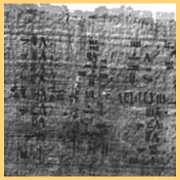
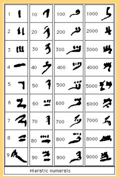
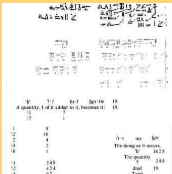
Mathematics in Ancient Egypt


1. Introduction
2. The Rosetta Stone
3. Papyrus
4. Scribes
5. The Moscow and Rhind Papyri
6. Multiplication in Ancient Egypt
7. Division in Ancient Egypt
8. Fractions (parts) in Ancient Egypt
9. Method of false position
10. Ideas of Length, Area and Volume
11. The Moscow Papyrus: Volume of the truncated pyramid
12. The Rhind Papyrus Problem 50: Area of the circle, Approximation of π
13. The Rhind Papyrus Problem 79: 7 houses, 49 cats, 343 mice. Math for math sake?
14. Conclusions

Introduction

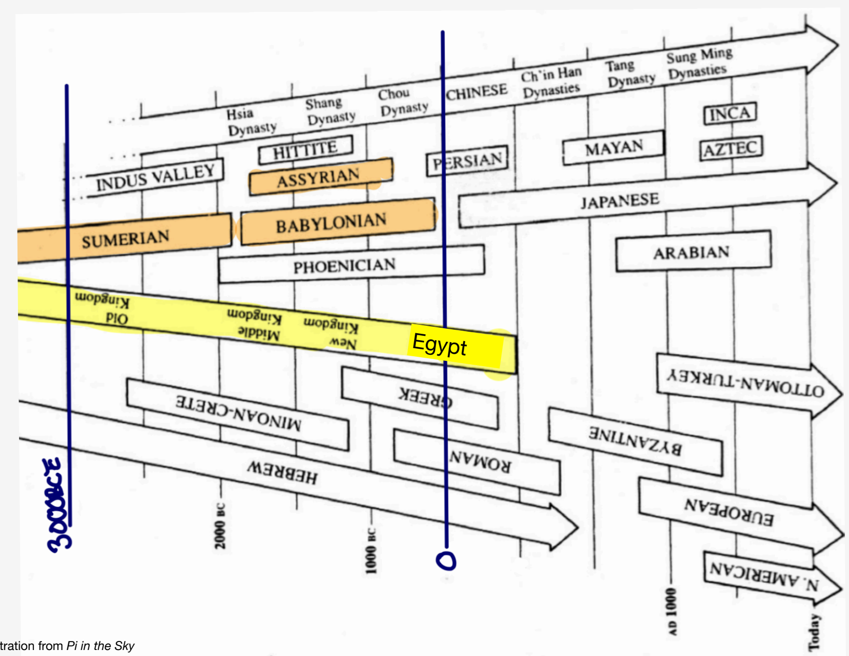
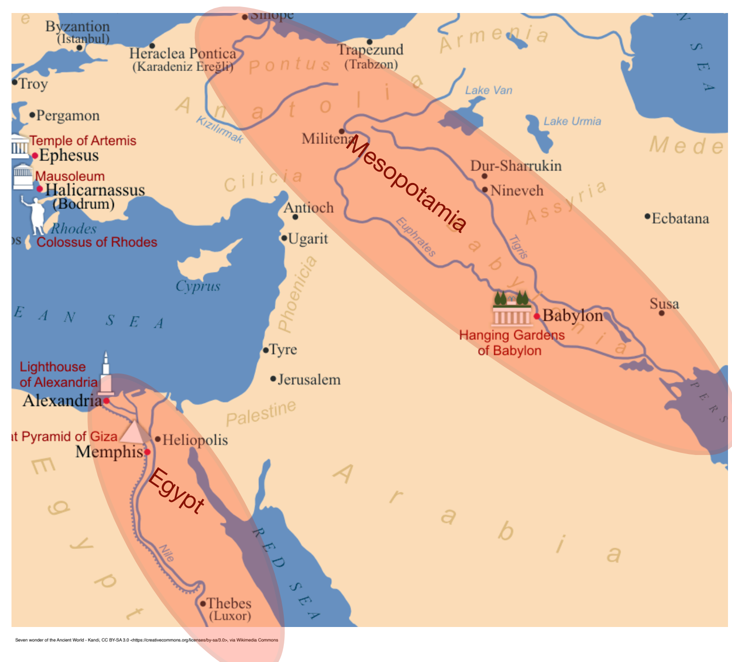


Illustration from *Pi in the Sky*
By Wendy Mass Little, Brown Books for Young Readers 2013



Ancient Egypt and Mesopotamia

Civilizational context

- Along major rivers, predictable flooding.
- Favorable climate, fertile lands.
- Developed writing systems.
- Strong centralized government.
- Strong religious life.

Mathematical Development

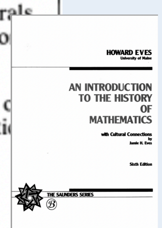
- **Motivation**: Administrative needs
  - Taxes, public works, calendar
- **Focus**: Arithmetic and mensuration
  - Lengths, areas, volumes
- **Evolution**:
  - From practical applications to some abstraction.
  - In later years, perhaps math for its own sake.

Ancient Egypt 101 | National Geographic https://youtu.be/hO1tzmi1V5g

Write down one idea from the video related to mathematics.

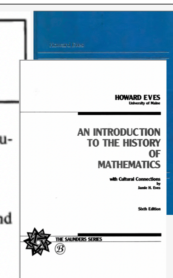
Find one word or one sentence in this table that could not have been written by a neutral observer.
Optional: What assumption does that word or sentence make?

EGYPTIAN AND BABYLONIAN (3000 B.C. to A.D. 260)	GREEK (600 B.C. to A.D. 450)	CHINESE (1030 B.C. to A.D. 1644)
Essentially empirical, or inductive, mathematics Introduction of early numeral systems (decimal and sexagesimal) Simple arithmetic, practical geometry Mathematical tables, collections of mathematical problems Chief primary	Significant introduction, then development, of deductive geometry (Thales, 600 B.C.; Pythagoras, 540 B.C.) Start of number theory (Pythagorean School, 540 B.C.) Discovery of incommensurable magnitudes (Pythagorean School, before 340 B.C.)	Largely isolated from the mainstream of mathematical development Decimal numeral system, rod numerals magic squares (fro earliest time) *Chou-peï*, oldest c Chinese mathemati classics (300 B.C.?) *Arithmetic in Nine Sections* (100 B.C.?)



An Introduction to the History of Mathematics - Saunders College Publishing - 1990

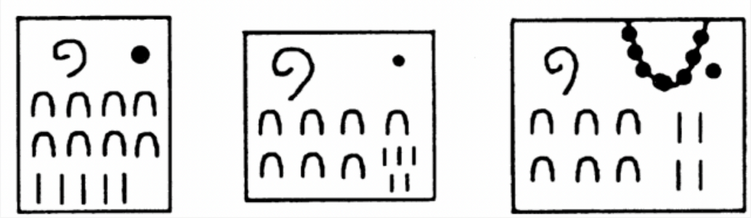
EGYPTIAN AND BABYLONIAN (3000 B.C. to A.D. 260)	GREEK (600 B.C. to A.D. 450)	CHINESE (1030 B.C. to A.D. 1644)	HINDU (200 B.C. to A.D. 1250)
Essentially empirical, or inductive, mathematics Introduction of early numeral systems (decimal and sexagesimal) Simple arithmetic, practical geometry Mathematical tables, collections of mathematical problems Chief primary sources: Moscow (1850 B.C.), Rhind (1650 B.C.), and other Egyptian papyri; Babylonian cuneiform tablets (2100 B.C. to 1600 B.C. and 600 B.C. to A.D. 300)	Significant introduction, then development, of deductive geometry (Thales, 600 B.C.; Pythagoras, 540 B.C.) Start of number theory (Pythagorean School, 540 B.C.) Discovery of incommensurable magnitudes (Pythagorean School, before 340 B.C.) Systematization of deductive logic (Aristotle, 340 B.C.) Axiomatic development of geometry (Euclid, 300 B.C.) Germs of the integral calculus (Archimedes, 225 B.C.) Geometry of conic sections (Apollonius, 225 B.C.)	Largely isolated from the mainstream of mathematical development Decimal numeral system, rod numerals magic squares (from earliest time) *Chou-peï*, oldest of Chinese mathematical classics (300 B.C.?) *Arithmetic in Nine Sections* (100 B.C.?) Horner's method (Ch'in Kiu-Shoo, 1247) Pascal's arithmetic triangle, binomial theorem (Chu Shï-kié, 1303) Jesuit missionaries infiltrated China in early 1600s	Introduction of Hindu-Arabic numeral system (before A.D. 250) Negative numbers and invention of zero symbol (early centuries A.D.) Development of early computing algorithms (A.D. 900–1000) Syncopated algebra, indeterminate equations (Brahmagupta, A.D. 628; Bhāskara, A.D. 1150) **ARABIAN** (A.D. 650 to 1200) Preservers of Hindu arithmetic and Greek geometry (encouraged by caliph patrons of learning, such as Harun al-Rashid, A.D. 790)



