2) The ODE is \( y' = -ky^2 \). Solving, we get:
\[ y(t) = Ae^{-kt} \]
\[ y(0) = y_0 \Rightarrow A = y_0 \Rightarrow y(t) = y_0 e^{-kt} \]

But \( y' = y_0 e^{-kt} \). We integrate, and get:
\[ x(t) = \frac{y_0}{k} e^{-kt} + B \]
\[ x(0) = x_0 \Rightarrow \frac{y_0}{k} + B = x_0 \Rightarrow B = x_0 - \frac{y_0}{k} \]
\[ \Rightarrow \left( x(t) = x_0 \frac{y_0}{k} \left( 1 - e^{-kt} \right) \right) \]

Q.E.D.

4) Given \( y' = -k y^2 \)
\[ \frac{dy}{dt} = -k \]
Integrating, we get:
\[ \Rightarrow y(t) = \frac{1}{kt + C} \]
\[ y(0) = y_0 \Rightarrow C = \frac{1}{y_0} \]
\[ \Rightarrow y(t) = \frac{1}{kt + \frac{1}{y_0}} \]
Thus, \( z = \frac{x}{k + \alpha}. \)

We integrate, and get:

\[
\begin{align*}
  x(t) &= \frac{x_0}{k + \alpha} - \frac{1}{\alpha} \ln |k + \alpha t + 1| + C \\
  x(0) &= x_0 \\
  \Rightarrow x(t) &= x_0 + \frac{x_0}{k + \alpha} - \frac{1}{\alpha} \ln |k + \alpha t + 1|
\end{align*}
\]

In problem 2, resistance is \( \frac{dx}{dt} = -kv \).

In problem 4, resistance is \( \frac{dx}{dt} = -kv^3 \).

Thus, if \( |v(t)| < 1 \), then \( |v| < |v| \), as there will be less resistance in the case of problem 6, if the object is moving slowly.

\( \lim_{t \to \infty} v(t) = 0 \) in both cases.

so if it is very large, then \( |v(t)| < 1 \), and we see that the object travels an infinite distance in problem 6, but only a finite distance in problem 2. QED.
2.3 problems (continued)

11) ODE: \( y' = y - y^2 \)

- \( 0 \leq t \leq 20 \):
  - \( 0.5 \) years, many \( y = 0 \)
  - \( y(0) = \frac{1}{2} e^{-\frac{t}{2}} - \frac{1}{2} \)
  - \( y \geq 32.2 \) ft/s, \( p = 0.15 \)
  - \( y(20) = 203.28 \) ft/s

- \( t \geq 5 \) gives:
  - \( y(t) = 10000 + \frac{1}{2} t - \frac{1}{2} (1 - e^{-\frac{t}{2}}) \)
  - \( v = -\frac{1}{2} \Rightarrow \frac{v}{p} \approx 714.7 \) ft/s

- \( y(20) = 214.7 \times 20 + \frac{1}{0.15} \cdot 214.7 (1 - e^{-0.15 \times 20}) \)

\[ \Rightarrow y(20) = 2066.1 \text{ ft/s} \]
Let $I = 1.70$, $p = 1.5$ (different from before).

$v = 7066.1$ ft/s,

$v_0 = 263.38$ ft/s,

$v_r = 71.5$ ft/s.

Using eq. 9, we want to find $t$ at $y(t)$-wise.

$0 = 7066.1 - 21.5 t + \frac{1}{1.5} \left( -206 + 21.5/1.5 \cdot e^{-1.5t} \right)$

Solving using command `fsolve` in Maple:

$t \approx 323.9$.

$\Rightarrow t = 323 + 20 = 343.0 \approx 5\text{min.} 43.0$. 

2.5 problems (continued)
22) \( p = 1.085 \) (in fps units, with \( g = 32 \text{ ft/s}^2 \))

Terminal speed

\[
\frac{v_t}{f} = \frac{1.085}{0.005} = 217.8 \text{ ft/s}
\]

\[v(t) = \frac{1.085}{\sqrt{0.005}} \tanh(0.005t)\]

\[v(0) = 10000 \cdot \frac{1}{0.005} \ln | \cosh(0.005t) | \approx 0\]

Thus, using Maple’s `fsolve` command:

\[ t \approx 4.855 \approx 8 \text{ min 50 sec} \]