MAT 211 List of review problems

(1) Find the projection of the vector $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ onto the vector $\vec{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

(2) Find a single equation for the plane $\mathcal{P}$ in $\mathbb{R}^3$ which passes through the $P = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and has normal vector $\vec{n} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$.

(3) Write the equations of the plane passing through P with direction vectors $\vec{u} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$, in a) parametric and b) vector form.

(4) Find the equation of the line passing through $P = \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix}$ and $Q = \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}$, where

(5) Find the vector form of the line in $\mathbb{R}^2$ passing through $P = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and perpendicular to the line with general equation $3x - 2y = 33$.

(6) Find the distance from the point $P = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ to the line ℓ of equation $3x + y = 1$

(7) Determine which of the equations below are linear. If any equation is not linear, explain why.

(a) $x + y + \sin z = 3$.

(b) $x + y^2 = 0$.

(c) $2x + 2y = 2$.

(d) $x.y = 2$.

(8) Determine geometrically whether each of the systems has a unique solution, infinitely many solutions, or no solutions. Then solve the systems algebraically to confirm your answer.

(a) $x + y = 1$
$x - 2y = 1$

(b) $x + y = 1$
$2x + 2y = 2$

(c) $x + y = 1$
$2x + 2y = -2$

(9) If possible, give examples of the following types of systems of equations. If it is not possible, explain why.

(a) 2 variables, 2 equations, infinitely many solutions.

(b) 2 variables, 1 equations, infinitely many solutions.

(c) 2 variables, 2 equations, a unique solution.

(d) 2 variables, 1 equations, a unique solution.

(e) 2 variables, 2 equations, no solutions.

(f) 2 variables, 1 equations, no solutions.

(10) Solve the given system of equations.

$x + y + 2z = 0$
$x + z = -1$
$4x - 2y + z = 2$

(11) Determine by inspection (that is, without performing any calculations) whether a linear system with each of the given augmented matrices has a unique solution, no solution or infinitely many solutions. Justify your answer.

(a) $\begin{pmatrix} 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 0 & 1 & | & 1 \\ 0 & 1 & 0 & | & -2 \\ 1 & 0 & 0 & | & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & -2 \\ 0 & 0 & 0 & | & 2 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 1 & 1 & | & 1 \\ 0 & 1 & 1 & | & -2 \\ 0 & 0 & 1 & | & 2 \end{pmatrix}$

(12) (a) Show that $\mathbb{R}^3 = \text{span}(\vec{u}, \vec{v}, \vec{w})$, where $\vec{u} = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$.

(b) Write $z = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$ as a linear combination of $\vec{u}, \vec{v}$ and $\vec{w}$.

(13) Describe the span of $v = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ and $w = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ a) geometrically b)algebraically.

(14) Determine whether the vectors $\vec{u} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}$ are linearly independent.

(15) Given $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ find all matrices B such that $AB = BA$,

(16) Given the matrix $a = \begin{pmatrix} 1 & 1 \\ 2 & -4 \end{pmatrix}$.

(a) Find A^{-1}.

(b) Use A^{-1} found in (a). to solve the system $A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(c) Find the inverse of the given elementary matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(d) Find the inverse of the matrix. $\begin{pmatrix} 1 & 2 & 3 \\ -2 & -1 & -2 \\ 0 & 4 & 5 \end{pmatrix}$