An Introduction to the History of Mathematics - Saunders College Publishing - 1990

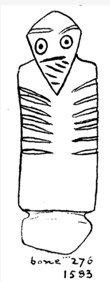
Note the point of view on this table

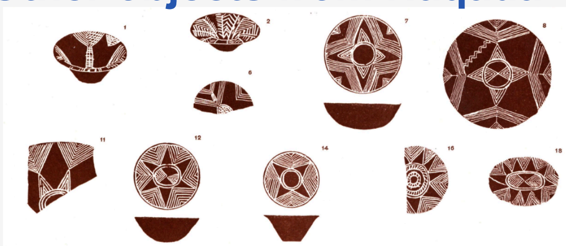
Naqada labels/tags - c. 3300–3000 BCE



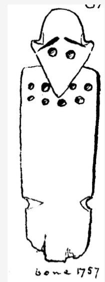
The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook By Annette Imhausen, Eleanor Robson, Victor J. Katz, Victor J. Katz, Annette Imhausen · 2007

Other objects from Naqada





https://brittlebooks.library.illinois.edu/brittlebooks_open/Books2009-08/petrwi0001naqbal/petrwi0001naqbal.pdf



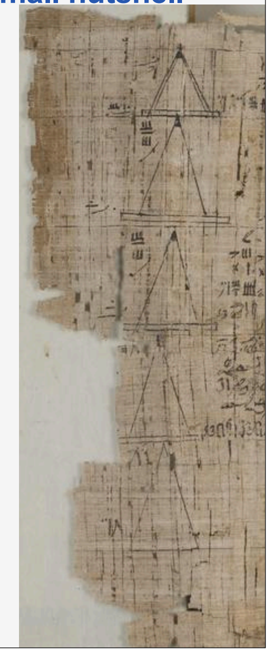
Ancient Egyptian Mathematics in a very small nutshell

Sources & Context:

- Very few surviving sources (papyrus is fragile!)
- Practical problems:
  - Example: area and volume
  - Example: fair division of loaves of bread.

Key Mathematical Features

- Linear equations solved by "false position" method
- Doubling/halving as basic arithmetic operations
- Egyptian fractions: written as sums of unit fractions (1/n)
- Surprisingly accurate π approximation.
- Some problems with theoretical interest
  - Adding $7 + 7^2 + 7^3 \ldots + 7^5$



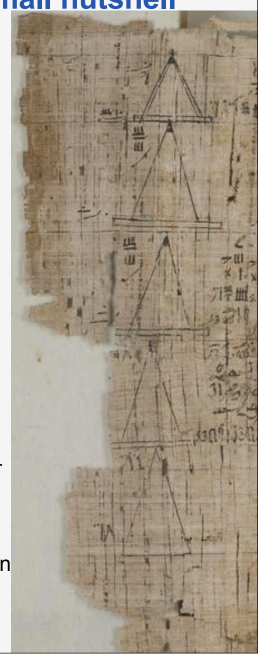
Ancient Egyptian Mathematics in a very small nutshell

Sources & Context:

- Problems and solutions to concrete algebraic and geometric problems
  - finding the area or volume of certain shapes,
  - fair division of loaves of bread.
  - feeding animals and storage of grain
  - solutions of linear equations with one unknown - false position.

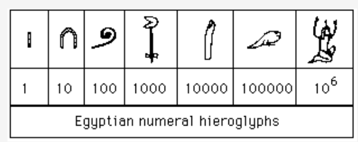
Key Mathematical Features

- Some Problems with theoretical interest
  - Adding $7 + 7^2 + 7^3 \ldots + 7^5$
- Examples (as opposed to rules); how (as opposed to why)
- Doubling and halving were the basic arithmetic operations.
- Two number systems: hieroglyphic and a ciphered (used for different purposes).
- Intriguingly accurate approximation to π
- Fractions were written as a sum of *parts* (fractions of form 1/n
- Development of calendar.



Two number (and writing) systems in Ancient Egypt

Hieroglyphic numerals

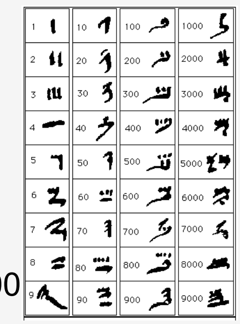


Example of a hieroglyphic number from a tomb inscription.

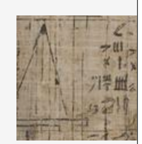
Hieroglyphic and Hieratic

- Two parallel systems used for around 2000 years
- **Hieroglyphic**: Formal script carved or painted in stone monuments
- **Hieratic**: Cursive script written on papyrus for daily use
- Both evolved over time
- Mathematical texts primarily used hieratic (practical documents)

Hieratic numerals



Credit tables: https://mathshistory.st-andrews.ac.uk/HistTopics/Egyptian_numerals/

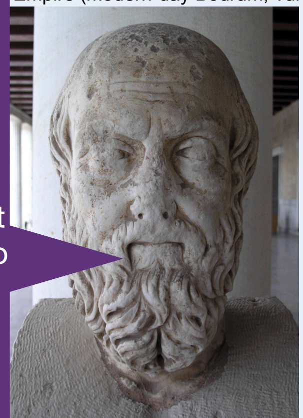


Rhind Papyrus Section British Museum

How was geometry invented, according to Herodotus?

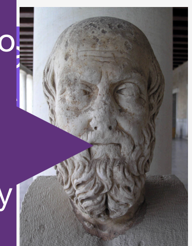
"It was this king, moreover, who divided the country among all the Egyptians by giving each an equal square parcel of land, and made this his source of revenue, appointing the payment of a yearly tax. And any man who was robbed by the river of a part of his land would come to Sesostris and declare what had befallen him; then the king would send men to look into it and measure the space by which the land was diminished, so that thereafter it should pay in proportion to the tax originally imposed. **Perhaps this was the way in which the art of measuring land (geometry) was invented, and passed afterwards into Greece"**

Herodotus (~400BC) was an ancient Greek historian who was born in Halicarnassus in the Persian Empire (modern-day Bodrum, Turkey).

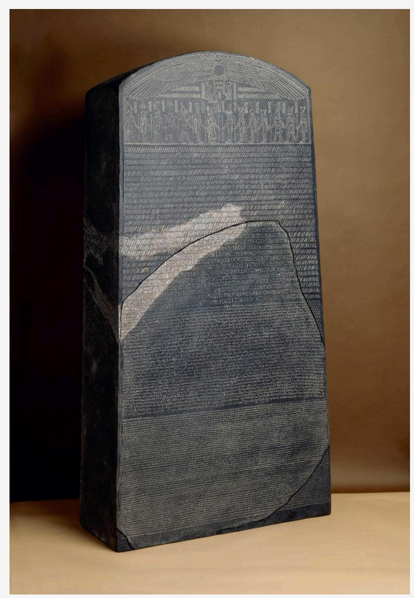


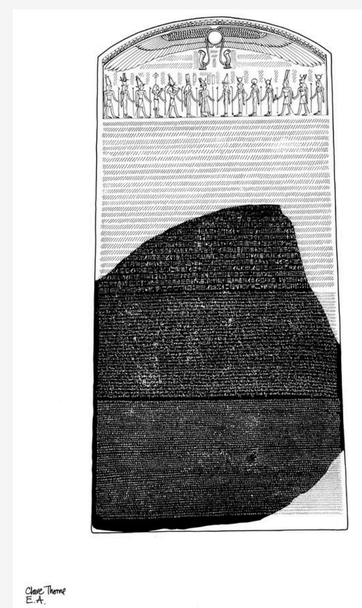
Bust of Herodotus. 2nd century AD. Roman copy after a Greek original. On display along the portico of the Stoa of Attalus, which houses the Ancient Agora Museum in Athens.

