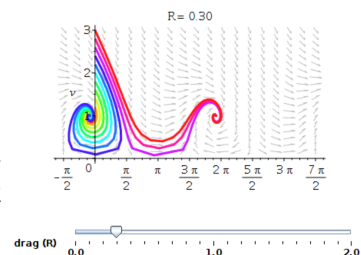


22. (*expires Mar 25*) Some time ago we made an animation of solutions to the Phugoid model as the drag parameter R changes. Modify this to produce a plot of the corresponding picture in the (θ, v) -plane where the value of R is controlled by a slider, similar to Exercise 12. Your plot should change as the slider is moved, and look something like the image at right. You might want to refer to the worksheet [sliderfit.mw](#)



23. (*expires Mar 25*) In this problem we will study the Lotka–Volterra predator–prey equations. In a very simple ecosystem there are two populations, whose numbers at a time t (with t in, say, years) are given by $f(t)$ (foxes) and $r(t)$ (rabbits). The evolution of these quantities obeys the system

$$\begin{cases} \dot{f}(t) = G_f f(t) + E f(t)r(t), \\ \dot{r}(t) = G_r r(t) - E f(t)r(t), \end{cases}$$

where G_f and G_r are the growth rates for the foxes and the rabbits, respectively, in the absence of each other. E is the probability of a fatal encounter between a fox and a rabbit (normalized per number of foxes and rabbits).

First, write some words to explain why these equations make sense. Then, fix $G_f = 0.4$, $G_r = 2.4$ (it is notorious that rabbits have the tendency to reproduce quickly) and $E = 0.01$. For a few initial conditions of your choice, plot the trajectories in the (f, r) -plane (say, with $0 \leq f \leq 1000$ and $0 \leq r \leq 1000$). For the same initial conditions, plot the actual solutions too (i.e., $f(t)$ against t , and $r(t)$ against t). Write some comments interpreting how the behavior of the solutions relates to what happens to the two species. (Here, to plot $f(t)$ against t , you can use the `scene` argument to `DEplot`, or you can use `dsolve` and maybe `plots[odeplot]`.)

Finally, repeat the same procedure with $G_f = -1.1$. Things change substantially. As above, explain how the solution behavior relates to the populations of foxes and rabbits. What does having $G_f = -1.1$ mean in the context of rabbit and fox populations?

24. (*expires Mar 25*) Consider the differential equation corresponding to the vector field

$$F(x, y) = (-x(x^4 + y^4) - y, x - y(x^4 + y^4)).$$

Use Maple to draw either the direction field or the vector field, together with some well-chosen solution curves. (I would use `DEplot` here, but you can use a combination of `fieldplot`, `dsolve` (with the `numeric` option), `plots[odeplot]`, and `plots[display]` if you prefer.)

Then prove that the origin is a global attractor in the future, i.e., for every solution $z(t) = (x(t), y(t))$, we have

$$\lim_{t \rightarrow +\infty} z(t) = 0.$$

Note: The proof is not long, but requires a mathematical argument, not a Maple calculation. The proof may depend on something you calculated in Maple, but more will be needed. Polar coordinates can be your friend.