**2.4.4** $y' = x - y \,, \quad y(0) = 1$

$h = 0.25$ :

$x_0 = 0 \qquad y_0 = 1$

$x_1 = 0.25 \qquad y_1 = 1 + 0.25(-1) = 0.75$

$x_2 = 0.5 \qquad y_2 = 0.75 + 0.25(-0.5) = 0.625$

$h = 0.1$

$x_0 = 0 \qquad y_0 = 1$

$x_1 = 0.1 \qquad y_1 = 1 + 0.1(-1) = 0.9$

$x_2 = 0.2 \qquad y_2 = 0.9 + 0.1(-0.8) = 0.82$

$x_3 = 0.3 \qquad y_3 = 0.82 + 0.1(-0.62) = 0.758$

$x_4 = 0.4 \qquad y_4 = 0.758 + 0.1(-0.458) = 0.7122$

$x_5 = 0.5 \qquad y_5 = 0.7122 + 0.1(-0.3122) = 0.68098$

Actual value: $y(0.5) = 2e^{-0.5} + 0.5 - 1 \approx 0.714$

---

2.4.8 $y' = e^{-y} \,, \quad y(0) = 0$

$h = 0.25$ :

$x_0 = 0 \qquad y_0 = 0$

$x_1 = 0.25 \qquad y_1 = 0 + 0.25(1) = 0.25$

$x_2 = 0.5 \qquad y_2 = 0.25 + 0.25(-0.77880) \approx 0.445$

$h = 0.1$ :

$x_0 = 0 \qquad y_0 = 0$

$x_1 = 0.1 \qquad y_1 = 0 + 0.1(1) = 0.1$

$x_2 = 0.2 \qquad y_2 = 0.1 + 0.1(0.90484) \approx 0.19048$

$x_3 = 0.3 \qquad y_3 = 0.19048 + 0.1(0.82656) \approx 0.27314$

$x_4 = 0.4 \qquad y_4 = 0.27314 + 0.1(0.76099) \approx 0.34924$

$x_5 = 0.5 \qquad y_5 = 0.34924 + 0.1(0.70522) \approx 0.41976$

Actual value: $y(0.5) = \ln(0.5 + 1) \approx 0.405$

## 2.4.13

$h = 0.01$

$x_0 = 1$ $\qquad$ $y_0 = 3$

$x_1 = 1.01$

$\vdots$ $\qquad$ $\vdots$

$x_{100} = 2.00$ $\qquad$ $y_{100} \approx 4.8890$

$h = 0.005$

| $i$ | $x_i$ | approx. $y_i$ | approx. $y(x_i)$ | approx. % error |
|---|---|---|---|---|
| 0 | 1 | 3 | 3 | 0% |
| 40 | 1.2 | 3.1729 | 3.1739 | 0.03% |
| 80 | 1.4 | 3.4390 | 3.4412 | 0.06% |
| 120 | 1.6 | 3.8117 | 3.8149 | 0.08% |
| 160 | 1.8 | 4.2967 | 4.3009 | 0.10% |
| 200 | 2.0 | 4.8940 | 4.8990 | 0.10% |

$$yy' = 2x^3$$

$$\tfrac{1}{2}y^2 = \tfrac{1}{2}x^4 + C$$

$$y(1) = 3 \Rightarrow \frac{9}{2} = \frac{1}{2} + C \Rightarrow C = 4$$

$$\boxed{y = (x^4+8)^{1/2}}$$