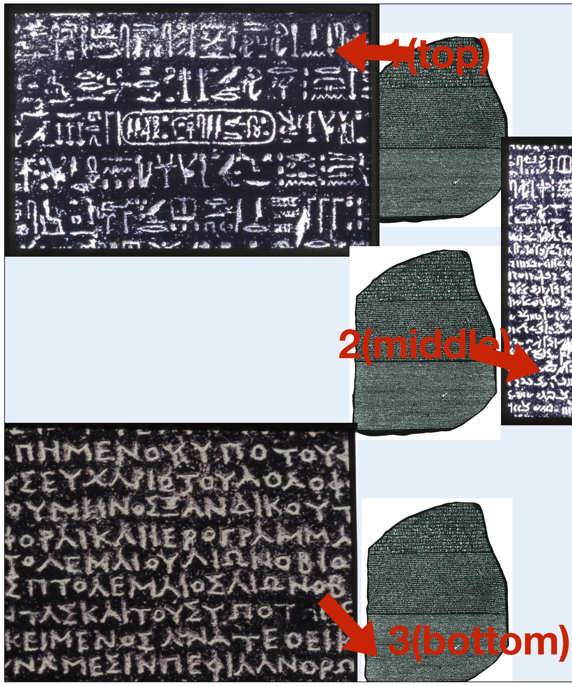
… Cheops became king over them and brought them to every kind of evil: (….) he then bade all the Egyptians work for him. So some were appointed to draw stones from the stone-quarries in the Arabian mountains to the Nile, and others he ordered to receive the stones after they had been carried over the river in boats, and to draw them to those which are called the Libyan mountains; and **they worked by a hundred thousand men at a time, for each three months continually.** Of this oppression there passed ten years (..) For this they said, the ten years were spent, and for the underground he caused to be made as **sepulchral chambers for himself** in an island, having conducted thither a channel from the Nile. **For the making of the pyramid itself there passed a period of twenty years.**



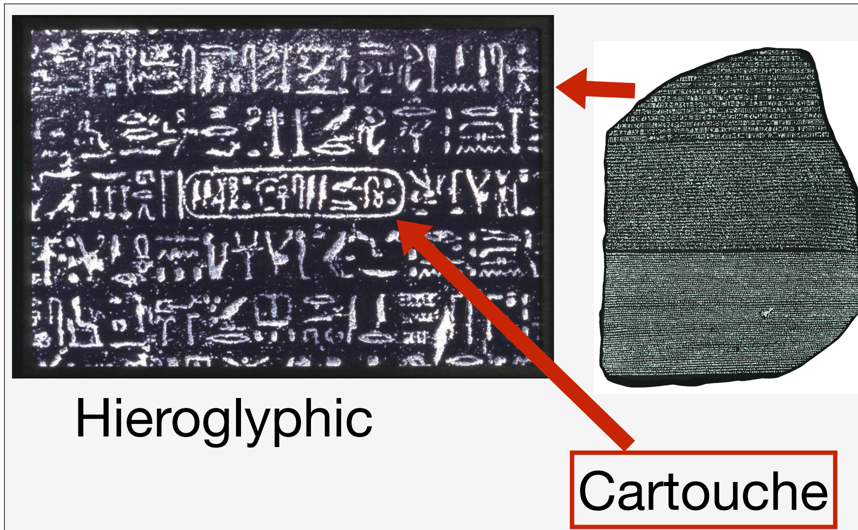
The Rosetta Stone





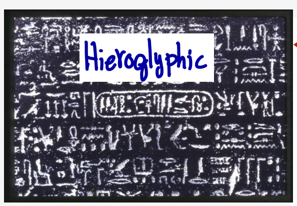


What languages are used in the Rosetta stone?

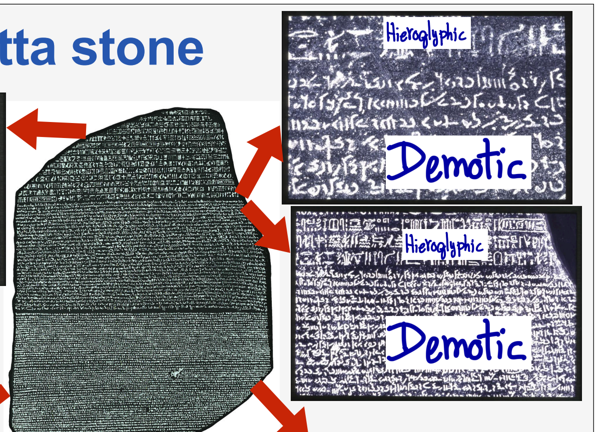


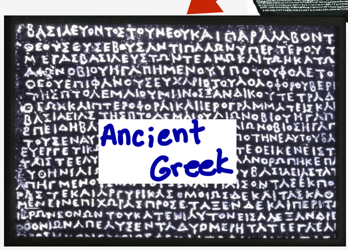
Royal names were written inside cartouches (oval shape) to set them apart.

The Rosetta stone



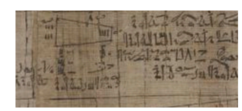
Royal names written inside cartouches were the first words deciphered.





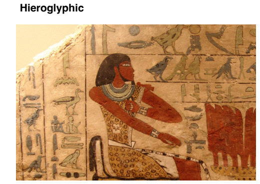
Over their long history, Egyptians developed more than one writing system. Some were used at the same time.

Hieratic - Rhind Papyrus



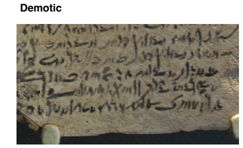
https://www.britishmuseum.org/collection/object/Y_EA10057

Hieroglyphic

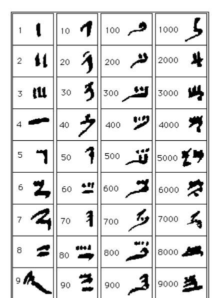


By Djehouty - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=90222138

Demotic

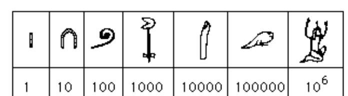


https://commons.wikimedia.org/wiki/File:Demotic_Ostrakon.jpg



Hieratic numerals

https://mathshistory.st-andrews.ac.uk/HistTopics/Egyptian_numerals/



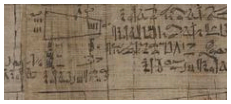
Egyptian numeral hieroglyphs

Early labels → Hieroglyphic → Hieratic → Demotic → Coptic

Egyptian Writing - Rough Chronology

- **c. 3300–3000 BCE** First writing: Naqada III labels/tags, numbers (administrative)
- **c. 3000** BCE Hieroglyphic (monumental) + Hieratic (cursive, everyday)
- **c. 700 BCE** Demotic (a very simplified, later cursive development of hieratic, administrative)
- **c. 0 CE Coptic** (Greek alphabet + a few signs from Demotic)
- **c. 400–500 CE** Last hieroglyphic/demotic inscriptions
- **c. 1000 CE** Coptic survives (mainly liturgical)

Hieratic - Rhind Papyrus

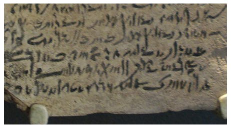


https://www.britishmuseum.org/collection/object/Y_EA10057

Hieroglyphic

By Djehouty - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=90222138

Demotic



https://commons.wikimedia.org/wiki/File:Demotic_Ostrakon.jpg

Early labels → Hieroglyphic → Hieratic → Demotic → Coptic

Decipherment of the Rosetta Stone

- **1799** Rosetta Stone discovered during the French campaign in Egypt (led by Napoleon); Greek text used as key to Egyptian texts
- **Early 1820s** Champollion: cartouches (Ptolemy, Cleopatra) show hieroglyphs include phonetic values (not only ideographic elements)
- **Early 1820s** Young: makes major progress on demotic using Rosetta Stone + parallel Greek/demotic texts
- **1825–1850** Further progress; system established; decipherment largely secured

Rough History of the Rosetta Stone

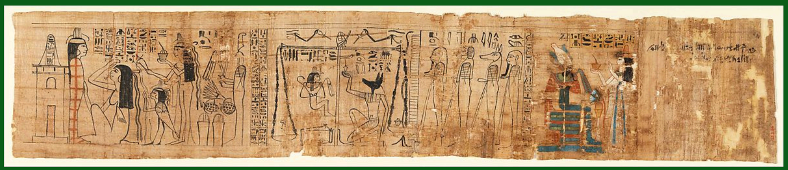
- **196 BCE:** decree issued by Egyptian priests for Ptolemy V (a teenager)
  → declares him a living god (standard for pharaohs)
  → result of political negotiation with powerful priesthood
- Decree written in **three scripts/languages** (hieroglyphic, demotic, Greek)
- Survived ~2000 years unread under many rulers (Romans, Byzantines, Persians, Arabs, Ottomans)
- **1799**: found in Rosetta during French campaign (Napoleon)
- **1801**: taken by the British after the French defeat
  → inscription: “Captured by the British Army… Presented by King George III”
- Today is in the British Museum (London)

https://www.bbc.co.uk/programmes/articles/3dtD0Nr8tJPgH7pXwt302Ys/episode-transcript-episode-33-rosetta-stone

Men Splitting Papyrus (public domain)
A.D. 1914–1916, original ca. 1479–1458 B.C

This is a photo of Papyrus growing wild along the banks of the Nile River in Uganda. It was taken by Michael Shade in the fall of 2006.

Papyrus



Book of the Dead Papyrus of Tiyeca. 975–945 B.C. Discovered tucked inside her hollow wooden "Osiris figure" (now Cairo JE 49164) this papyrus was designed to assist Tiye in her successful transition from death to eternal life. - (Public domain)

Papyri, reading, writing and math

- Fragile and survived unevenly.
- Production was labor-intensive and writing materials were valuable.
- Only about 10 mathematical papyri have survived
- Most are fragmentary; the main substantial exceptions are the Rhind and Moscow papyri.
- Ancient Egyptian literacy was probably low overall, but estimates vary and are debated.

Aristotle writes (Metaphysics):
“Thus the mathematical sciences originated in the neighborhood of Egypt, because there the priestly class was allowed leisure.”

