solutions to 1.5 problems :

2) $\begin{cases} y' - 2y = 3e^{2x} \\ y(0) = 0 \end{cases}$

Integr. fact. $= e^{\int -2\,dx}$

$= e^{-2x}$

$e^{-2x} y' - 2e^{-2x} y = 3$

$\Rightarrow \left(e^{-2x} y\right)' = 3$

Integrate.

$e^{-2x} y = 3x + C$

$\Rightarrow \boxed{y = e^{2x}(3x + C)}$

$y(0) = 0$

$\Rightarrow 0 = 1(0 + C)$

$\Rightarrow \boxed{C = 0}$

$\Rightarrow \boxed{y = 3x e^{2x}}$

12) $\begin{cases} xy' + 3y = 2x^5 \\ y(2) = 1 \end{cases}$

$y' + \frac{3}{x} y = 2x^4$

Int. fact. $= e^{\int \frac{3}{x} dx}$

$= e^{3 \ln x}$

$= \left(e^{\ln x}\right)^3$

$= x^3$

$\Rightarrow x^3 y' + 3x^2 y = 2x^7$

$\Rightarrow (x^3 y)' = 2x^7$

Integrate:

$x^3 y = \frac{x^8}{4} + C$

$\Rightarrow \boxed{y = \frac{1}{x^3}\left(\frac{x^8}{4} + C\right)}$

$y(2) = 1$

$\Rightarrow 1 = \frac{1}{8}\left(\frac{2^8}{4} + C\right)$

$= \cancel{32}\ 8 + \frac{C}{8}$

$\Rightarrow \boxed{\cancel{C = -248}} \Rightarrow \boxed{C = -56}$

$\Rightarrow \boxed{y = \frac{1}{x^3}\left(\frac{x^8}{4} - \cancel{248}\ 56\right)}$

Solutions to 1.5 problems (continued)

24) $\begin{cases} (x^2+4)y' + 3xy = x \\ y(0) = 1 \end{cases}$

$$y' + \frac{3x}{x^2+4} y = \frac{x}{x^2+4}$$

Int. fact. $= e^{\int \frac{3x}{x^2+4} dx}$

$= e^{\frac{3}{2} \ln|x^2+4|}$

$= (x^2+4)^{\frac{3}{2}}$

$\Rightarrow (x^2+4)^{\frac{3}{2}} y' + 3x(x^2+4)^{\frac{1}{2}} y = x(x^2+4)^{\frac{1}{2}}$

$\Rightarrow \left((x^2+4)^{\frac{3}{2}} y\right)' = x(x^2+4)^{\frac{1}{2}}$

Integrate:

$(x^2+4)^{\frac{3}{2}} y = \frac{1}{2} \cdot \frac{2}{3} (x^2+4)^{\frac{3}{2}} + C$

$= \frac{1}{3}(x^2+4)^{\frac{3}{2}} + C$

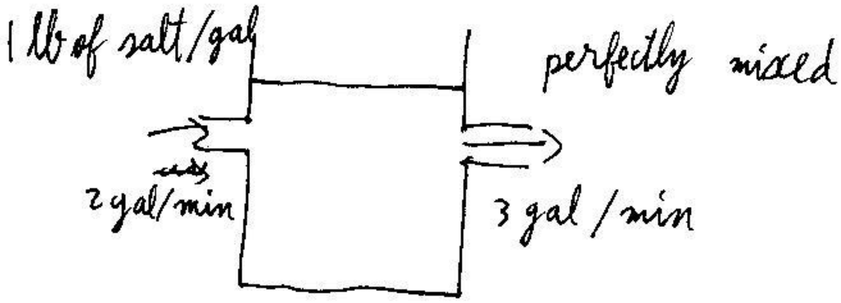
$\Rightarrow \boxed{y = \frac{1}{3} + C(x^2+4)^{-\frac{3}{2}}}$

$y(0) = 1$

$\Rightarrow 1 = \frac{1}{3} + \frac{C}{8}$

$\Rightarrow \boxed{C = \frac{16}{3}}$ $\Rightarrow \boxed{y = \frac{1}{3} + \frac{16}{3}(x^2+4)^{-\frac{3}{2}}}$

36)



$V(0) = 60$ gal

Let $x(t)$ = amount of salt in tank at time t (in lb)

$V(t)$ = volume of water in tank at time t (in gal)

$V(t) = 60 - t$ gal ($t \to$ min.)

(because over all, water is leaving tank at the rate of $3 - 2 = 1$ gal/min)

For the time interval $[t, t+\Delta t]$, for Δt small, the amount of salt in tank increases approximately by:

$\Delta x \simeq$ amount of salt coming in $-$ amount of salt going out

$\simeq 1 \cdot \underbrace{2\Delta t}_{\Delta V_1 \text{ (vol. of water coming in)}} - (\text{concentration of salt}) \cdot \underbrace{3\Delta t}_{\Delta V_2 \text{ (vol. of water} \to \text{out)}}$

$$\Rightarrow \Delta x \simeq 2\Delta t - \frac{x(t)}{V(t)} 3\Delta t$$

Dividing by Δt and letting $\Delta t \to 0$, we get:

$$\frac{dx}{dt} = 2 - \frac{3x}{60-t}$$

$$\Rightarrow \quad x' + \frac{3x}{60-t} = 2$$

$$\text{int. fact.} = e^{\int \frac{3\,dt}{60-t}}$$

$$= e^{-3\ln|60-t|}$$

$$= \left(e^{3\ln(60-t)}\right)^{-1} \qquad (\text{assuming } 60-t>0)$$

$$= \frac{1}{(60-t)^3}$$

$$\Rightarrow \quad \frac{x'}{(60-t)^3} + \frac{3x}{(60-t)^4} = \frac{2}{(60-t)^3}$$

$$\Rightarrow \quad \left(\frac{x}{(60-t)^3}\right)' = \frac{2}{(60-t)^3}$$

Integrate:

$$\frac{x}{(60-t)^3} = -2 \times \frac{1}{-2}(60-t)^{-2} + C = (60-t)^{-2} + C$$

$$\Rightarrow \quad \boxed{x(t) = 60-t + C(60-t)^3}$$

$x(0) = 0$ (initially pure water => no salt)

$\Rightarrow \quad 0 = 60 + C\, 60^3$

$\Rightarrow \quad \boxed{C = -\frac{1}{3600}}$

$\Rightarrow \quad \boxed{x(t) = 60 - t - \frac{1}{3600}(60-t)^3}$