1.3.14 $\quad \dfrac{dy}{dx} = \sqrt[3]{y} \qquad y(0) = 0$

$\dfrac{dy}{dx} = f(x,y)$ where $f(x,y) = \sqrt[3]{y}$

- $f$ is continuous everywhere
- $D_y f = \frac{1}{3} y^{-2/3}$ is continuous for all $(x,y)$ except when $y = 0$.

Theorem 1 does not guarantee existence nor uniqueness to the given initial value problem. Nonetheless, solutions do exist such as

$$y(x) = 0 \quad \text{or} \quad y(x) = \begin{cases} 0 & \text{for } x \le 0, \\ \left[\frac{2}{3}x\right]^{3/2} & \text{for } x > 0. \end{cases}$$

---

1.3.18 $\quad y\dfrac{dy}{dx} = x - 1 \qquad y(1) = 0$

$\dfrac{dy}{dx} = f(x,y)$ where $f(x,y) = \dfrac{x-1}{y}$

- $f$ is continuous everywhere except when $y = 0$
- $D_y f = \dfrac{1-x}{y^2}$ is continuous everywhere except when $y = 0$.

Theorem 1 does not guarantee existence nor uniqueness. Two solutions do exist however,

$$y_1(x) = x - 1 \quad \text{and} \quad y_2(x) = 1 - x$$

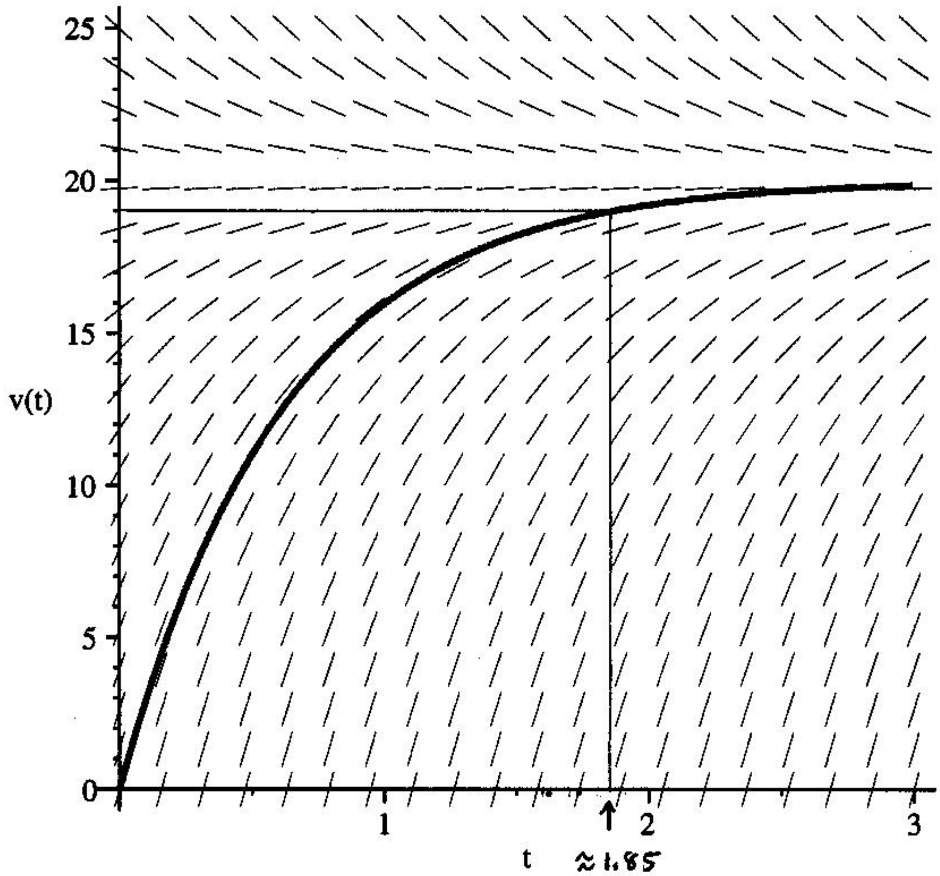
1.3.25 The following was plotted using Maple computer software.