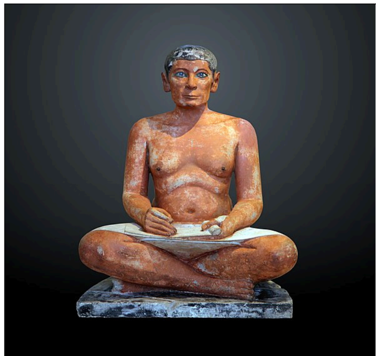
Scribes

Funerary model of a granary with a scribe recording amounts of grain stored or issued, Egypt, Middle Kingdom (about 2055–1795 BC).

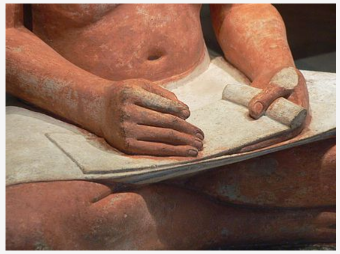
Scribes were trained in reading, writing, and calculation.
They were essential to administration in ancient Egypt.

Funerary model of a granary - British Museum

https://www.britishmuseum.org/blog/learn-maths-egyptian-secrets-rhind-mathematical-papyrus



The seated scribe ~2500 BCE



mage https://commons.wikimedia.org/

Scribes recorded goods, labor, and taxes.

They were readers, writers, and accountants.

In tomb scenes and models, scribes often appear recording quantities.

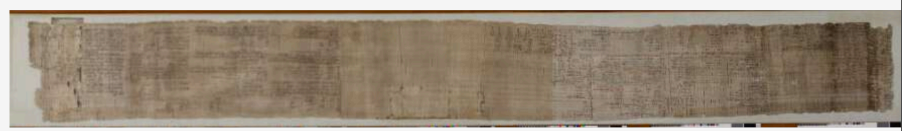
Katz, Victor J., ed. The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook. Princeton University Press, 2007.

A fragment from Papyrus Anastasi - A fictional letter, which forms part of a debate between two scribes

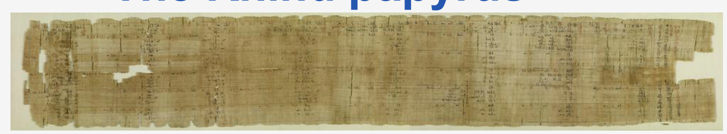
You are told: "**Empty the magazine that has been loaded with sand** under the monument for your lord—may he live, prosper, and be healthy—which has been brought from the Red Mountain. **It makes 30 cubits stretched upon the ground with a width of 20 cubits,** passing chambers filled with sand from the riverbank. The **walls of its chambers have a breadth of 4 to 4 to 4 cubits.** It has a **height of 50 cubits in total.** [...] You are commanded to find out what is before it. **How many men will it take to remove it in 6 hours if their minds are apt?** Their desire to remove it will be small if (a break at) noon does not come. You shall give the troops a break to receive their cakes, in order to establish the monument in its place. One wishes to see it beautiful.

The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook By Annette Imhausen, Eleanor Robson, Victor J. Katz, Victor J. Katz, Annette Imhausen · 2007, page 11

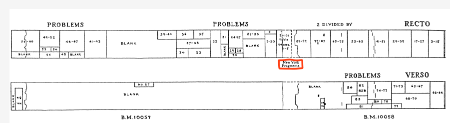
The Moscow Rhind and Papyri



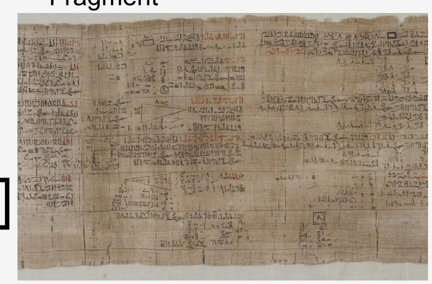
The Rhind papyrus



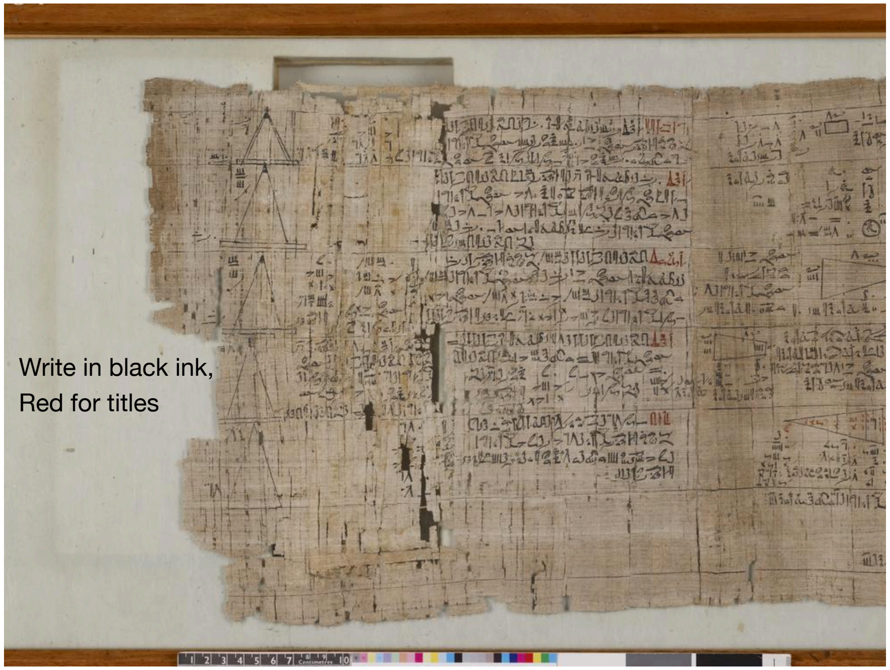
Fragment



13 inches high and 18 feet long!



https://www.britishmuseum.org/collection/object/Y_EA10057



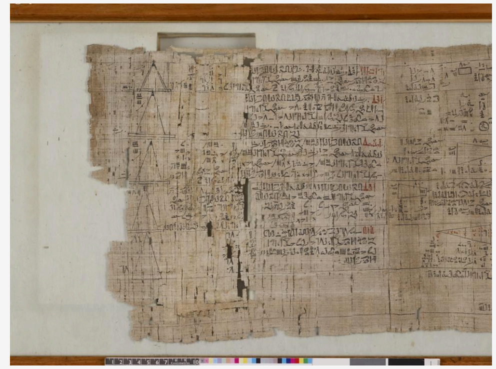
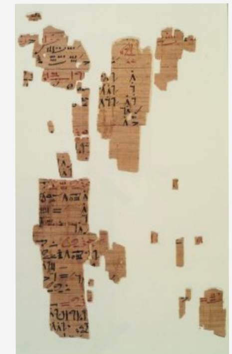
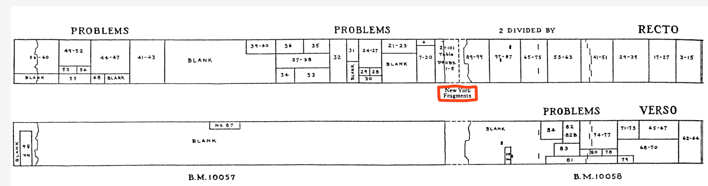


Image from the British Museum
https://www.britishmuseum.org/collection/image/366145001



Fragments of Rhind Mathematical Papyrus, ca. 1493-1481 B.C.E. Papyrus, ink, Largest Fragment: 6 5/16 x 3 3/8 in. (16 x 8.5 cm). Brooklyn Museum, Charles Edwin Wilbour Fund, 37.1784Ea-b (Photo: Brooklyn Museum, 37.1784E_negA_bw_IMLS.jpg)



Rhind papyrus recent history

- It was acquired by the Scottish lawyer A.H. Rhind in Thebes in about 1858.
- Evidence indicate that these fragments were found in a chamber of a ruined building
- The two sections in the British Museum were linked by a missing section about 18 cm long; the original may have been cut in half by modern robbers to increase its sale value.
- Fragments which partly fill this gap were identified in 1922, in the collection of the **New York Historical Society,** which had acquired them from Edwin Smith. Smith also acquired a surgical papyrus of about the same date as the Rhind Papyrus, suggesting that these two documents could have come from a cache of early New Kingdom manuscripts.

