MAT 360 Topology

Problem Set 8

due Thursday, April 8

Problem 1. Prove that any isometry S is a bijective mapping. As you know from MAT 200, this has two parts:
(a) Prove that S is injective. (This is easy once you remember the definitions.)
(b) Prove that S is surjective. This can be done as follows: pick any two points A, B, consider their images $S(A) = A'$, $S(B) = B'$. To prove surjectivity, you need to find, for any point X' in the plane, a point X such that $S(X) = X'$. We know that in this case we would have $XA = X'A'$, $XB = X'B'$. Explain how to use this to construct possible candidates for X, and show that your X is indeed mapped to X'.

Problem 2. Let the line l be perpendicular to the segment AB. Let R_l be the reflection through l, $T_{\overrightarrow{AB}}$ translation by $\overrightarrow{AB}$. Find compositions $R_l \circ T_{\overrightarrow{AB}}$ and $T_{\overrightarrow{AB}} \circ R_l$ of these two isometries (in the first composition, translation is followed by reflection; in the second, reflection is performed first). Prove your answers.

Problem 3. Prove that the composition of two translations is a translation (Theorem 3 in the lecture notes).

Problem 4. Prove Theorem 5 in the lecture notes.

Problem 5. Given lines l, m and a point A (not on these lines), find points $B \in l$ and $C \in m$ such that the perimeter of the triangle ABC is the smallest possible

Hint: Reflect the point A through lines l and m, that is, consider the points $B' = R_l(A)$ and $C' = R_m(A)$. Use these auxiliary points to find B and C, and prove that the resulting triangle indeed has the smallest perimeter.