MAT 211 - Review Problems, Fall 2018

(1) Let A be the matrix $A = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 3 & 1 & -1 & -2 \\ -7 & 1 & 3 & 4 \end{pmatrix}$

(a) Find basis for the row space, and nullspace of A.

(b) Find the dimensions of the row space, nullspace and column space of A.

(c) Determine whether $\vec{v}$ is in the nullspace of A, where $\vec{v} = \begin{bmatrix} -4 \\ 7 \\ -25 \\ 10 \end{bmatrix}$

(2) Find all the possible values of $rank(A)$as a varies. $\begin{pmatrix} 1 & a & 0 & 1 \\ 3 & 1 & -a & 2 \\ 3 & 1 & -1 & 2a \end{pmatrix}$

(3) Find the matrix of a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$, such that

$$T\left(\begin{bmatrix} 3 \\ -3 \end{bmatrix}\right) = \begin{bmatrix} 10 \\ 3 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 3 \\ -3 \end{bmatrix}\right) = \begin{bmatrix} 10 \\ 3 \end{bmatrix}$$

(4) Determine whether T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$, such that

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x - y \\ 2x + y + 1 \end{bmatrix}$$

(5) Find the matrices of the following linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$.

(a) Counterclockwise rotation through 90^o followed by the reflection on the line $y = 2x$.

(b) Counterclockwise rotation through 37^o followed by counterclockwise rotation through 53^o.

(6) For the following matrices, find the eigenvalues and eigenspaces of A, and determine whether A is diagonalizable or not, If A is not diagonalizable, explain why not. If A is diagonalizable, find and invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$

(a) A is the matrix of a reflection in the y-axis in $\mathbb{R}^2$.

(b) If A is a 2×2 matrix, with eigenvalues $\lambda_1 = -1$ and $\lambda_2 = 3$, and the corresponding eigenvectors $\vec{v_1} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$ and $\vec{v_2} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ find $A^8.u$, where $\vec{u} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

(c) $A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 2 \\ 4 & 0 & 1 & 4 \end{pmatrix}$

(7) If A is a square matrix and $A^4 = A$ what are the possible values of the eigenvalues of A?

(8) If $A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 2 \\ 4 & 0 & 1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 2 & 3 & 0 & 2 \\ 0 & 1 & 5 & 2 \\ 4 & 0 & 1 & 4 \end{pmatrix}$, and $C = \begin{pmatrix} 10 & 10 & 0 & 10 \\ 2 & 3 & 0 & 2 \\ 0 & 10 & 50 & 20 \\ 4 & 0 & 1 & 4 \end{pmatrix}$

(a) Compute $\det(A^8 B^3 A^7)$.

(b) Compute det(C)

(9) Find all the (real) values of k that make the matrix A invertible, $A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & k & 0 & 2 \\ 0 & 1 & k & 2 \\ 4 & 0 & 1 & 4 \end{pmatrix}$

(10) Find all possible 3×3 diagonal matrices with eigenvalues 1 (with geometric multiplicity 2) and -7.

(11) Compute A^k where $A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & -2 & 2 \\ 0 & 1 & 1 \end{pmatrix}$

(12) Find the values of k that make A diagonalizable, where $A = \begin{pmatrix} 0 & 3 & 0 \\ 3 & k & 0 \\ 0 & 0 & 4 \end{pmatrix}$

(13) Mark each of the following statements true or false. If they are false, give a counterexample.

(a) If A, B, X are square matrices, $A.X = B$ and B is invertible then $X = BA^{-1}$.

(b) For all square matrices A, $\det(A) = -det(A^{-1})$

(c) if A is an $n \times n$ matrix with n distinct eigenvalues then A is diagonalizable.

(d) Two vectors corresponding to different eigenvalues are linearly dependent.

(e) Every matrix is diagonalizable.

(f) If two matrices are row equivalent they they have the same nullspace.

(g) If two matrices A and B are row equivalent then $dim(row(A)) = dim(col(B))$,