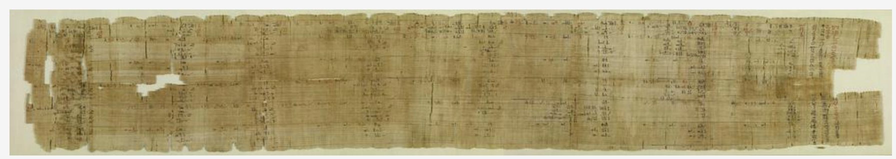
Adapted Fromm the British Museum curator's comments about the Ahmes or Rhind Papyrus

https://www.britishmuseum.org/collection/object/Y_EA10058

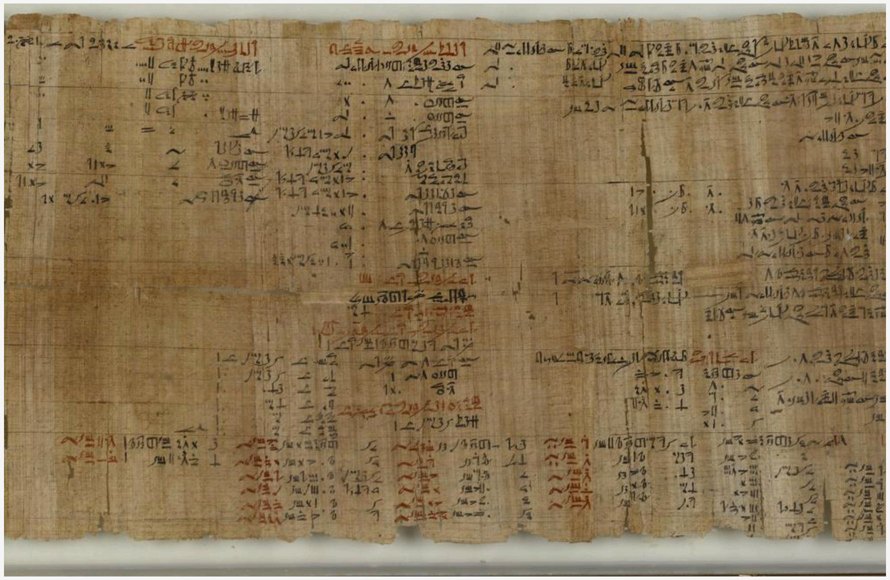
The Rhind papyrus beginning

Accurate reckoning for inquiring into things, insight into all that exists, knowledge of all obscure secrets. This book was copied in regnal year 33, month 4 of Akhet, under the majesty of the King of Upper and Lower Egypt, Awserre, given life, from an ancient copy made in the time of the King of Upper and Lower Egypt Nimaatre. The scribe Ahmose writes this copy.

thorough study of all things, insight into all that exists, knowledge of all obscure secrets.



- "Ahmes" or "Ahmose " is writing it about 1550 BC but that he had copied it from "ancient writings" (1800 BC or earlier).
- "Ahmes" or "Ahmose " is the earliest personal name known to us on the history of mathematics.



Three types of problem

- pure mathematical problems teaching basic techniques
- practical problems, containing an additional layer of knowledge from their respective practical setting
- non-utilitarian problems, which are phrased with a pseudo-daily life setting without having a practical application (only very few examples extant)

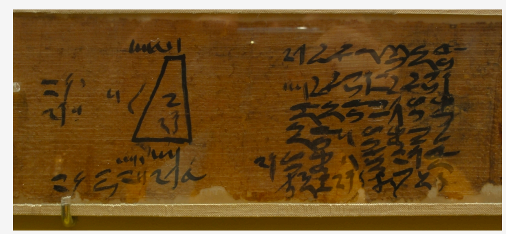
  - No symbols (like + or -)
  - No variables (like x)
  - Algorithmic: a list of concrete instructions to solve them

The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook By Annette Imhausen, Eleanor Robson, Victor J. Katz, Victor J. Katz, Annette Imhausen · 2007

- Table 2/n as a sum of parts, n=3 to 103
- divisions of a certain number of loaves of bread among 10 men
- addition of fractions, summing up to 1.
- solution of linear equations (but not as we understand them)
- unequal distribution of goods and other problems
- find the volume of cylindrical and rectangular granaries.
- show how to compute an assortment of areas
- slopes (of pyramids.)
- multiplications of fractions.
- Value, fair exchange and feeding

The Rhind Papyrus (in present language)

The Moscow Papyrus



Moscow Mathematical PapyrusnnnFotography by Carles Dorce
https://themathematicaltourist.wordpress.com/2012/10/19/moscow-mathematical-papyrus/

14th problem of the Moscow Mathematical Papyrus

Dimensions of the Moscow Papyrus
Length: 5.5 metres (18 ft)
Width: 3.8 to 7.6 cm (1.5 to 3 in)

Multiplication in Ancient Egypt

Explain what it means to multiply a positive integer A by another positive integer B. (If you prefer, you may explain it with an example, say A=20 and B=45)

Ancient Egyptian Multiplication

Find the product of A and B

- Set up two columns: write 1 in the left column and B in the right column.
- Double both numbers until the left column exceeds A.
- Mark the rows in the left column whose values sum to A, and mark the corresponding right-column values (hint: start from the bottom row).
- Add the marked right-column values; their sum is the product A·B.

A.B

1	B
2	2B
4	4B
…	…
2^n	2^nB

n is the largest integer such that $2^n < A$

EXAMPLE: A=45, B=20

Ancient Egyptian Multiplication

Find the product of A and B

- Set up two columns: write 1 in the left column and B in the right column.
- Double both numbers until the left column exceeds A.
- Mark the rows in the left column whose values sum to A, and mark the corresponding right-column values (hint: start from the bottom row).
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A.B

1	B
2	2B
4	4B
…	…
2^n	2^nB

n is the largest integer such that $2^n < A$

EXAMPLE: A=45, B=20

1	20
2	40
4	80
8	160
16	320
32	640
STOP	

20.45 = 640+160+80+20 = 900

Mark all the numbers in the 1-column that add up to B??

Exercise: Multiply 20 by 45 as we just did, but exchanging the roles of A and B.

EXAMPLE:

1	45

Multiply 25 by 26

ANOTHER EXAMPLE:

Recall that **a number and the representation of the number** in a number system are two different concepts. There are many ways to represent a given number, but each number is "unique".

In a similar way, **multiplication and how multiplication is performed are different concepts**. Again, there are many algorithms, that is, many ways to multiply two numbers. But the meaning of multiplication is only one.

what ≠ how

However,

Rhind Mathematical Papyrus, problem 69

.	80
\ 10	800
2	160
\ 4	320
Total	1120

The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook By Annette Imhausen, Eleanor Robson, Victor J. Katz, Victor J. Katz, Annette Imhausen · 2007, page 11

Division in Ancient Egypt

Ancient Egyptian division

Example: Find the quotient of 130 divided by 10 using the Egyptian method. Think of it as solving 10 × x = 130.

- Set up two columns: write 1 in the left column and 10 in the right.
- Double both numbers until the right-column value exceeds 130.
- Mark the rows in the right column whose values sum to 130; mark the corresponding rows in the left column.
- Add the marked numbers in the left column. This sum is the quotient 130 ÷ 10.

Ancient Egyptian division

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- Double both numbers until the right-column value exceeds 130.
- Mark the rows in the right column whose values sum to 130; mark the corresponding rows in the left column.
- Add the marked numbers in the left column. This sum is the quotient 130 ÷ 10.

1	10
2	20
4	40
8	80

80< 130 but
2.80=160 >130

130= 80 + 40 +10
130/10= 8+4+1

Ancient Egyptian division

Find the quotient of A divided by B using the Egyptian method. Think of it as solving B × x = A.

- Set up two columns: write 1 in the left column and B in the right.
- Double both numbers until the right-column value exceeds A.
- Mark the rows in the right column whose values sum to A; mark the corresponding rows in the left column.
- Add the marked numbers in the left column. This sum is the quotient A ÷ B.

1	B
2	2B
4	4B
…	…
2^n	2^nB

n is the largest integer such that $2^nB<A$

Exercise: Find the quotient of 420 divided by 35

EXAMPLE:

1	35
2	70
4	140
8	280

420=
280+140

420/35=
8+4=12

Find the quotient of 561 divided by 17.

ANOTHER EXAMPLE:

The fundamental operations of Egyptian arithmetic are adding and doubling.

Fractions (parts) in Ancient Egypt

Examples of fractions: 1/2, 3/2, 1/3, 2/3, 1/4, 20/501….

Examples of parts: 1/2, 1/3, 1/4, 1/5,…

Ancient Egyptian Fractions: Parts

- A **part** is a fraction of the form 1/n
  - *Examples*: 1/2, 1/3, 1/10.
- Egyptians used
  - whole numbers (1,2,3..),
  - parts (1/2, 1/3, 1/4..), and
  - the special fraction 2/3.
- All other fractions were written as a sum of distinct parts.

Example:
5/6 was written as 1/2 + 1/3 (with no + sign!)

For clarity, we will write parts as 1/n with + signs.

