

3.1.4

$$y'' + 25y = 0$$

$$y(0) = 10$$

$$y'(0) = -10$$

$$y_1 = \cos 5x$$

$$y_2 = \sin 5x$$

$$y_1' = -5 \sin 5x$$

$$y_2' = 5 \cos 5x$$

$$y_1'' = -25 \cos 5x$$

$$y_2'' = -25 \sin 5x$$

$$y_1'' + 25y_1 = -25 \cos 5x + 25 \cos 5x = 0 \quad \checkmark$$

$$y_2'' + 25y_2 = -25 \sin 5x + 25 \sin 5x = 0 \quad \checkmark$$

$$y = c_1 \cos 5x + c_2 \sin 5x$$

$$y' = -5c_1 \sin 5x + 5c_2 \cos 5x$$

$$\left. \begin{array}{l} y(0) = 10 \\ y'(0) = -10 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} 10 = c_1 \\ -10 = 5c_2 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} c_1 = 10 \\ c_2 = -2 \end{array} \right.$$

$$y = 10 \cos 5x - 2 \sin 5x$$

3.1.10

$$y'' - 10y' + 25y = 0, \quad y(0) = 3 \quad y'(0) = 13$$

$$y_1 = e^{5x}$$

$$y_2 = xe^{5x}$$

$$y_1' = 5e^{5x}$$

$$y_2' = (1+5x)e^{5x}$$

$$y_1'' = 25e^{5x}$$

$$y_2'' = (10+25x)e^{5x}$$

$$y_1'' - 10y_1' + 25y_1 = 25e^{5x} - 10 \cdot 5e^{5x} + 25e^{5x} = 0 \quad \checkmark$$

$$\begin{aligned} y_2'' - 10y_2' + 25y_2 &= (10+25x)e^{5x} - 10(1+5x)e^{5x} + 25xe^{5x} \\ &= 10e^{5x} - 10e^{5x} + 25xe^{5x} - 50xe^{5x} + 25xe^{5x} \\ &= 0 \quad \checkmark \end{aligned}$$

$$y = c_1 e^{5x} + c_2 x e^{5x}$$

$$y' = 5c_1 e^{5x} + c_2 (1+5x)e^{5x}$$

$$\begin{cases} y(0) = 3 \\ y'(0) = 13 \end{cases} \Rightarrow \begin{cases} 3 = c_1 \\ 13 = 5c_1 + c_2 \end{cases} \Rightarrow \begin{cases} c_1 = 3 \\ c_2 = -2 \end{cases}$$

$$y = 3e^{5x} - 2xe^{5x}$$

3.1.14

$$x^2 y'' + 2xy' - 6y = 0 \quad y(2) = 10 \quad y'(2) = 15$$

$$y_1 = x^2$$

$$y_2 = x^{-3}$$

$$y_1' = 2x$$

$$y_2' = -3x^{-4}$$

$$y_1'' = 2$$

$$y_2'' = 12x^{-5}$$

$$x^2 y_1'' + 2x y_1' - 6y_1 = 2x^2 + 4x^2 - 6x^2 = 0 \quad \checkmark$$

$$x^2 y_2'' + 2x y_2' - 6y_2 = 12x^{-3} - 6x^{-3} - 6x^{-3} = 0 \quad \checkmark$$

$$y = c_1 x^2 + c_2 x^{-3}$$

$$y' = 2c_1 x - 3c_2 x^{-4}$$

$$\left. \begin{array}{l} y(2) = 10 \\ y'(2) = 15 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} 10 = 4c_1 + \frac{1}{8}c_2 \\ 15 = 4c_1 - \frac{3}{16}c_2 \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} 5 = \frac{-5}{16}c_2 \\ 30 = 10c_1 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} c_1 = 3 \\ c_2 = -16 \end{array} \right.$$

$$\boxed{y = 3x^2 - 16x^{-3}}$$

3.1.34

$$y'' + 2y' - 15y = 0$$

$$r^2 + 2r - 15 = 0 \Rightarrow (r+5)(r-3) = 0$$

$$\Rightarrow r = -5 \text{ or } +3$$

$$y = c_1 e^{3x} + c_2 e^{-5x}$$

3.1.40

$$9y'' - 12y' + 4y = 0$$

$$9r^2 - 12r + 4 = 0 \Rightarrow 9r^2 - 6r - 6r + 4 = 0$$

$$\Rightarrow 3r(3r-2) - 2(3r-2) = 0$$

$$\Rightarrow (3r-2)(3r-2) = 0$$

$$\Rightarrow r = \frac{2}{3} \text{ (a repeated root)}$$

$$y = c_1 e^{\frac{2}{3}x} + c_2 x e^{\frac{2}{3}x}$$

3.1.46

$$y(x) = c_1 e^{10x} + c_2 e^{100x}$$

Find ~~the~~ polynomial with roots 10 and 100: ~~the polynomial~~

$$p(r) = (r-10)(r-100) = r^2 - 110r + 1000$$

Then  $p(r) \stackrel{!}{=} 0$  is characteristic equation, for

$$y'' - 110y' + 1000y = 0$$