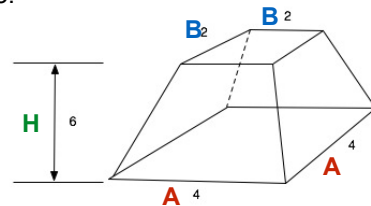
Problem 14 of the Moscow Mathematical Papyrus

Conjecture: to find the formula the truncated pyramid was broken into pieces.
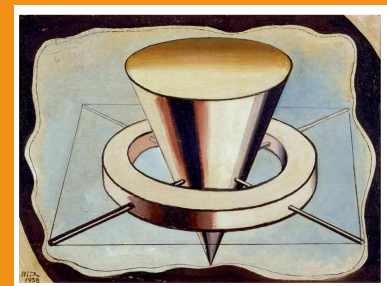
- Note: This 3D "cut and paste" is a fundamental property of volume.

The Rhind Papyrus
Problem 50
Area of the circle
Approximation of π

Squaring the circle

La Quadrature, 1938. Oil on panel Man Ray

Problem 50 of Rhind or Ahmes papyrus

Example of a round field of diameter 9 khet What is its area"

Take away ⅑ of the diameter, namely 1; the remainder is 8. Multiply 8 times 8; it makes 64. Therefore it contains 64 *setat* of land.

Do it thus:

1	9
⅑	1;

this taken away leaves 8

1	8
2	16
4	32
\8	• 64.

Its area is 64 *setat*.

A *khet* is unit of length (about 50 meters.)	A *setat* is a unit of area (one khet squared)

Problem 50 of Rhind or Ahmes papyrus

Find the formula of the area of the circle that the scribe would have obtained by starting with a circle of diameter d, instead of a circle of diameter 9. (Hint: Start by taking away 1/9 of the diameter, that is d/9.)

Example of a round field of diameter 9 khet. *What is its area?*

Take away ⅑ of the diameter, namely 1; the remainder is 8. Multiply 8 times 8; it makes 64. Therefore it contains 64 *setat* of land.

Do it thus:

1	9
⅑	1;

this taken away leaves 8

1	8
2	16
4	32
\8	• 64.

Its area is 64 *setat*.

Problem 50 of Rhind or Ahmes papyrus

The area of a disk is a constant (π) times the radius of the circle squared. What is the value of π that Egyptians assumed in their computation of the area of the disk? (in the problem we are discussing)

More about the Rhind Papyrus Problem 50: Conjectures about how the area of the circle was found.

1) How many small disks are inside the large disk?
2) Along the center line of the large disk, how many small-disk diameters fit across the diameter?
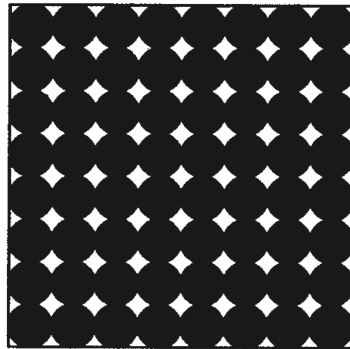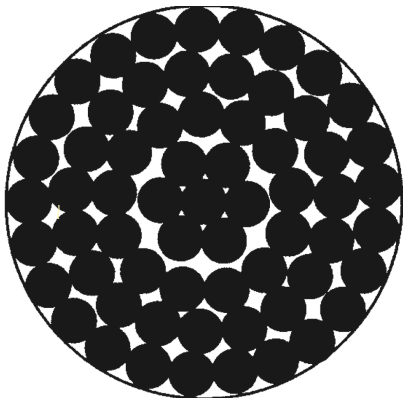

1) Using the same 64 small disks, how many small-disk diameters are along one side of the square built from them?

2) What relationship do you see between the circle's diameter and the side of the square made from the same disks?

The previous discussion suggests: the circle's area equals the area of a square whose side is 8/9 of the diameter

Problem 50 of Rhind or Ahmes papyrus
Another conjecture of how the area was found

Design found in several places in Africa, for instance
carved in wooden doors in Nigeria.
in fabrication of baskets in Egypt
Also in burial sites in Ancient Egypt.

Gerdes, Paulus. "Three alternate methods of obtaining the ancient Egyptian formula for the area of a circle." *Historia Mathematica* 12.3 (1985): 261-268. https://www.sciencedirect.com/science/article/pii/0315086085900242

The Rhind Papyrus
Problem 79: 7 houses, 49 cats, 343 mice.
Math for math sake?

As I was going to Saint Ives,
I met a man with seven wives.
Every wife had seven sacks,
Every sack had seven cats,
Every cat had seven kits;
Kits, cats, sacks and wives,
How many were there going to
Saint Ives?

Mother Goose

https://youtu.be/7vtszdW8MTs?t=11. 2:30 minutes

ST. IVES
As I was going to St. Ives
I met a Man with Seven Wives.
The Seven Wives had Seven Sacks.
The Seven Sacks had Seven Cats.
The Seven Cats had Seven Kits.
Kits, Cats, Sacks and Wives,—
How Many were there going to St. Ives?
?

Image credit: https://www.cornwalls.co.uk/st-ives/as-i-was-going-to-st-ives

There are seven old women on the road to Rome.
Each woman has seven mules;
each mule carries seven sacks;
each sack contains seven loaves;
with each loaf are seven knives;
and each knife is in seven sheaths.
Women, mules, sacks, loaves, knifes, and sheaths,
how many are there in all on the road to Rome?
(Translation from Fibonacci's Liber Abacci)

$7 + 7^2 + 7^3 + 7^4 + 7^5 =$

$7(1 + 7 + 7^2 + 7^3 + 7^4) =$

$7 . 2801 = 19,607$

Rhind Papyrus
Problem 79

Possible interpretation

In each of the seven houses there are seven cats: each cat kills seven mice; each mouse would have eaten seven sheaves of wheat; and each sheaf of wheat was capable of yielding seven hekat measures of grain. How much grain was thereby saved?

A house inventory

		houses	7
1	2,801	cats	49
2	5,602	mice	343
4	11,204	spelt	2,301
Total	19,607	hekat	16,807
		Total	19,607
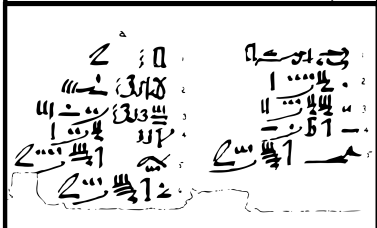
Conclusions

A careful study of the Rhind Papyrus convinced me several years ago that this work is not a mere selection of practical problems especially useful to determine land values, and that the Egyptians were not a nation of shopkeepers, interested only in that which they could use. Rather I believe that they studied mathematics and other subjects for their own sakes.

The Rhind Mathematical Papyrus. By A. B. Chace, L. Bull, and H. P. Manning. Vol. I 1927; Vol. II 1929. $20. (Mathematical Association of America.)

After working on the Rhind Papyrus, with which of the two paragraphs you agree more and why?

The Rhind and Moscow papyri are handbooks for the scribe, giving model examples of how to do things which were a part of his everyday tasks . . . The sheer difficulties of calculation with such a crude numeral system and primitive methods effectively prevented any advance or interest in developing the science for its own sake. It served the needs of everyday life . . . and that was enough.

Mathematics and Astronomy,   J. R. Harris (ed.): The Legacy of Egypt. Second Edition. Pp. xi+510; 24 pls. Oxford: Clarendon Press, 1971.