2/n table from the Rhind Mathematical Papyrus

2/3 = 1/2 + 1/6	2/5 = 1/3 + 1/15	2/7 = 1/4 + 1/28
2/9 = 1/6 + 1/18	2/11 = 1/6 + 1/66	2/13 = 1/8 + 1/52 + 1/104
2/15 = 1/10 + 1/30	2/17 = 1/12 + 1/51 + 1/68	2/19 = 1/12 + 1/76 + 1/114
2/21 = 1/14 + 1/42	2/23 = 1/12 + 1/276	2/25 = 1/15 + 1/75
2/27 = 1/18 + 1/54	2/29 = 1/24 + 1/58 + 1/174 + 1/232	2/31 = 1/20 + 1/124 + 1/155
2/33 = 1/22 + 1/66	2/35 = 1/30 + 1/42	2/37 = 1/24 + 1/111 + 1/296
2/39 = 1/26 + 1/78	2/41 = 1/24 + 1/246 + 1/328	2/43 = 1/42 + 1/86 + 1/129 + 1/301
2/45 = 1/30 + 1/90	2/47 = 1/30 + 1/141 + 1/470	2/49 = 1/28 + 1/196
2/51 = 1/34 + 1/102	2/53 = 1/30 + 1/318 + 1/795	2/55 = 1/30 + 1/330
2/57 = 1/38 + 1/114	2/59 = 1/36 + 1/236 + 1/531	2/61 = 1/40 + 1/244 + 1/488 + 1/610
2/63 = 1/42 + 1/126	2/65 = 1/39 + 1/195	2/67 = 1/40 + 1/335 + 1/536
2/69 = 1/46 + 1/138	2/71 = 1/40 + 1/568 + 1/710	2/73 = 1/60 + 1/219 + 1/292 + 1/365
2/75 = 1/50 + 1/150	2/77 = 1/44 + 1/308	2/79 = 1/60 + 1/237 + 1/316 + 1/790
2/81 = 1/54 + 1/162	2/83 = 1/60 + 1/332 + 1/415 + 1/498	2/85 = 1/51 + 1/255
2/87 = 1/58 + 1/174	2/89 = 1/60 + 1/356 + 1/534 + 1/890	2/91 = 1/70 + 1/130
2/93 = 1/62 + 1/186	2/95 = 1/60 + 1/380 + 1/570	2/97 = 1/56 + 1/679 + 1/776
2/99 = 1/66 + 1/198	2/101 = 1/101 + 1/202 + 1/303 + 1/606	

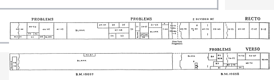

Image Credit: Wikipedia

2/n	Decomposition
2/3	1/2 + 1/6
2/5	1/3 + 1/15
2/7	1/4 + 1/28
2/9	1/6 + 1/18
2/11	1/6 + 1/66
2/13	1/8 + 1/52 + 1/104
2/15	1/10 + 1/30
2/17	1/12 + 1/51 + 1/68
2/19	1/12 + 1/76 + 1/114

2/n	Decomposition
2/21	1/14 + 1/42
2/23	1/12 + 1/276
2/25	1/15 + 1/75
2/27	1/18 + 1/54
2/29	1/24 + 1/58 + 1/174 + 1/232
2/31	1/20 + 1/124 + 1/155
2/33	1/22 + 1/66
2/35	1/30 + 1/42
2/37	1/24 + 1/111 + 1/296

It is the year 1500 BCE. You are a scribe in Egypt

1. Multiply 1/6 by 17.

2/n	Decomposition
2/3	1/2 + 1/6
2/5	1/3 + 1/15
2/7	1/4 + 1/28
2/9	1/6 + 1/18
2/11	1/6 + 1/66
2/13	1/8 + 1/52 + 1/104
2/15	1/10 + 1/30
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2/11	1/6 + 1/66
2/13	1/8 + 1/52 + 1/104
2/15	1/10 + 1/30
2/17	1/12 + 1/51 + 1/68
2/19	1/12 + 1/76 + 1/114

1	1/6
2	1/3
4	2/3=1/2+1/6
8	1+1/3
16	2+1/2+1/6
17(1/6)=	2+1/2+1/6+1/6
17(1/6)=	2+1/2+1/3

It is the year 1500 BCE. You are a scribe in Egypt

Multiply 1/5 by 17.

2/n	Decomposition
2/3	1/2 + 1/6
2/5	1/3 + 1/15
2/7	1/4 + 1/28
2/9	1/6 + 1/18
2/11	1/6 + 1/66
2/13	1/8 + 1/52 + 1/104
2/15	1/10 + 1/30
2/17	1/12 + 1/51 + 1/68
2/19	1/12 + 1/76 + 1/114

It is the year 1500 BCE. You are a scribe in Egypt

2. Multiply 1/5 by 17.

2/n	Decomposition
2/3	1/2 + 1/6
2/5	1/3 + 1/15
2/7	1/4 + 1/28
2/9	1/6 + 1/18
2/11	1/6 + 1/66
2/13	1/8 + 1/52 + 1/104
2/15	1/10 + 1/30
2/17	1/12 + 1/51 + 1/68
2/19	1/12 + 1/76 + 1/114

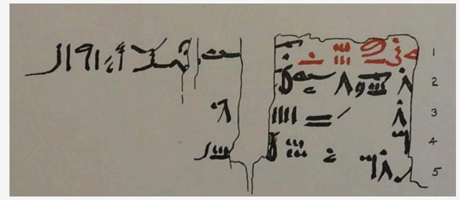
61

1	1/5
2	2/5=1/3+1/15
4	1/2+1/6+1/10+1/30
8	1+1/3+1/5+1/15
16	2+1/2+1/6+1/3+1/15+1/10+1/30
17(1/5)=	2+1/2+1/3+1/5+1/6+1/10+1/5+1/30

Problem 3 of the Rhind Mathematical Papyrus:

Divide 6 loaves among 10 men.

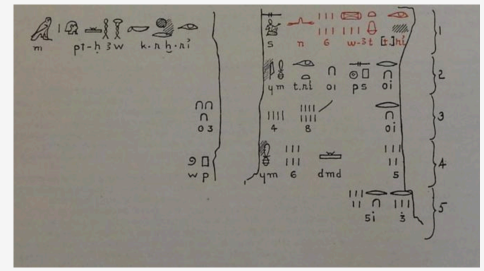
Copy of Sector of Rhind Papyrus (Problem 3)



https://archive.org/details/arnoldbuffumchaceludlowbullhenryparkermanningtherhindmathematicalpapyrus.volumei/page/n117/mode/1up

Bread bun in hieroglyphs

Translation to hieroglyphs of Problem 3



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1. Break 5 loaves in two. Each man gets 1/2
2. Break the remaining loaf in 10, each man gets 1/10.

Total: 1/2+1/10. Note **use of the parts!**

Method of false position

We will start with a computation we will need later.

Exercise: Find the “Egyptian” quotient of 19 divided by 8

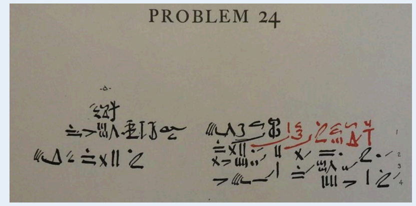
Exercise: Find the “Egyptian” quotient of 19 divided by 8

1	8
2	**16**
1/2	4
1/4	**2**
1/8	**1**

Quotient: 2+1/4+1/8

Problem 24 of the Rhind Mathematical Papyrus:

A quantity and its 1/7 added together become 19.
What is the quantity?

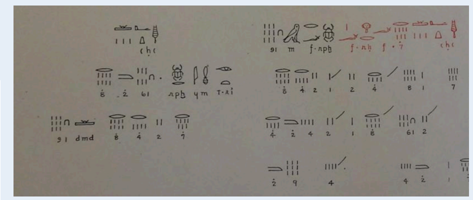


https://archive.org/details/arnoldbuffumchaceludlowbullhenryparkermanningtherhindmathematicalpapyrus.volumei/page/n141/mode/1up?view=theater

Using modern algebra, solve for the quantity.

Now, try to solve this problem without “x” or modern algebra

Translation to hieroglyphic notation



https://archive.org/details/arnoldbuffumchaceludlowbullhenryparkermanningtherhindmathematicalpapyrus.volumei/page/n141/mode/1up?view=theater

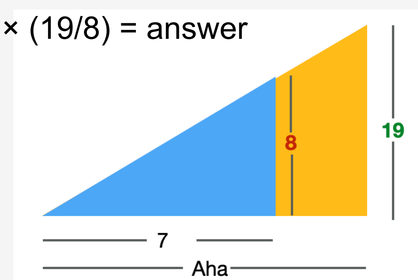
Method of False Position (aha)

Problem 24 of the Rhind Mathematical Papyrus:
A quantity and its 1/7 added together become **19**.
What is the quantity?

- **Step 1:** Pick a convenient number (here: 7)
- **Step 2**: Plug in 7 → 7 + 1/7 of 7 = **8**
- **Step 3**: Find the scale factor that turns **8** into **19** → 19/8.
- **Step 4:** Scale the guess. 7 × (19/8) = answer

21st century Hint:
$7/8 = x/19 \Rightarrow x = 7\cdot(19/8)$

ꜥḥꜣ (aha): Egyptian word for quantity or number.



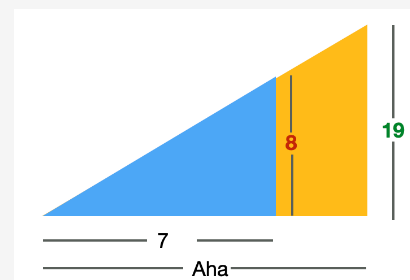
Method of False Position (aha)

Problem 24 of the Rhind Mathematical Papyrus:
A quantity and its 1/7 added together become **19**.
What is the quantity?

Step 1: Assume 7

7	7
1/7	1
Total	8

Hint: when one plugs in 7, gets 8.
What does one have to plug in to get 19?



Problem 24 of the Rhind Mathematical Papyrus:

A quantity and its 1/7 added together become 19. What is the quantity?

Assume 7.

