

3.6.4 $x'' + 25x = 90 \cos 4t$ $x(0) = 0$ $x'(0) = 90$

$m = 1$

$\omega_0 = \sqrt{\frac{k}{m}} = 5$

$k = 25$

$\omega = 4$

$F_0 = 90$

From equation (7),

$$x = c_1 \cos 5t + c_2 \sin 5t + 10 \cos 4t$$

$$x' = -5c_1 \sin 5t + 5c_2 \cos 5t - 40 \sin 4t$$

$$\left. \begin{array}{l} x(0) = 0 \\ x'(0) = 90 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} 0 = c_1 + 10 \\ 90 = 5c_2 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} c_1 = -10 \\ c_2 = 18 \end{array} \right.$$

~~alternatively~~

$$C = \sqrt{c_1^2 + c_2^2} = \sqrt{424}$$

$$\cos \alpha = \frac{-10}{\sqrt{424}} \left. \begin{array}{l} \\ \end{array} \right\} \Rightarrow \alpha \approx \begin{array}{l} 2.07789 \\ \cancel{1.06370} \end{array}$$

$$\sin \alpha = \frac{18}{\sqrt{424}}$$

Then equation (8) gives

$$x = \sqrt{424} \cos(5t - \alpha) + 10 \cos 4t$$

where α is given above.

3.6.19

$$m = 3.125$$

$$-100 = -k \frac{1}{12} \Rightarrow k = 1200$$

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{384} = 8\sqrt{6}$$

Resonance will occur when $\omega \approx 8\sqrt{6}$.