Problem 8.

A mass of 2kg is attached to an undamped spring (no gravity). The spring has spring constant of $3\text{kg}/\text{sec}^2$ and is at rest. At time $t = 1$, the spring is hit with a hammer, imparting a unit impulse force. Find the equation of the resulting motion and solve it.

$$m = 2 \quad , \quad k = 3$$

$$2x'' + 3x = 0$$

$$2r^2 + 3 = 0 \quad \Rightarrow \quad r = \pm \sqrt{\tfrac{3}{2}}\, i$$

$$x = c_1 \cos \sqrt{\tfrac{3}{2}}\, t + c_2 \sin \sqrt{\tfrac{3}{2}}\, t$$

$$\text{impulse force} = m x'(1) \quad \Rightarrow \quad -1 = 2x'(1)$$

$$\text{at rest} \quad \Rightarrow \quad x(1) = 0$$

$$0 = c_1 \cos \sqrt{\tfrac{3}{2}} + c_2 \sin \sqrt{\tfrac{3}{2}}$$

$$-\tfrac{1}{2} = -c_1 \sqrt{\tfrac{3}{2}} \sin \sqrt{\tfrac{3}{2}} + c_2 \sqrt{\tfrac{3}{2}} \cos \sqrt{\tfrac{3}{2}}$$

$$\Rightarrow \quad c_1 = \sqrt{\tfrac{1}{6}} \sin \sqrt{\tfrac{3}{2}} \quad \& \quad c_2 = -\sqrt{\tfrac{1}{6}} \cos \sqrt{\tfrac{3}{2}}$$

$$x = -\sqrt{\tfrac{1}{6}} \sin \sqrt{\tfrac{3}{2}} (t-1) \quad \left(\text{using trig identities} \right)$$