

9. (expires Feb 18) Given the set of points

$$(0, 3), (1, 0), (2, 4), (3, 4), (4, -1), (6, 4), (7, 0), (9, 2), (10, 10),$$

find the polynomial of degree 8 that passes through all of them. If you wish, you can use `CurveFitting[PolynomialInterpolation]`, or you can calculate all of the relevant equations.

Then find the polynomial of degree 9 which passes through these points and also has a critical point at $x = 6$.

Also find the polynomial of degree 10 passing through the points with critical points at both $x = 1$ and $x = 6$.

Finally, make a plot displaying all three graphs, together with the data points. Be sure that your plot shows the data points clearly (as points, not connected lines), and clearly distinguishes the three functions. Including a legend (see `plot, options`) is one good way to do this.

To avoid typing in the points, you can load them from the web at <https://www.math.stonybrook.edu/~mmovshev/mat331-spr2026/polydata.txt>, which defines a list called `polydata` containing them.

10. (expires Feb 18) Using the same data as the previous problem, find the “natural” cubic spline which interpolates the data.

Also calculate the cubic spline which has derivative 0 at $x = 0$ and $x = 10$, and then make a plot showing the data points and these two curves on the same axes. (You will probably have to read the help page on `Spline Continuity and End Conditions` to see how to adjust the derivatives at the endpoints.)

11. (expires Feb 18) Consider the set of six points (defined as `CSNY` on this page)

$$(-3.1, -1.94), (-2.2, 3.82), (-0.95, -4.05), (0.8, -0.05), (1.3, -0.221), (4.7, -1.30).$$

Find the function of the form

$$f(x) = a \cos(x) + b \sin(x) + k \cos(2x) + d \sin(2x) + g \cos(3x) + h \sin(3x)$$

with appropriate values of the constants a, b, k, d, g, h (correct to at least 6 significant figures) so that $f(x)$ passes through the given points.

Then plot the points and $f(x)$ on the same set of axes for $-2\pi < x < 2\pi$.

12. (expires Feb 18) The worksheet `sliderfit.mw` contains an interactive slider to demonstrate how the interpolating polynomial changes when the y-value of one of the data points is moved.

Modify this worksheet to add another slider which allows the user to also move the x-value of the same point.

Also modify it to include the graph of the corresponding cubic spline in the plot.

Hint: you will need to modify the “startup code” at the top, and right-clicking on the slider to select “Edit Value Changed Code...” will be useful.