\1	7
\⅐	1
Total	8.
1	8
\2	16
½	4
\¼	2
\⅛	1
Total 2 ¼ ⅛.	
\1	2 ¼ ⅛
\2	4 ½ ¼
\4	9 ½
Do it thus: The quantity is	16 ½ ⅛,
⅐	2 ¼ ⅛,
Total	19.

First, let's solve the problem in the "spirit of ancient Egypt" but using modern notation and language.
To obtain 8, the quantity is 7. (We choose 7 because it is easy to compute and then we obtain 8)

To obtain 19, what is the quantity?
We have the ratio,
19/8 = x/7 where x is the number we are trying to find.

By proportionally, we know that to obtain 19, x - the quantity we are looking for- is 7 multiplied by 19/8.
Before we found that 19/8 =2+1/4+1/8

Hence, the answer is
7(2+1/4+1/8)=16+1/2+1/8

Method of false position

Problem 25 of the Rhind Mathematical Papyrus:

A quantity and its 1/2 added together become 16.
What is the quantity?

Method of false position

Problem 31 of the Rhind Mathematical Papyrus:

A quantity, its 2/3, its 1/2, and its 1/7 added together become 33. What is the quantity?

The total is

14 1/4 1/56/ 1/97 1/194 1/388 1/679 1/776,

which multiplied by 1 2/3 1/2 1/7 makes 33.

Method of false position

Ideas of Length, Area and Volume

1.How can you find the length of a segment?
2.What does it mean to "*measure*" a segment?
3.(Optional) What does it mean for two curves to have the same length?

Answer 1. or 2.
Hint: use a concrete example.

Take 1 minute to write down your first thoughts, then turn to someone you never talked to before and compare ideas.

1. How do you measure a shape? (area or volume)
2. What does "measurement" mean?
3. What does it mean for two shapes to have the same measurements?

Choose Area or Volume and answer 1, 2, or 3.
You may use practical examples or procedures.

Take 1 minute to write down your first thoughts, then turn to someone you never talked to before and compare ideas.

The same amount unit squares can be rearranged to occupy both planar shapes

that they are the same size but can be different shapes

What does it mean for two planar shapes to have the same area?

That means we can put the same amount of water or some kind of unit into two planar shapes

They must be congruent.

identical

The measurement of area agrees

Here are some answers from students from previous years. Some right, some wrong, some do not answer the question..can you tell which is which?

Measuring a segment means comparing its length with that of a chosen unit, and finding how many times the unit "fits" into the segment. (The unit may not "fit" into the segment an integer or fractional number of times.)

Similarly, **measuring a planar figure** means finding how many times a given unit of area (and/or fractions of that unit) "fits" into the figure.

Finally, **measuring a solid** means finding how many times a given unit of volume (and/or fractions of that unit) "fits" into the solid.

What do you think the scissor congruence app shows?

https://dmsm.github.io/scissors-congruence/

Scissor CongruenceTheorem (Wallace–Bolyai–Gerwien): If two polygons have the same area, then it is possible to cut one of them into polygonal pieces and rearrange these pieces to form the other.

(Note: The number of pieces is finite here)

Two shapes of a given kind are ***scissor congruent*** if one can be cut into finitely many pieces of that kind, which can then be rearranged to form the other.

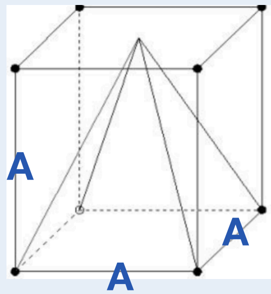
Measurement in Egypt: length, area, volume

- Volumes of
  - Cylindrical containers. 
  - Rectangular parallelepipedal containers. 
  - Truncated pyramid 
- Areas of
  - Rectangles
  - Circles
  - Triangles
  - Trapezoids
- Division of given area of land into equal-sized fields.
- A quantity related to what we now call slope

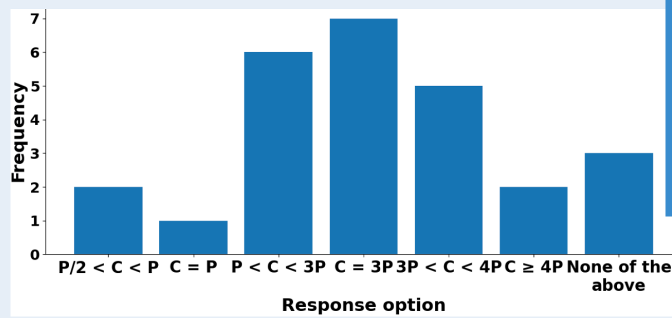
The Moscow Papyrus: Volume of the truncated pyramid

P = volume of a square pyramid with base side A and height A.
C = volume of a cube with side A (So C = A^3).
Question: Which statement is correct?

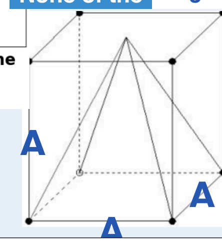
1. C < P/2
2. C = P/2
3. P/2 < C < P
4. C = P
5. P < C < 3P
6. C = 3P
7. 3P < C < 4P
8. C ≥ 4P
9. None of the above



P = volume of a square pyramid with base side A and height A.
C = volume of a cube with side A (So C = A^3).
Question: Which statement is correct?



Response	Freq
P/2 < C < P	2
C = P	1
P < C < 3P	6
C = 3P	7
3P < C < 4P	5
C ≥ 4P	2
None of the	3



Answers: Sp2026

What is the volume of a pyramid of square base?

Image credit: wikimedia, by Emőke Dénes

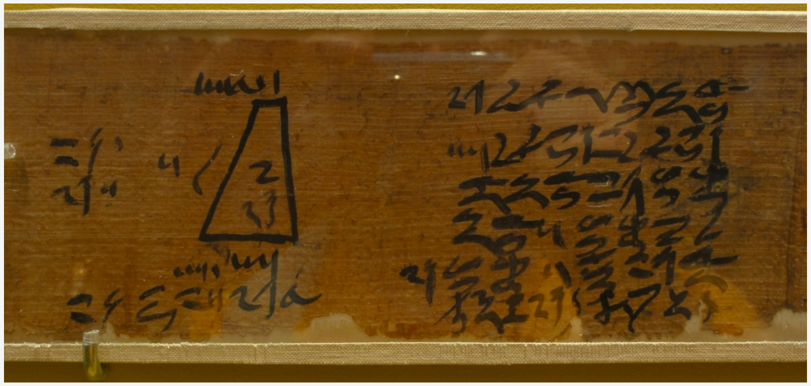
Demo

The Great Pyramid of Giza

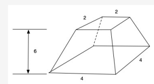
We did a demonstration in which we saw that a cube can be filled exactly with three squared pyramids (of square base of same side length and height as the cube).

Does the demonstration constitute a mathematical proof that the volume of the cube is three times the volume of the pyramid?

Problem 14 of the Moscow Mathematical Papyrus



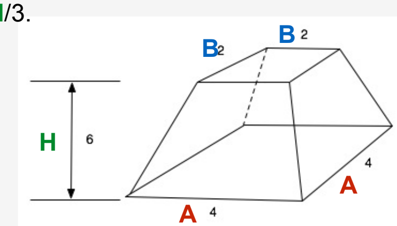
Carles Dorce photographed problem 14 of the Moscow Mathematical Papyrus



If someone says to you: a truncated pyramid of 6 for the height and by 4 on the base by 2 on the top. You are two square this 4; the result is 16. You are to double 4; the result is 8. You are to square this 2; the result is 4. You are to add the 16 and the 8 and the 4; the result is 28. You have to take 1/3 of the 6 the result is 2. You have to take 28 two times; the result is 56. Behold, the volume is 56. You will find that this is correct.

Problem 14 of the Moscow Mathematical Papyrus

1. If someone says to you:
2. a truncated pyramid of H for the height and by A on the base by B on the top.
3. You are two square this A; the result is A^2.
4. You are to multiply B by A; the result is A.B.
5. You are to square this B; the result is B^2
6. You are to add the A^2 and the A.B and the B^2; the result is $A^2 + A.B + B^2$.
7. You have to take 1/3 of the H the result is H/3
8. You have to multiply $(A^2 + A.B + B^2)$ by H/3; the result is $(A^2 + A.B + B^2)H/3$. Behold, the volume is $(A^2 + A.B + B^2)H/3$.
9. You will find that this is correct.

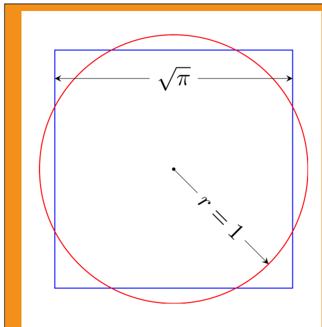


Problem 14 of the Moscow Mathematical Papyrus

Conjecture: to find the formula the truncated pyramid was broken into pieces.

