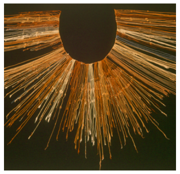
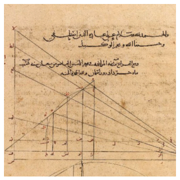
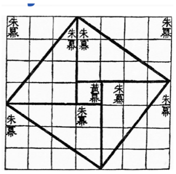
Counting: What It Is and Where It Comes From



What is Counting and How Do We Do It?

- What is counting (definitions, one-to-one correspondence)
- Recording methods (tally systems)
- The brain and counting - connection between fingers and numbers (parietal lobe)
- What is a number

What is counting?

~~What is counting?~~

What do you mean by counting?

What is counting? Explain it to someone who does not know what it is

By counting one (usually) means a process to determine the number of objects in a set.

Counting involves establishing a **one-to-one correspondence** between two sets.

- One set (let's call it the *standard set*) is a list of symbols, objects, etc.
- The other set is variable.

Image credit: https://creativekindergartenblog.com/one-to-one-correspondence-intervention-for-kindergarten/

Counting

If the standard set if composed of symbols, then each symbol

- has to be different from the others.
- has a prescribed order of use.

Body parts as numerals, by Geoffrey Saxe. https://www.jstor.org/stable/pdf/1129244.pdf

Counting

Tally

Early Latin *talea* = a cutting, rod, or stick

Late Latin *taliare* = to cut or split

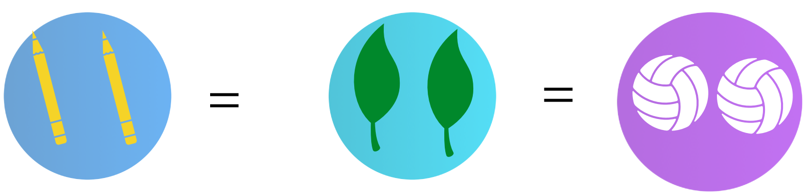
(same root as tailor)

- Objects:
  - sticks, rocks shells - Abacus
- Body parts:
  - fingers, toes

Determine how many

Record how many

assigning labels, language

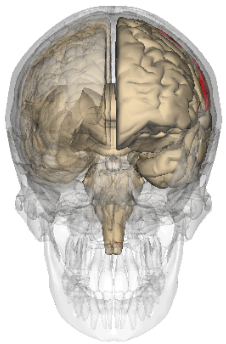


Counting and the brain

Counting and the human brain

Parietal lobe(red) in left hemisphere,

- Humans have innate number sense
- Small number words come from body parts
- Brain connects finger control with number processing
- **Hypothesis**: Evolutionary link between fingers and counting



from Anatomography, website maintained by Life Science Databases(LSDB).

Number sense is a short-hand for our ability to quickly understand, approximate, and manipulate numerical quantities

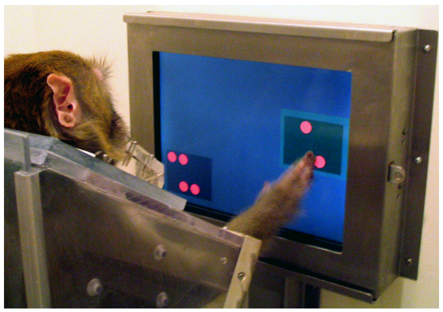
For more listen to the podcast: https://www.bbc.co.uk/programmes/p00545hk - with Ian Stewart and Brian Butterworth, author of The mathematical brain

Approximate number system - Object tracking system

To separate mental systems to represent number without symbols in humans an many animals

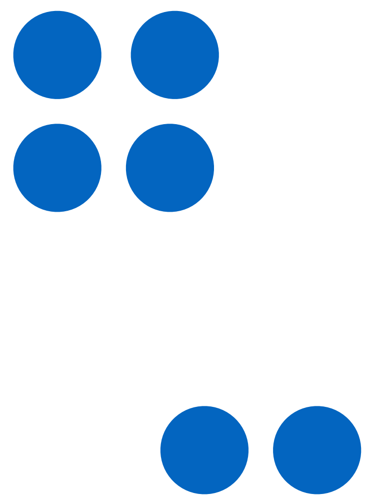
Approximate number system: a primitive, nonverbal cognitive system that allows humans and animals to estimate and rapidly compare quantities without relying on language or symbols.

- approximate
- non-verbal
- Represents and compares numerosities
- ratio is key.



Example: Experiment where a monkey selects between two groups of dots displayed on a screen, choosing the group with the smaller quantity.

To separate mental systems to represent number without symbols in humans an many animals



Object tracking system:allows grasping only small set sizes, from one to about four, in an unconscious but relatively precise way.

Note: **Grasping** (not counting!) up to four.

