EXTRA PROBLEMS FOR HOMEWORK 8

Consider the following initial-value problem for the wave equation for $t > 0$ and $0 < s < L$:

$$\begin{aligned} y_{tt} &= c^2 y_{ss} \\ y(0,t) &= 0 = y(L.t) \\ y(s,0) &= f_0(s) \\ y_t(s,0) &= f_2(s). \end{aligned}$$

1. For the case when $f_1(s)$ is a function whose graph is an isosceles triangle of width L and height h and $f_2(s) = 0$ find $y(s,t)$ for $s = 0.25L$ and $0.5L$ and for times $ct = 0, 0.2L, 0.4L, 0.8L, 1.4L$.

2. Let $y(s,t)$ be the solution for the case $f_1(s) = 0$ and

$$f_2(s) = \begin{cases} 0, & 0 < s < 0.4L, \\ 5c, & 0.4L < s < 0.6L, \\ 0, & 0.6L < s < L. \end{cases}$$

Sketch $y(s,t)$ as a function of s for times $ct = 0, 0.2L, 0.4L, 0.5L, L, 1.2L$.

3. Sketch the solution $y(s,t)$ at times $ct = 0, 0.1L, 0.3L, 0.4L, 0.5L, 0.6L$, if $f_2(s) = 0$ and

$$f_1(s) = \begin{cases} 0, & 0 < s < 0.4L, \\ 10h(s - 0.4L), & 0.4L < s < 0.5L, \\ 10h(0.6L - s), & 0.5L < s < 0.6L, \\ 0, & 0.6L < s < L. \end{cases}$$