

2.4.4

$$y' = x - y, \quad y(0) = 1$$

$h = 0.25$:

$$x_0 = 0$$

$$x_1 = 0.25$$

$$x_2 = 0.5$$

$$y_0 = 1$$

$$y_1 = 1 + 0.25(-1) = 0.75$$

$$y_2 = 0.75 + 0.25(-0.5) = 0.625$$

$h = 0.1$

$$x_0 = 0$$

$$x_1 = 0.1$$

$$x_2 = 0.2$$

$$x_3 = 0.3$$

$$x_4 = 0.4$$

$$x_5 = 0.5$$

$$y_0 = 1$$

$$y_1 = 1 + 0.1(-1) = 0.9$$

$$y_2 = 0.9 + 0.1(-0.8) = 0.82$$

$$y_3 = 0.82 + 0.1(-0.62) = 0.758$$

$$y_4 = 0.758 + 0.1(-0.458) = 0.7122$$

$$y_5 = 0.7122 + 0.1(-0.3122) = 0.68098$$

Actual value: $y(0.5) = 2e^{-0.5} + 0.5 - 1 \approx 0.714$

2.4.8 $y' = e^{-y}, \quad y(0) = 0$

$h = 0.25$:

$$x_0 = 0$$

$$x_1 = 0.25$$

$$x_2 = 0.5$$

$$y_0 = 0$$

$$y_1 = 0 + 0.25(1) = 0.25$$

$$y_2 = 0.25 + 0.25(-0.77880) \approx 0.445$$

$h = 0.1$:

$$x_0 = 0$$

$$x_1 = 0.1$$

$$x_2 = 0.2$$

$$x_3 = 0.3$$

$$x_4 = 0.4$$

$$x_5 = 0.5$$

$$y_0 = 0$$

$$y_1 = 0 + 0.1(1) = 0.1$$

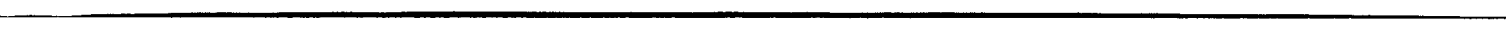
$$y_2 = 0.1 + 0.1(0.90484) \approx 0.19048$$

$$y_3 = 0.19048 + 0.1(0.82656) \approx 0.27314$$

$$y_4 = 0.27314 + 0.1(0.76099) \approx 0.34924$$

$$y_5 = 0.34924 + 0.1(0.70522) \approx 0.41976$$

Actual value: $y(0.5) = \ln(0.5 + 1) \approx 0.405$



2.4.13

$$h = 0.01$$

$$x_0 = 1$$

$$x_1 = 1.01$$

⋮

$$x_{100} = 2.00$$

$$y_0 = 3$$

⋮

$$y_{100} \approx 4.8890$$

$$h = 0.005$$

i	x_i	approx. y_i	approx. $y(x_i)$	approx. % error
0	1	3	3	0%
40	1.2	3.1729	3.1739	0.03%
80	1.4	3.4390	3.4412	0.06%
120	1.6	3.8117	3.8149	0.08%
160	1.8	4.2967	4.3009	0.10%
200	2.0	4.8940	4.8990	0.10%

$$yy' = 2x^3$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^4 + C$$

$$y(1) = 3 \Rightarrow \frac{9}{2} = \frac{1}{2} + C \Rightarrow C = 4$$

$$y = (x^4 + 8)^{1/2}$$