The object tracking system is thought to be responsible for a "**subitizing**" effect, from the Latin word subitus ("suddenly"). It describes an effortless, fast, and accurate process to judge a small number of items

To separate mental systems to represent number without symbols in humans an many animals

Just to illustrate that animals have also numbers sense (although feeling their chicks involves many systems…)

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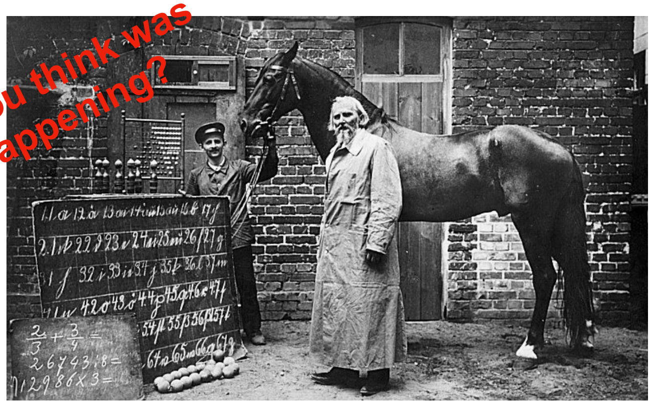
Clever Hans, the horse who could count?

- Around 1900 Von Osten claimed his horse, Clever Hans could do arithmetic.
- Von Osten would show Clever Hans written problems or ask questions aloud
- The horse would tap out the answer with its hoof.

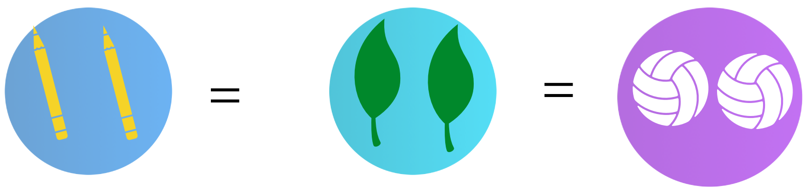
What do you think was really happening?



Animals do have counting abilities but sometimes these are exaggerated!!

What is a natural (or counting) number?

(And why is this such a hard question)



Russell’s paradox Other forms

A man of Sevilla is shaved by the Barber of Sevilla if and only if the man does not shave himself.

Does the barber shave himself? Why or why not?

Proposed by Bertrand Russell
More about this paradox here.

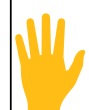
Consider S the set of all sets.
So, Ø (the empty set) belongs to S.
The set containing the empty {Ø} set belongs to S.
And so on.
Define R = {x in S : x does not belong to x}.

Does R belong to R? Why or why not?

The beginnings of a rigorous definition of (counting) number

- **Cantor** (late 1800s):
  - Beginnings of set theory.
  - Defined “two infinite sets have the same *power* if the elements of the sets can be put into **one-to-one correspondence**.”
- **Frege** (~1900) applied that this idea of the one-to-one correspondence of elements to **finite** sets to **define cardinality.**
  - Two finite sets are said to have the same **cardinal number**—that is, to be equal—if the elements in either class can be put into one-to-one

Example: Cardinality of set of fingers in



is the set of all sets that can be put in one to one correspondence with the fingers in this hand.

Boyer, C. B., & Merzbach, U. C. (2011). "A History of Mathematics."

Russell's paradox

Define R = {x : x does not belong to x}.
then R belongs to R iff R does not belong to R.

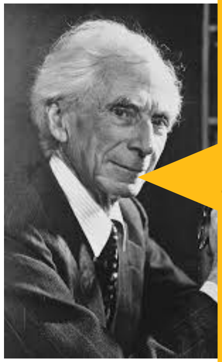
- Russell wrote to Frege explaining his paradox in 1902.
- Russell's paradox showed that the axioms Frege was using to formalize his logic were inconsistent.
- Russell's letter arrived just as the second volume of Frege's *Grundgesetze der Arithmetik* (*The Basic Laws of Arithmetic*, 1893, 1903) was in press.
- Frege eventually felt forced to abandon many of his views about logic and mathematics.

Thus, giving a rigorous definition of "number" is quite difficult

Stanford Encyclopedia of Philosophy

https://plato.stanford.edu/entries/russell-paradox/#HOTP

Bertrand Russell.



As I think about acts of integrity and grace, I realise that there is nothing in my knowledge to compare with Frege's dedication to truth. His entire life's work was on the verge of completion, much of his work had been ignored to the benefit of men infinitely less capable, his second volume was about to be published, and upon finding that his fundamental assumption was in error, he responded with intellectual pleasure clearly submerging any feelings of personal disappointment. It was almost superhuman and a telling indication of that of which men are capable if their dedication is to creative work and knowledge instead of cruder efforts to dominate and be known. (Quoted in van Heijenoort (1967), 127)

Stanford Encyclopedia of Philosophy

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