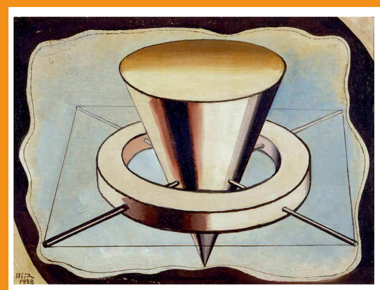
- Note: This 3D "cut and paste" is a fundamental property of volume.

The Rhind Papyrus Problem 50 Area of the circle Approximation of π



By 蔡蜜 - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=132030577

Squaring the circle



La Quadrature, 1938. Oil on panel Man Ray

Problem 50 of Rhind or Ahmes papyrus

Example of a round field of diameter 9 khet What is its area"

Take away ⅑ of the diameter, namely 1; the remainder is 8. Multiply 8 times 8; it makes 64. Therefore it contains 64 *setat* of land.

Do it thus:

1	9
⅑	1;

this taken away leaves 8

1	8
2	16
4	32
\8	64.

Its area is 64 *setat.*

A ***khet*** is unit of length (about 50 meters.)

A ***setat*** is a unit of area (one khet squared)

Problem 50 of Rhind or Ahmes papyrus

Find the formula of the area of the circle that the scribe would have obtained by starting with a circle of diameter d, instead of a circle of diameter 9. (Hint: Start by taking away 1/9 of the diameter, that is d/9.)

Example of a round field of diameter 9 khet. *What is its area?*

Take away ⅑ of the diameter, namely 1; the remainder is 8. Multiply 8 times 8; it makes 64. Therefore it contains 64 *setat* of land.

Do it thus:

1	9
⅑	1;

this taken away leaves 8

1	8
2	16
4	32
\8	64.

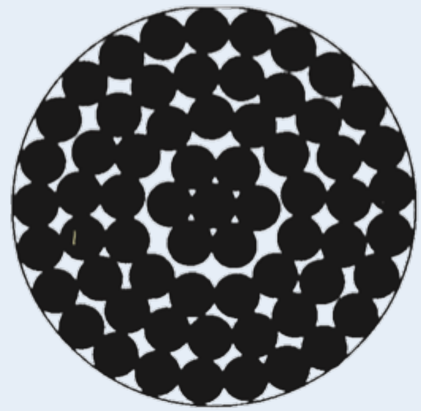
Its area is 64 *setat.*

Problem 50 of Rhind or Ahmes papyrus

The area of a disk is a constant (π) times the radius of the circle squared. What is the value of π that Egyptians assumed in their computation of the area of the disk? (in the problem we are discussing)

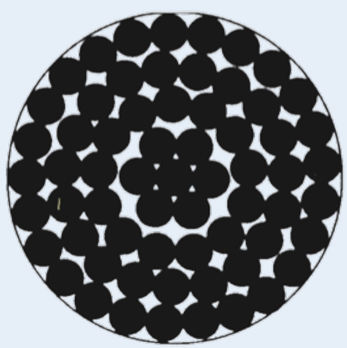
More about the Rhind Papyrus Problem 50: Conjectures about how the area of the circle was found.

1) How many small disks are inside the large disk?
2) Along the center line of the large disk, how many small-disk diameters fit across the diameter?

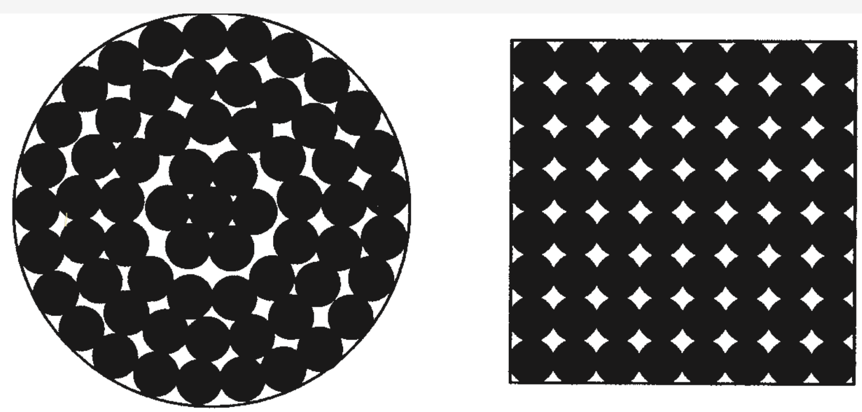


1) Using the same 64 small disks, how many small-disk diameters are along one side of the square built from them?

2) What relationship do you see between the circle's diameter and the side of the square made from the same disks?

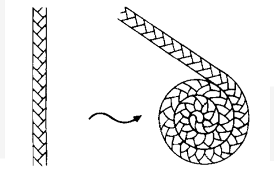


The previous discussion suggests: the circle’s area equals the area of a square whose side is 8/9 of the diameter



Problem 50 of Rhind or Ahmes papyrus
Another conjecture of how the area was found

Design found in several places in Africa, for instance
 carved in wooden doors in Nigeria.
in fabrication of baskets in Egypt
Also in burial sites in Ancient Egypt.



Gerdes, Paulus. "Three alternate methods of obtaining the ancient Egyptian formula for the area of a circle." *Historia Mathematica* 12.3 (1985): 261-268. https://www.sciencedirect.com/science/article/pii/0315086085900242

The Rhind Papyrus Problem 79: 7 houses, 49 cats, 343 mice. Math for math sake?

As I was going to Saint Ives,
I met a man with seven wives.
Every wife had seven sacks,
Every sack had seven cats,
Every cat had seven kits;
Kits, cats, sacks and wives,
How many were there going to Saint Ives?

Mother Goose

https://youtu.be/7vtszdW8MTs?t=11. 2:30 minutes

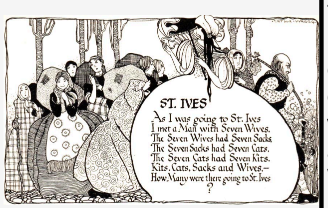


Image credti: https://www.cornwalls.co.uk/st-ives/as-i-was-going-to-st-ives

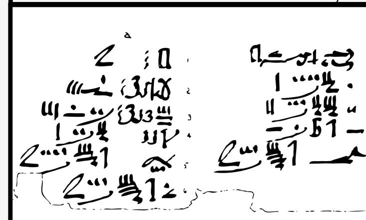
There are seven old women on the road to Rome.
Each woman has seven mules;
each mule carries seven sacks;
each sack contains seven loaves;
with each loaf are seven knives;
and each knife is in seven sheaths.
Women, mules, sacks, loaves, knifes, and sheaths, how many are there in all on the road to Rome?
(Translation from Fibonacci's Liber Abacci)

$7 + 7^2 + 7^3 + 7^4 + 7^5 =$
$7(1 + 7 + 7^2 + 7^3 + 7^4)=$
$7 . 2801 =19{,}607$

Rhind Papyrus Problem 79

A house inventory

		houses	7
1	2,801	cats	49
2	5,602	mice	343
4	11,204	spelt	2,301
Total	19,607	hekat	16,807
		Total	19,607



Possible interpretation

In each of the seven houses there are seven cats: each cat kills seven mice; each mouse would have eaten seven sheaves of wheat; and each sheaf of wheat was capable of yielding seven hekat measures of grain. How much grain was thereby saved?

Conclusions

A careful study of the Rhind Papyrus convinced me several years ago that this work is not a mere selection of practical problems especially useful to determine land values, and that the Egyptians were not a nation of shopkeepers, interested only in that which they could use. Rather I believe that they studied mathematics and other subjects for their own sakes.

The Rhind Mathematical Papyrus. By A. B. Chace, L. Bull, and H. P. Manning. Vol. I 1927; Vol. II 1929. $20. (Mathematical Association of America.)

After working on the Rhind Papyrus, with which of the two paragraphs you agree more and why?

The Rhind and Moscow papyri are handbooks for the scribe, giving model examples of how to do things which were a part of his everyday tasks . . . The sheer difficulties of calculation with such a crude numeral system and primitive methods effectively prevented any advance or interest in developing the science for its own sake. It served the needs of everyday life . . . and that was enough.

Mathematics and Astronomy, J. R. Harris (ed.): The Legacy of Egypt. Second Edition. Pp. xi+510; 24 pls. Oxford: Clarendon Press, 1971.

Extracts from https://www.open.edu/openlearn/science-maths-technology/mathematics-and-statistics/mathematics/egyptian-mathematics/content-section-1.1.3

Answer one or more of the following questions.

1. **What role did writing materials (papyrus, scribes) play in how Egyptians recorded and worked with numbers? (Hint: was papyrus accessible to everyone?)**
2. **What do you think Egyptians needed to calculate areas and volumes for?**
3. **We write $V = (1/3)h(a^2 + ab + b^2)$ for a truncated pyramid's volume (base side b, top side a, height h). How did Egyptians explain how to find this volume?**
4. **How did Egyptians think about multiplication differently than we do? What made their method work?**

Answer one or more of the following questions.

1. **Is there an assumption about ancient mathematics that you had before this unit that turned out to be wrong? If so, which one?**
2. **Egyptian fractions used only unit fractions (1/n). Give a plausible example of how this restriction change the way they had to think about parts.**
3. **Did working on ideas of area and volume change (or not change) your understanding of what these measurements mean?**