MAT 360 Topology

Problem Set 10
due Thursday, April 28

Problem 1. Prove that the composition of a central symmetry and a reflection is a glide reflection.
Use the same method as for Theorem 10 (composition of rotations) in the notes on isometries.

Problem 2. Prove that
(a) A figure similar to a circle is a circle, i.e. any similarity transformation maps every circle to a circle.
(b) Any circle can be mapped to any other by a similarity transformation, i.e. any two circles are similar.

Problem 3. Prove that
(a) A figure similar to an angle is an angle, i.e. any similarity transformation maps every angle to an angle.
(b) Two angles are similar if and only they are congruent.

Problem 4. Prove the Pythagorean theorem by using similarity, as follows. Let ABC be a right triangle, AD its altitude dropped from the vertex of the right angle A to the hypotenuse BC.
(a) Prove that the triangles ABC, ABD, and ADC are all similar.
(b) Conclude that
$$\frac{BD}{AB} = \frac{AB}{BC}, \qquad \frac{DC}{AC} = \frac{AC}{BC}.$$
(c) Manipulate these equalities using algebra, derive the Pythagorean theorem: $|AB|^2 + |AC|^2 = |BC|^2$.

Please also do problem 393 from the book. Use method of homothety, sections 181-182. (We'll discuss this in class on Tuesday.)