*with*(*DEtools*) :

*ode* := *diff*(*v*(*t*), *t*) = 32 − 1.6·*v*(*t*);

$$\frac{d}{dt} v(t) = 32 - 1.6\, v(t) \tag{1}$$

*DEplot*(*ode*, *v*(*t*), *t* = 0 ..3, *v* = 0 ..25, [*v*(0) = 0], *arrows* = *line*);




Limiting velocity : $V_\infty = 20$ ft/s

95% of limiting velocity is $0.95 \cdot 20 = 19$

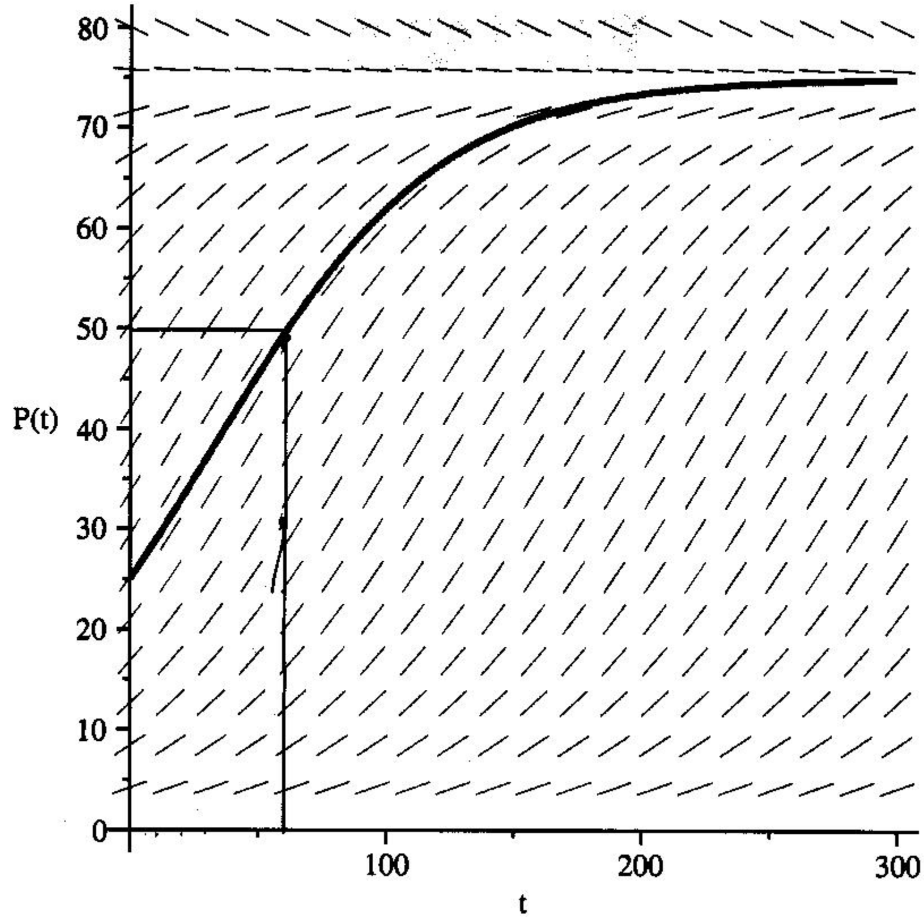
It will take approximately 1.85 seconds to achieve this velocity.

1.3.26

*with*(*DEtools*) :
*ode* := *diff*(*P*(*t*), *t*) = 0.0225·*P*(*t*) − 0.0003·*P*(*t*)²;

$$\frac{d}{dt} P(t) = 0.0225\, P(t) - 0.0003\, P(t)^2 \qquad (1)$$

*DEplot*(*ode*, *P*(*t*), t = 0 ..300, P = 0 ..80, [*P*(0) = 25], *arrows* = *line*);




It will take approximately 60 months for the population to double.

The limiting deer population is 75 deer.