

Solutions to 1.4 problems

1/3

$$4) (1+x) \frac{dy}{dx} = 4y$$

$$\Rightarrow \left( \frac{dy}{4y} = \frac{dx}{x+1} \right)$$

$$\Rightarrow \frac{1}{4} \ln|y| = \ln|x+1| + C$$

$$\Rightarrow \ln y^4 = \ln|x+1| + C$$

$$\Rightarrow \boxed{y^4 = k|x+1|}$$

$$12) yy' = x(y^2+1)$$

$$\Rightarrow \left( \frac{2yy' dx}{y^2+1} = \int 2x dx \right)$$

$$\Rightarrow \ln|y^2+1| = x^2 + C$$

$$\Rightarrow \boxed{y^2+1 = ke^{x^2}}$$

$$22) \begin{cases} y' = 4x^3y - y \\ y(1) = -3 \end{cases}$$

$$\Rightarrow y' = y(4x^3 - 1)$$

$$\Rightarrow \left( \frac{y'}{y} dx = (4x^3 - 1) dx \right)$$

$$\Rightarrow \ln|y| = x^4 - x + C$$

$$\Rightarrow |y| = ke^{x^4 - x}$$

$$\Rightarrow \boxed{y = B e^{x^4 - x}}, B \in \mathbb{R}$$

## Solutions to 1.4 problems (continued)

2/3

$$46) a) P' = -0.2P$$

$$\Rightarrow P(x) = k e^{-0.2x}$$

$$P(0) = 29.92$$

$$\Rightarrow \boxed{P(x) = 29.92 e^{-0.2x}}$$

$$\begin{aligned} \Rightarrow P(10,000) &= 29.92 e^{-0.2 \times 10000} \\ &= 29.92 e^{-2000} \text{ in. of m/c.} \end{aligned}$$

$$b) P(h) = 15$$

$$29.92 e^{-0.2h} = 15$$

$$\Rightarrow e^{-0.2h} = \frac{15}{29.92}$$

$$\Rightarrow -0.2h = \ln\left(\frac{15}{29.92}\right)$$

$$\Rightarrow \boxed{h = 5 \ln\left(\frac{29.92}{15}\right) \text{ mi}}$$

solutions to problems from 1.4 (continued)

3/3

$$(48) \quad m_{235}(0) = m_{238}(0) = m$$

$$m_{238}(t_{\text{now}}) = 137.7 m_{235}(t_{\text{now}})$$

$$t_{1/2; 238} = 4.51 \times 10^9 \text{ y}$$

$$t_{1/2; 235} = 7.10 \times 10^8 \text{ y}$$

$$m(t) = m(0) \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$m(t) = m(0) 2^{-\frac{t}{t_{1/2}}}$$

$$\Rightarrow m_{235}(t) = m 2^{-\frac{t}{t_{1/2; 235}}}$$

$$m_{238}(t) = m 2^{-\frac{t}{t_{1/2; 238}}}$$

$$\frac{m_{238}(t_{\text{now}})}{m_{235}(t_{\text{now}})} = 137.7$$

$$\Rightarrow \frac{2^{-\frac{t_{\text{now}}}{t_{1/2; 238}}}}{2^{-\frac{t_{\text{now}}}{t_{1/2; 235}}}} = 137.7$$

$$\Rightarrow 2^{t_{\text{now}} \left( \frac{1}{t_{1/2; 235}} - \frac{1}{t_{1/2; 238}} \right)} = 137.7$$

$$\Rightarrow t_{\text{now}} \left( \frac{1}{t_{1/2; 235}} - \frac{1}{t_{1/2; 238}} \right) = \log_2(137.7) \approx 7.1054$$

$$\Rightarrow t_{\text{now}} = \dots \quad (\text{I don't have a calculator})$$

$$\approx 5.99 \text{ billion years}$$

(probably not very accurate...)