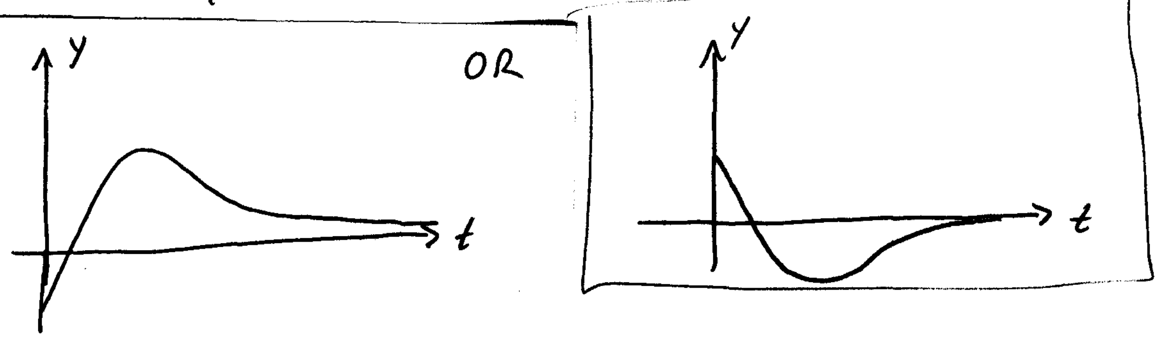
Problem 5. For what value of the constant c the differential equation $y'' + cy' + 10y = 0$ is underdamped, critically damped, or overdamped.

- Sketch a typical solution in the critically damped case.
- Pick c such the system is underdamped. Find a general solution.
- Rewrite the function $\cos(2x) + \sqrt{3}\sin(2x)$ in the form $A\cos(\omega x - b)$.
- Write the general solution un the underdapmed case in the form $Ae^{rt}\cos(\omega t - b)$ for suitable A, b, ω, b.

- $r^2 + cr + 10 = 0 \implies r = \dfrac{-c \pm \sqrt{c^2 - 40}}{2}$

underdamped: $c^2 < 40$
critically damped: $c^2 = 40$
overdamped: $c^2 > 40$



- Let $c = 2$. Then $r = -1 \pm 3i$

$$y = e^{-x}(c_1 \cos 3x + c_2 \sin 3x)$$

- $1^2 + (\sqrt{3})^2 = c^2 \implies c = 2$

Find θ s.t. $\cos\theta = \frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$. $\theta = \frac{\pi}{3}$

Then $\cos(2x) + \sqrt{3}\sin(2x) = 2\left[\frac{1}{2}\cos(2x) + \frac{\sqrt{3}}{2}\sin(2x)\right]$
$= 2\cos(2x - \frac{\pi}{3})$

- Put $A = \sqrt{c_1^2 + c_2^2}$

Then $y = Ae^{-x}\cos(3x - b)$